



On components in temporal graphs

the case of journey-based components

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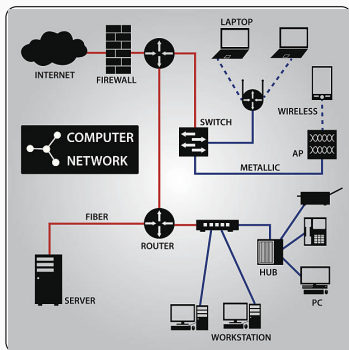


¹Supported by the RIN Tremplin Région Normandie project DynNet.

Temporal graphs

Temporal graphs

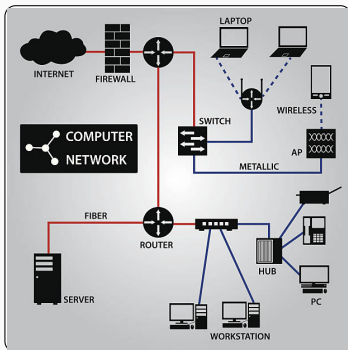
- faulty networks



Context

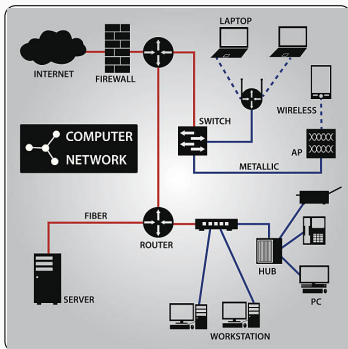
Temporal graphs

- faulty networks
- swarms of drones



Temporal graphs

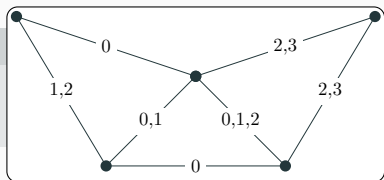
- faulty networks
- swarms of drones
- interacting population



Preliminaries

Definition temporal graph

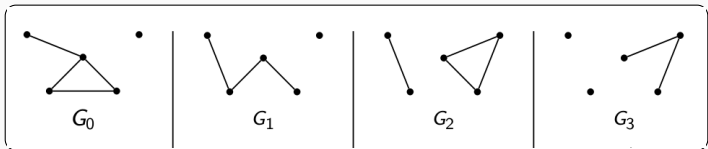
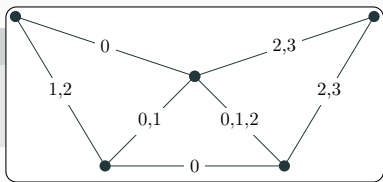
Edges appear and disappear
over the lifetime of the graph



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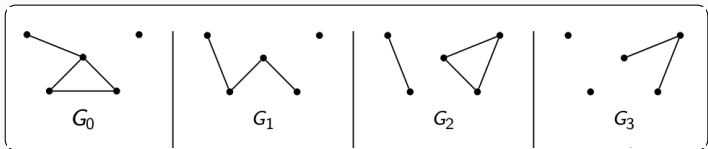
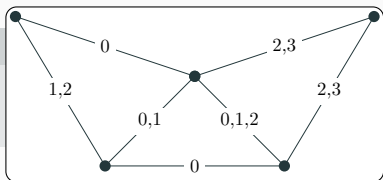
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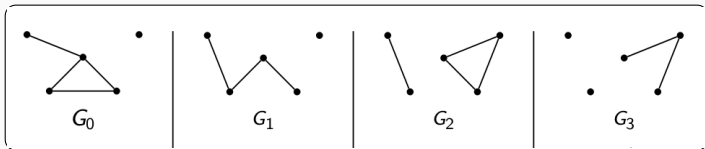
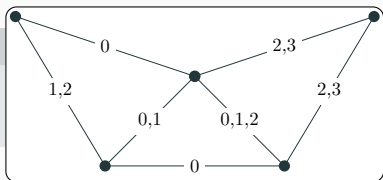
Definition footprint (or underlying graph)

Union of all snapshots G_i

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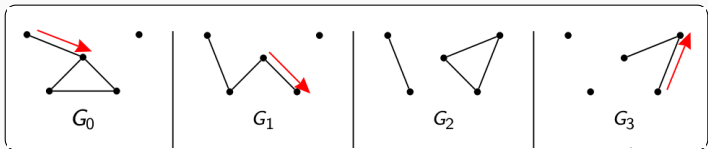
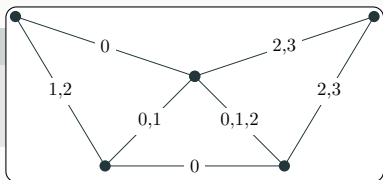
Definition journey (or temporal path)

Path in footprint with non-decreasing labels on the edges

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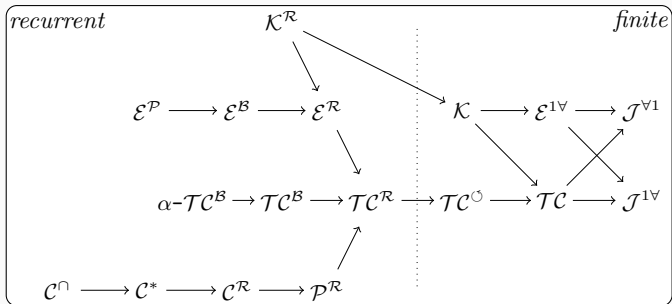
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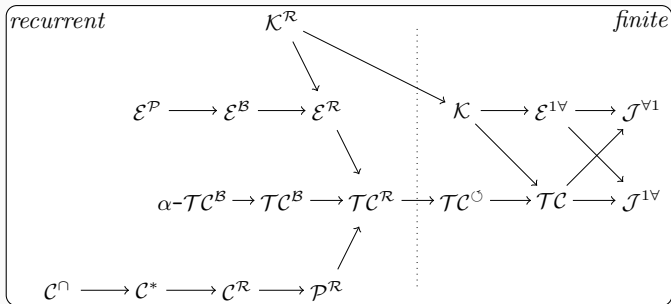
Definition temporal connectivity hierarchy [Casteigts *et al.*][2, 3]



Preliminaries

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Multiple properties for “connected temporal graphs”

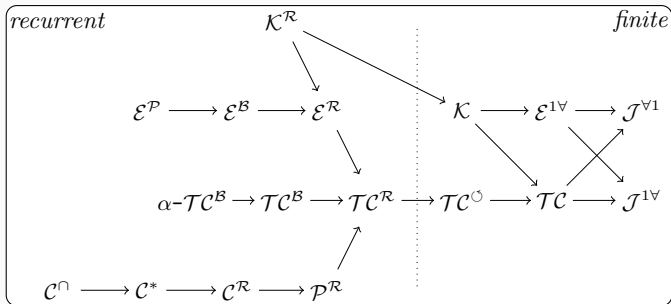


Preliminaries

Definition temporal connectivity hierarchy [Casteigts *et al.*][2, 3]

Multiple properties for “connected temporal graphs” e.g.

- temporal source ($\mathcal{J}^{1\forall}$)

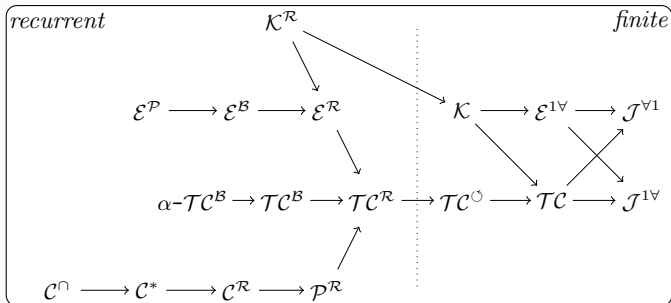


Preliminaries

Definition temporal connectivity hierarchy [Casteigts *et al.*][2, 3]

Multiple properties for “connected temporal graphs” e.g.

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- temporal connectivity (\mathcal{TC} or $\mathcal{J}^{\forall\forall}$)

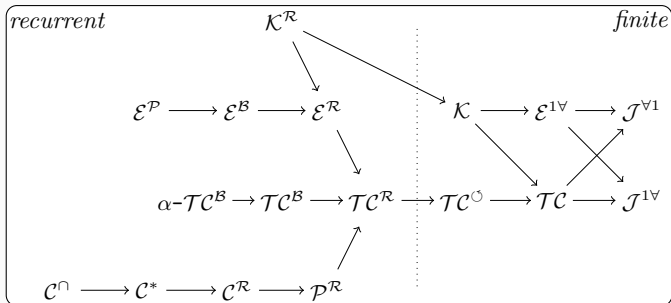


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Definition temporal connectivity hierarchy [Casteigts *et al.*][2, 3]

Multiple properties for “connected temporal graphs” e.g.

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- constant connectivity (\mathcal{C}^*)

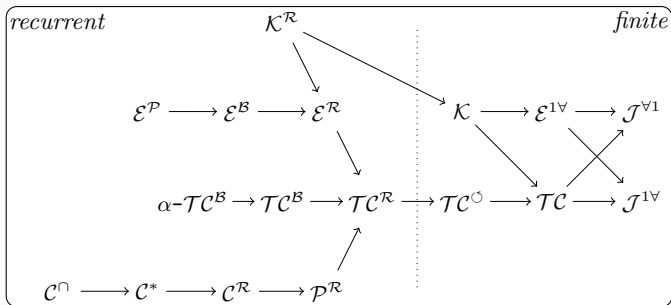


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Definition \mathcal{X} component

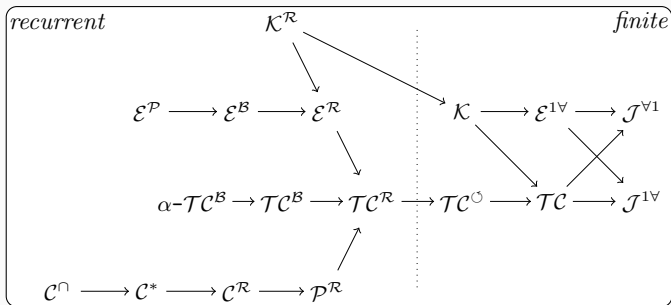
Maximal subset of vertices such that property \mathcal{X} is respected

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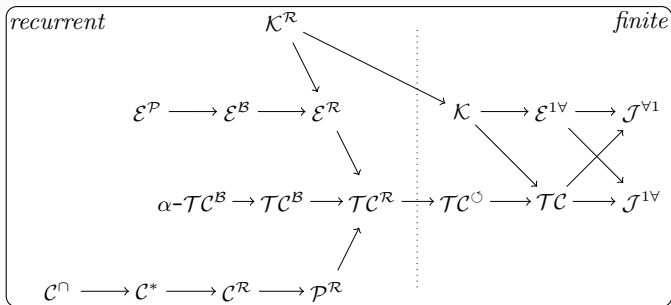
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Definition \mathcal{X} component

Maximal subset of vertices such that property \mathcal{X} is respected

\mathcal{S} and \mathcal{T} components

Definition \mathcal{S} component

Maximal subset of vertices V' such that $\exists s \in V', \forall v \in V', s \rightsquigarrow v$

Definition \mathcal{T} component

Maximal subset of vertices V' such that $\exists t \in V', \forall v \in V', v \rightsquigarrow t$

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Observation temporal sources vs temporal sinks

Temporal sources (*resp.* sinks) for $\mathcal{G} = (G_1, G_2, \dots, G_T)$ correspond to temporal sinks (*resp.* sources) in $\mathcal{G}' = (G_T, G_{T-1}, \dots, G_1)$

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W.l.o.g. we thus focus on \mathcal{S} components
(and later on $\mathcal{S}^{\mathcal{B}}$ and $\mathcal{S}^{\mathcal{D}}$ components)

\mathcal{S} components

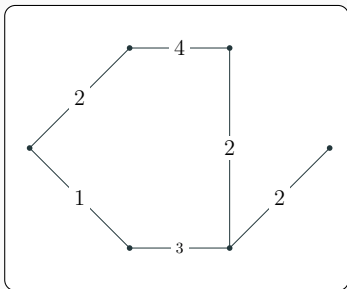
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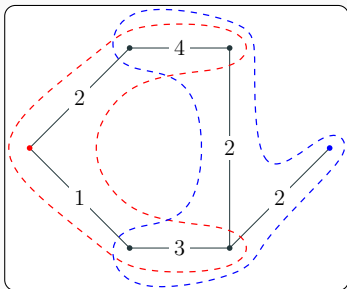
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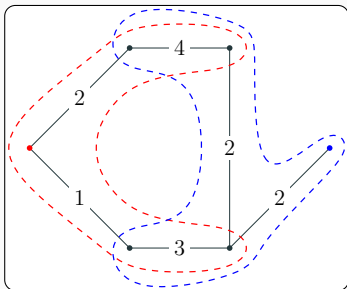
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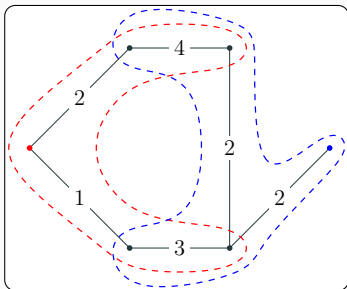


Results [Balev, Pigné, Sanlaville, S.]

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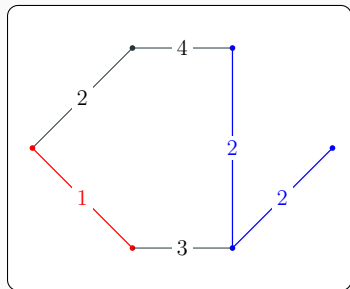
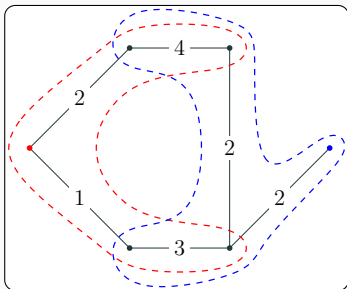
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at most n \mathcal{S} components (all isolated vertices)

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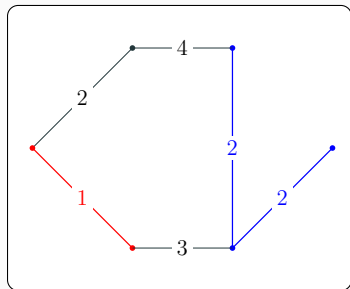
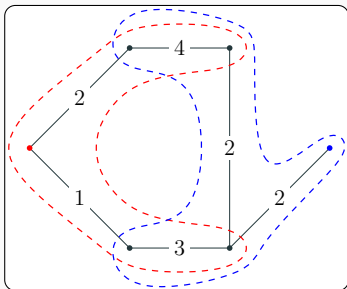
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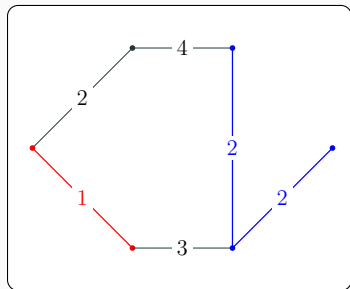
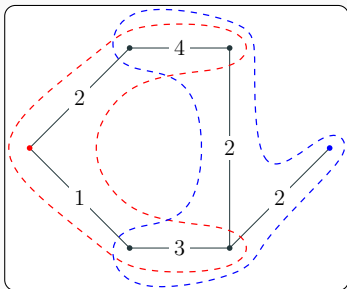
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at most $\lfloor \frac{n}{2} \rfloor$ \mathcal{S} components (no isolated vertices)

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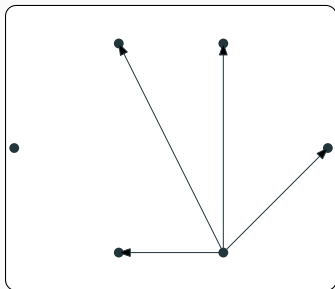
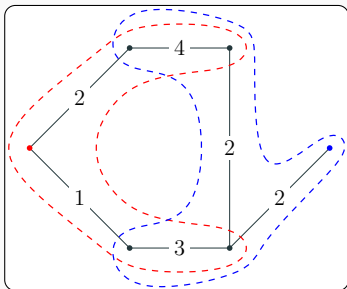
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decision problem \mathcal{S} COMPONENT (\exists a \mathcal{S} component of size $\geq k$):

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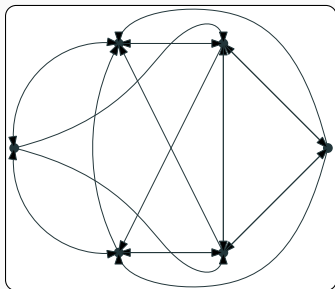
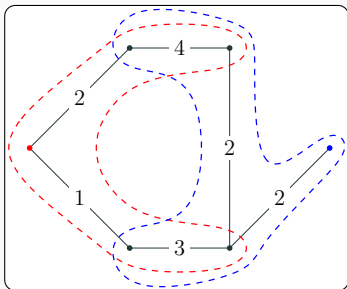
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decision problem \mathcal{S} COMPONENT (\exists a \mathcal{S} component of size $\geq k$):

- construct transitive closure and check if max outdegree $\geq k - 1$
- solvable in time $O(n(m \log T + n \log n))$

$\mathcal{S}^{\mathcal{B}}$ components

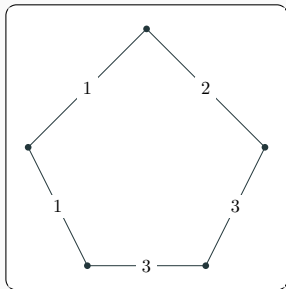
Definition $\mathcal{S}^{\mathcal{B}}$ component (of window size Δ)

Maximal subset of vertices V' such that $\exists s \in V', \forall v \in V', s \rightsquigarrow v$
in all windowed graphs $\mathcal{G}_{[t, t+\Delta-1]}$

\mathcal{S}^B components

Definition \mathcal{S}^B component (of window size Δ)

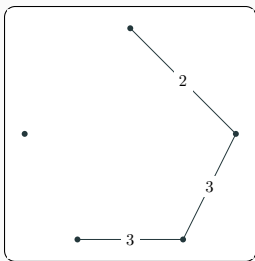
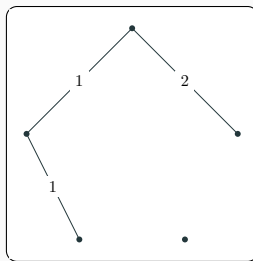
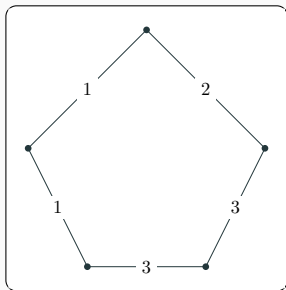
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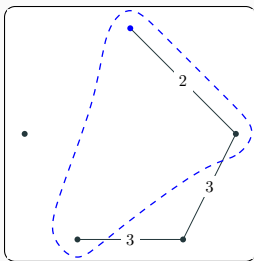
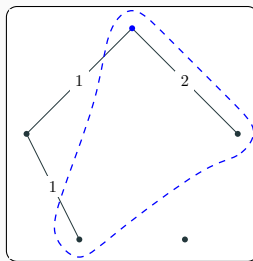
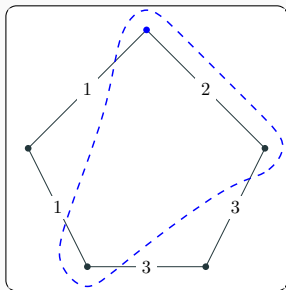
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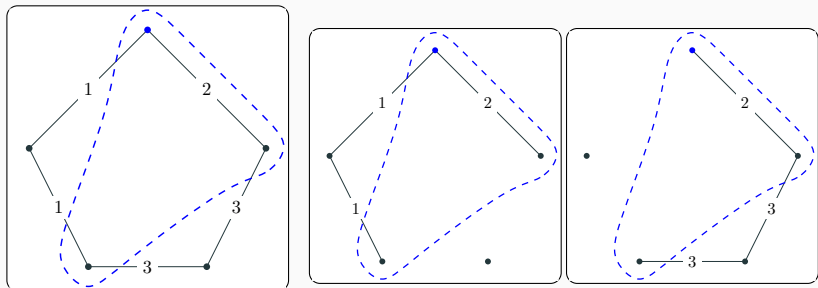
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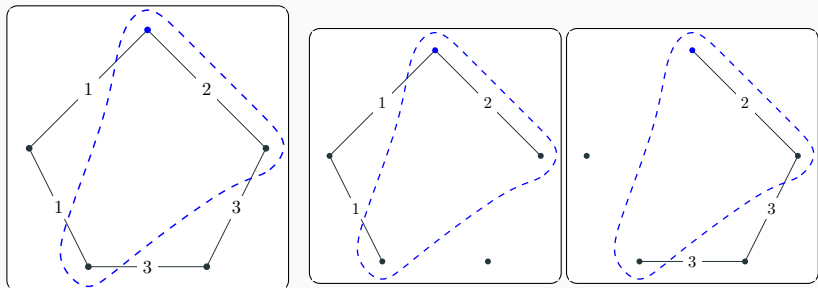


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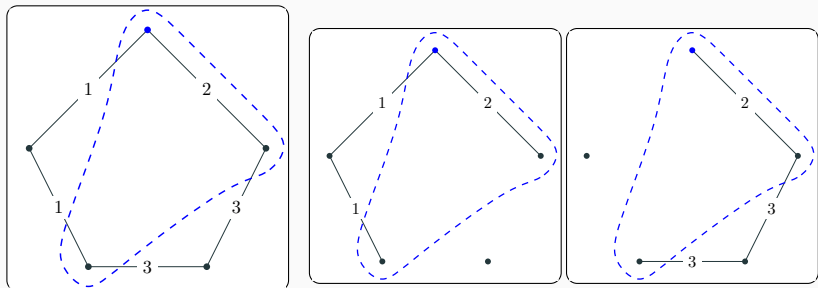
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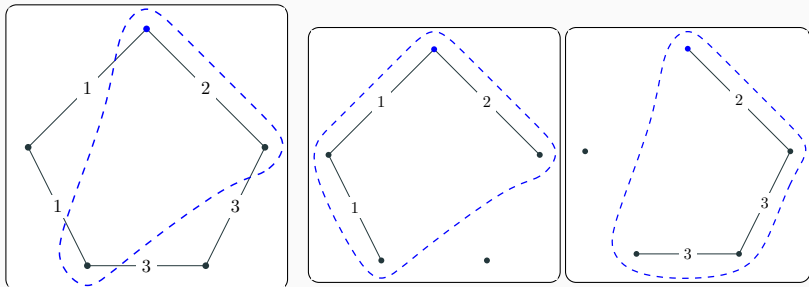
at most n \mathcal{S}^B components

decision problem \mathcal{S}^B COMPONENT:

\mathcal{S}^B components

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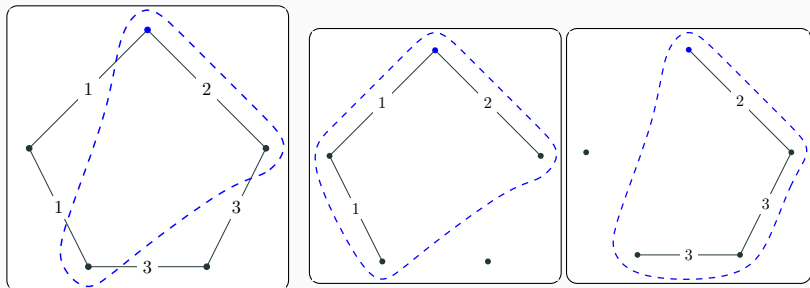
decision problem \mathcal{S}^B COMPONENT:

- compute \mathcal{S} components for all windows and take intersection

\mathcal{S}^B components

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Maximal subset of vertices V' such that $\exists s \in V', \forall v \in V', s \rightsquigarrow v$ in all windowed graphs $\mathcal{G}_{[t, t+\Delta-1]}$



Results [Balev, Pigné, Sanlaville, S.]

at most n \mathcal{S}^B components

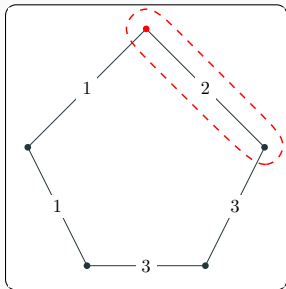
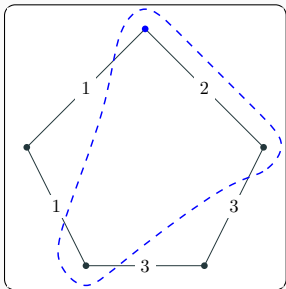
decision problem \mathcal{S}^B COMPONENT:

- compute \mathcal{S} components for all windows and take intersection
- computable in time $O((T - \Delta)n(m \log \Delta + n \log n))$

\mathcal{S}^B components

Definition closed \mathcal{S}^B component (of window size Δ)

Maximal subset of vertices V' such that $\exists s \in V', \forall v \in V', s \rightsquigarrow v$
in all windowed **induced subgraphs** $\mathcal{G}[V']_{[t, t+\Delta-1]}$



Results [Balev, Pigné, Sanlaville, S.]

$\leq n$ components

\mathcal{S}^B COMPONENT:

- $O((T - \Delta)$
 $n(m \log \Delta + n \log n))$

$\leq n$ components

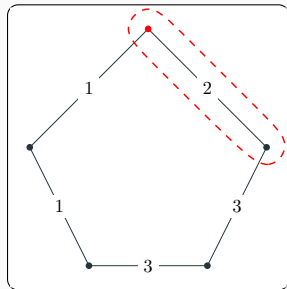
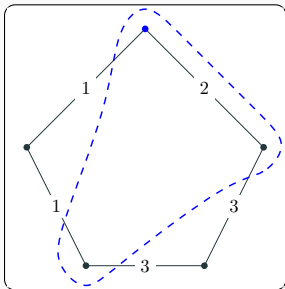
CLOSED \mathcal{S}^B COMPONENT:

- $O((n - k)(T - \Delta)$
 $n(m \log \Delta + n \log n))$

\mathcal{S}^B components

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Results [Balev, Pigné, Sanlaville, S.]

$\leq n$ components

\mathcal{S}^B COMPONENT:

- $O((T - \Delta)$
 $n(m \log \Delta + n \log n))$

$\leq n$ components

CLOSED \mathcal{S}^B COMPONENT:

- $O((n - k)(T - \Delta)$
 $n(m \log \Delta + n \log n))$

$\mathcal{S}^{\mathcal{D}}$ components

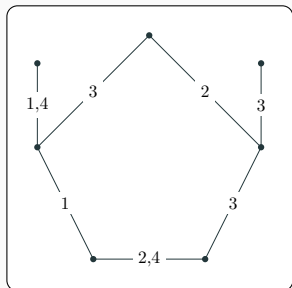
Definition $\mathcal{S}^{\mathcal{D}}$ component (of window size Δ)

Maximal subset of vertices V' such that $\forall t, \exists s \in V', \forall v \in V', s \rightsquigarrow v$
in windowed graphs $\mathcal{G}_{[t, t+\Delta-1]}$

\mathcal{S}^D components

Definition \mathcal{S}^D component (of window size Δ)

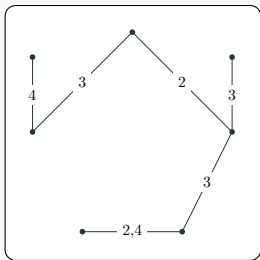
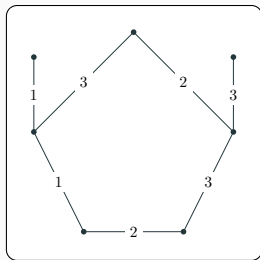
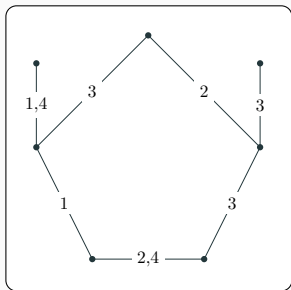
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\mathcal{S}^D components

Definition \mathcal{S}^D component (of window size Δ)

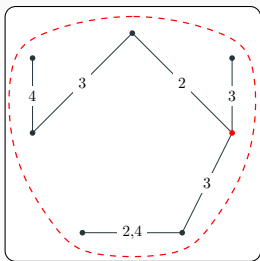
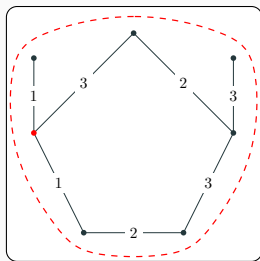
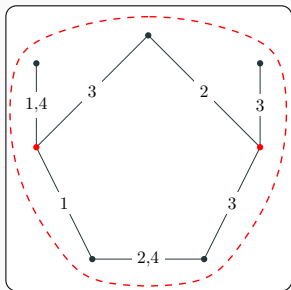
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\mathcal{S}^D components

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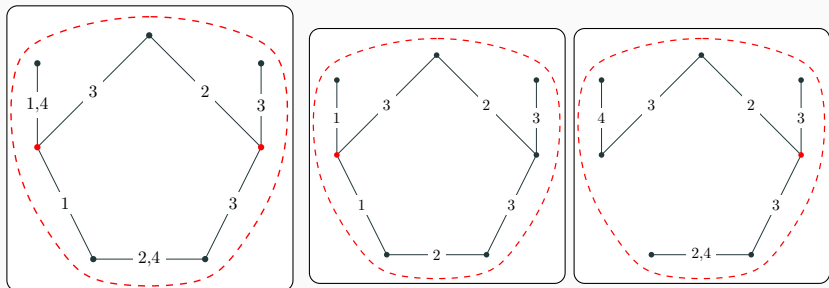
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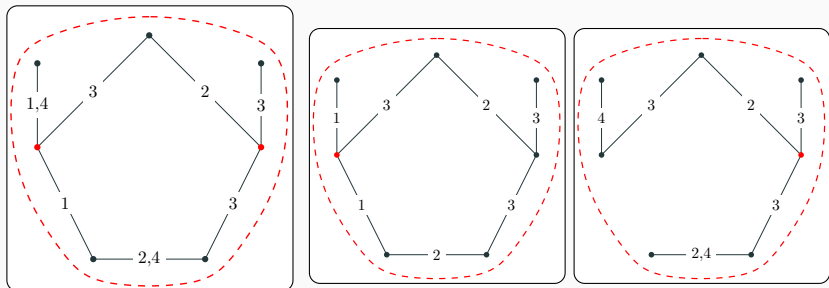


Results [Balev, Pigné, Sanlaville, S.]

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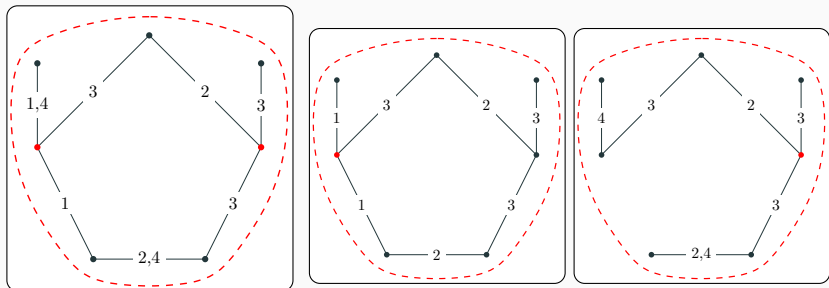
Results [Balev, Pigné, Sanlaville, S.]

at most $\min(2^n, n^{T-\Delta})$ \mathcal{S}^D components

\mathcal{S}^D components

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Results [Balev, Pigné, Sanlaville, S.]

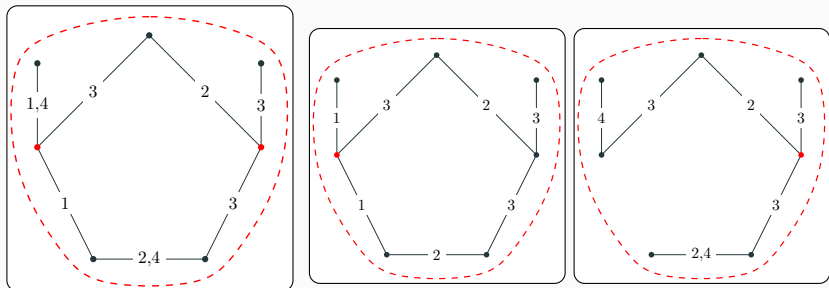
at most $\min(2^n, n^{T-\Delta})$ \mathcal{S}^D components

decision problem \mathcal{S}^D COMPONENT:

\mathcal{S}^D components

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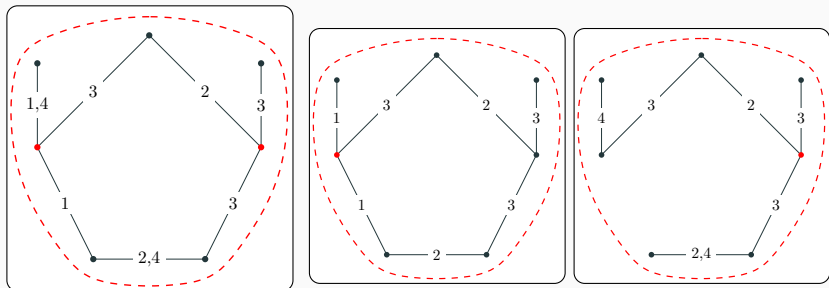
decision problem \mathcal{S}^D COMPONENT:

- exhaustive search or adapting algorithm for \mathcal{S}^B COMPONENT

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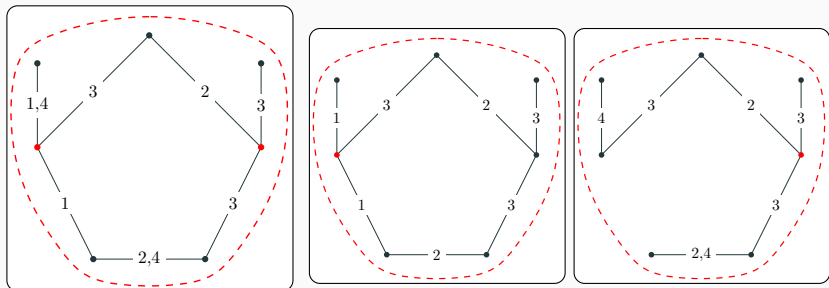
decision problem \mathcal{S}^D COMPONENT:

- exhaustive search or adapting algorithm for \mathcal{S}^B COMPONENT
- NP-complete (reduction from CLIQUE)

\mathcal{S}^D components

Definition closed \mathcal{S}^D component (of window size Δ)

Maximal subset of vertices V' such that $\forall t, \exists s \in V', \forall v \in V', s \rightsquigarrow v$
in windowed induced subgraphs $\mathcal{G}[V']_{[t, t+\Delta-1]}$



Results [Balev, Pigné, Sanlaville, S.]

$\leq \min(2^n, n^{T-\Delta})$ components

\mathcal{S}^D COMPONENT:

- NP-complete

$\leq \min(2^n, n^{T-\Delta})$ components

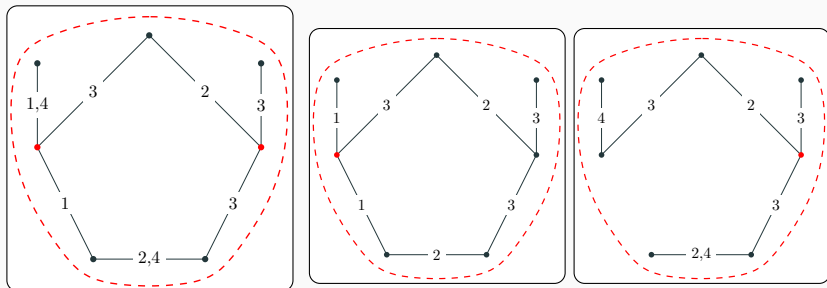
CLOSED \mathcal{S}^D COMPONENT:

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\mathcal{S}^D components

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Results [Balev, Pigné, Sanlaville, S.]

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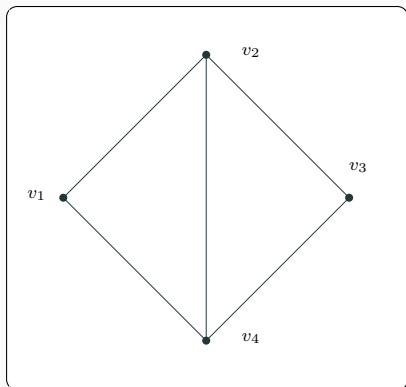
\mathcal{S}^D COMPONENT:

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CLOSED \mathcal{S}^D COMPONENT:

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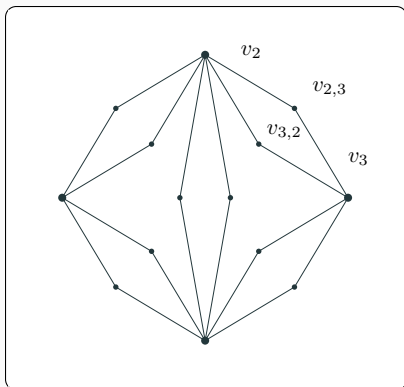
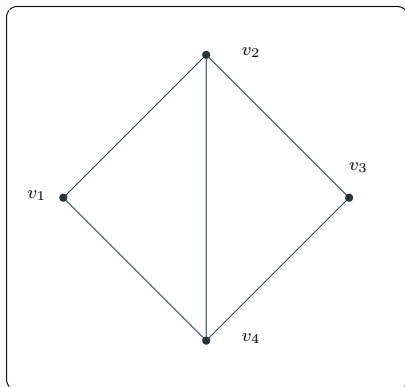
Reduction from CLIQUE to $\mathcal{S}^{\mathcal{D}}$ COMPONENT



Lemma

There exists a clique of size k in G if and only if there exists a $\mathcal{S}^{\mathcal{D}}$ component (with $\Delta = 3$) of size $k + 2m + n$ in \mathcal{G}

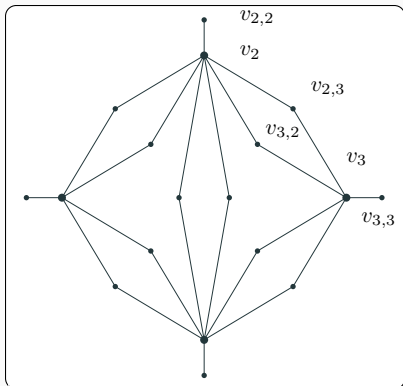
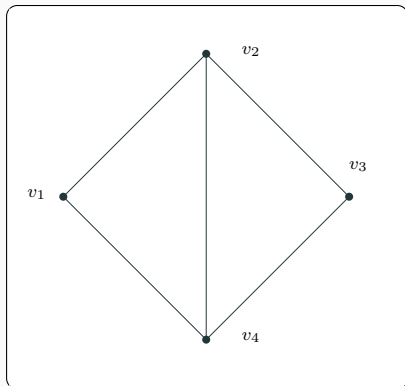
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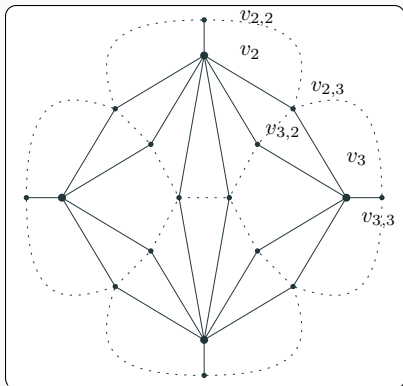
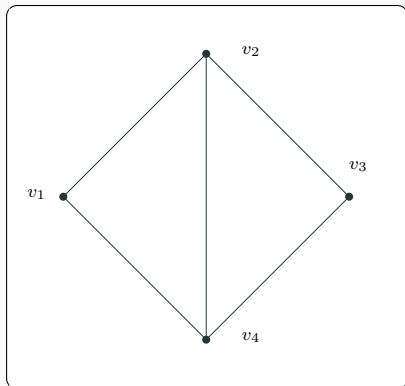
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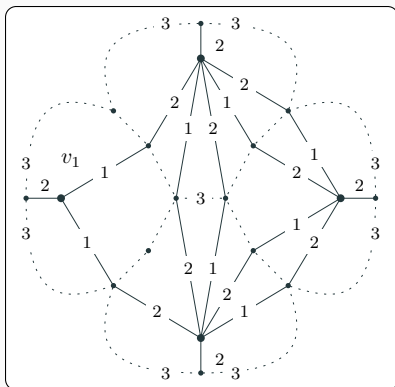
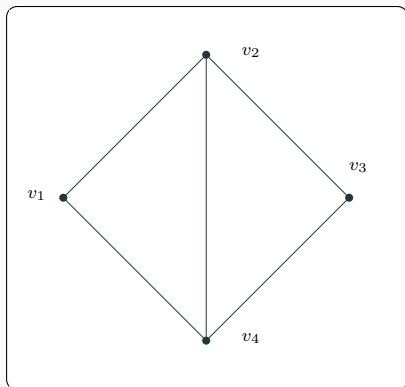
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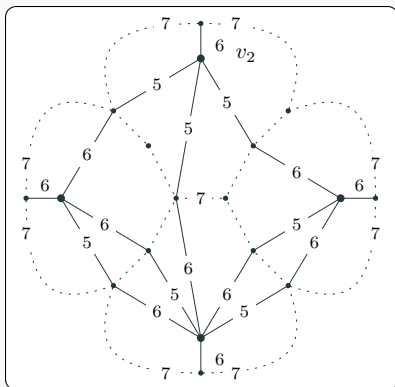
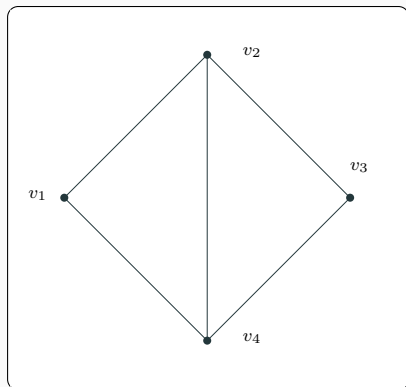
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\mathcal{TC} components and alike: hardness *w.r.t.* graph lifetime

Definition \mathcal{TC} component [Bhadra, Ferreira, 2012][1]

Maximal subset of vertices V' such that $\forall u, v \in V', u \rightsquigarrow v$

\mathcal{TC} components and alike: hardness *w.r.t.* graph lifetime

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\mathcal{TC}° COMPONENT is NP-complete for all lifetimes $T > 2$, and polynomial-time solvable else wise

Theorem [Balev, Pigné, Sanlaville, S.]

$\text{semi}\mathcal{TC}$ COMPONENT is NP-complete for all lifetimes $T > 2$, and polynomial-time solvable for $T = 1$

Conclusion




Conclusion

We have studied a multitude of components defined through journeys, and presented structural and algorithmic results.

Future work

- finish paper (details, proofs, ...)
- finish twin paper on path-based components
- what about strict journeys?

Thank you for your attention.

-  S. BHADRA AND A. FERREIRA, Computing multicast trees in dynamic networks and the complexity of connected components in evolving graphs, *Journal of Internet Services and Applications*, 3 (2012), pp. 269–275.
-  A. CASTEIGTS, A Journey through Dynamic Networks (with Excursions), habilitation à diriger des recherches, Université de Bordeaux, June 2018.
-  A. CASTEIGTS, P. FLOCCHINI, W. QUATTROCIOCCHI, AND N. SANTORO, Time-varying graphs and dynamic networks, *International Journal of Parallel, Emergent and Distributed Systems*, 27 (2012), pp. 387–408.