

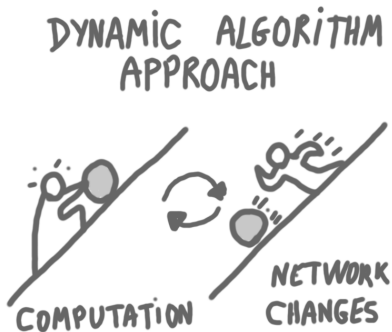
When Should You Wait Before Updating?

Toward a Robustness Refinement

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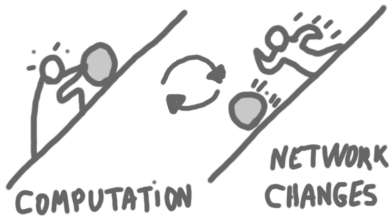
ANR TEMPOGRAL meeting 2023 · Ile du Saussay

Dynamic algorithms and our approach



Dynamic algorithms and our approach

DYNAMIC ALGORITHM
APPROACH

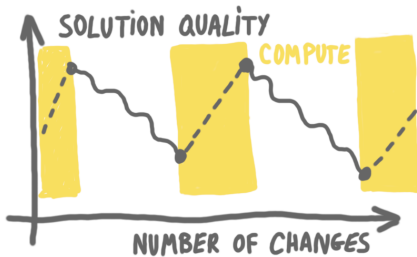


OUR APPROACH

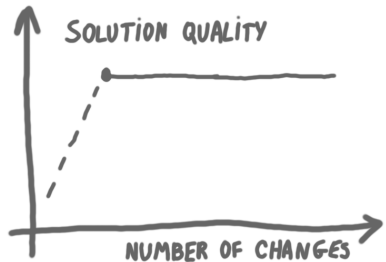


Dynamic algorithms and our approach

DYNAMIC ALGORITHM
APPROACH



OUR APPROACH



General goal

Identify instances and solutions
that are more robust than others

Necessary restrictions

In some situations there is no hope to get any sort of robustness.

→ Very sensitive problems, eg maintain the number of edges

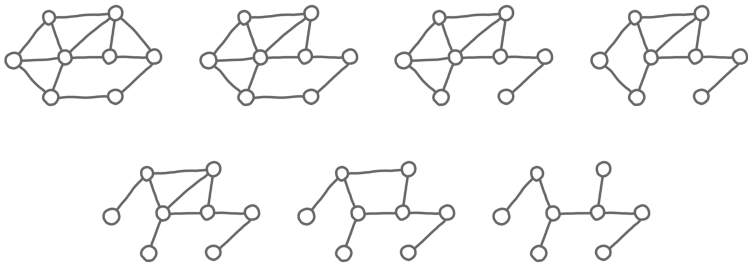
→ Arbitrary dynamic.



Restriction of the dynamic

Assumption on the dynamic:

- ▶ Edge removals only (static vertex set, no edge additions)
- ▶ Keeping connectivity.



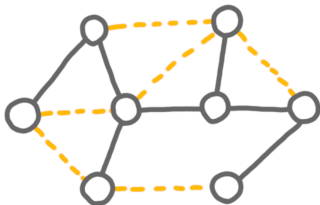
Restriction of the dynamic

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Motivations:

- ▶ Decaying networks
- ▶ More promising
- ▶ Eventual footprint.

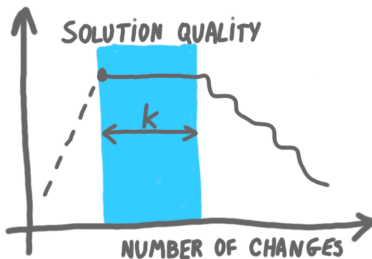


[CDPR20] Robustness: A new form of heredity motivated by dynamic networks by Casteigts, Dubois, Petit, Robson. 2020.

Granularity of edge-removals

New parameter: $k = \text{number of edges removed}$.

A solution is k -robust if it is still correct after k edge removals.



Focus on local problems

Locally checkable problems (LCL): the solution is correct if and only if it is locally correct everywhere.

Our three problems:

MINIMAL
DOMINATING
SET



MAXIMAL
MATCHING



MAXIMAL
INDEPENDENT
SET



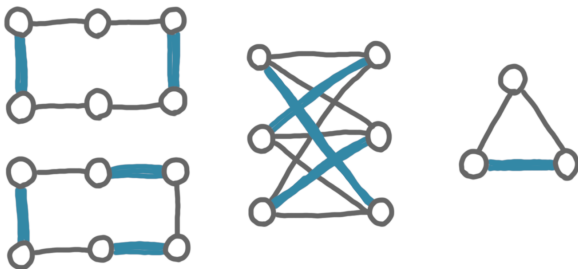
Two basic questions

For a problem P and a parameter k :

Universal question: What is the set of graphs such that all solutions are k -robust, \mathcal{U}_k^P ?

Existential question: What is the set of graphs such that there exists a solution that is k -robust, \mathcal{E}_k^P ?

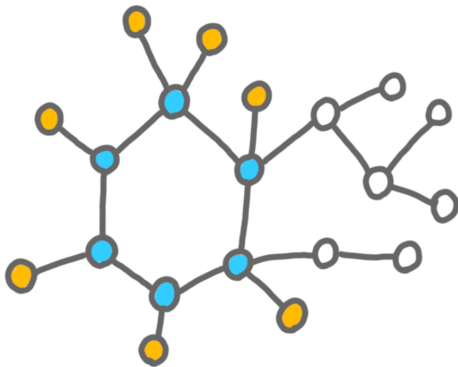
→ Examples with $P =$ maximal matching and $k = 1$.



Basic building blocks



Trees



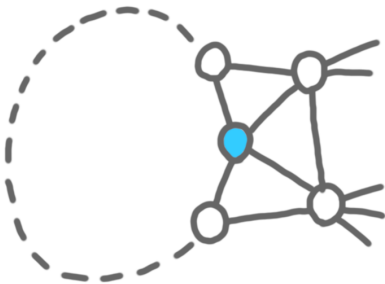
Sputnik graphs

Minimal dominating set results

Theorem: $\forall k, \mathcal{U}_k^{MDS} = \{\text{Sputnik graphs}\}$.

Proof idea:

- ▶ Suppose there is a node v of a cycle with no antenna.
- ▶ (Case 1) There is no neighbor u of v is such that $N(u) \subseteq N(v)$.
- ▶ Build a MDS by avoiding the yellow area.
- ▶ Then the blue-pink edge is “weak”.

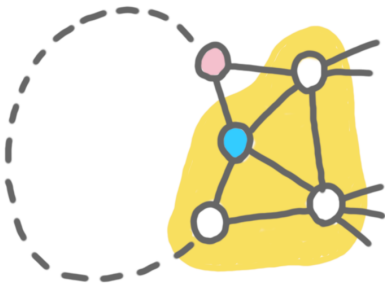


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Maximal matchings results

Theorem:

$\mathcal{U}_1^{MM} =$ _____ and trees.

$\mathcal{U}_{k>1}^{MM} =$ _____, and trees.

Core of the proof: proving that " $G \in \mathcal{U}_1^{MM}$ " is equivalent to "all maximal matchings in G are perfect matchings" (except for trees).

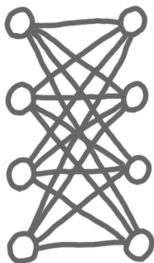
→ A question for you: What are the graphs where all maximal matchings are perfect?

Maximal matchings results

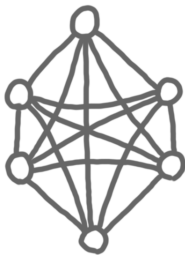
Theorem:

$\mathcal{U}_1^{MM} =$ balanced bicliques ($K_{t,t}$), even cliques (K_{2n}), and trees.

$\mathcal{U}_{k>1}^{MM} =$ cycle C_4 , and trees.



$K_{t,t}$



K_{2n}



tree

Maximal matchings results

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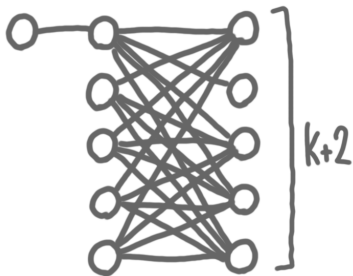
Theorem [Summer79]: Randomly matchable graphs = $\{K_{tt}, K_{2n}\}$.

Maximal independent set results

→ No exact characterization of \mathcal{U}_k^{MIS} . :-)

Theorem: A strict hierarchy: $\mathcal{U}_{k+1}^{MIS} \subsetneq \mathcal{U}_k^{MIS}$.

Proof: $G_k \in \mathcal{U}_k^{MIS} \setminus \mathcal{U}_{k+1}^{MIS}$.

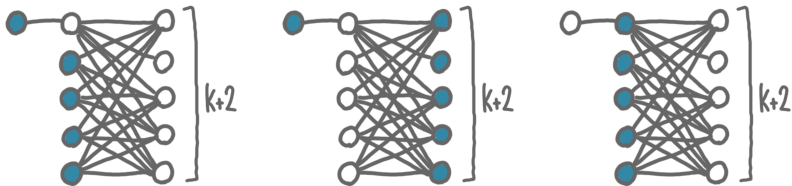


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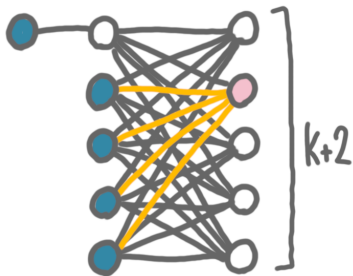


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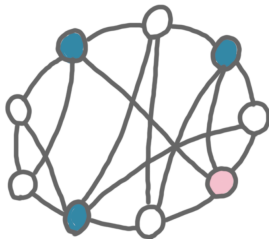


About the existential question

Theorem [CDPR20]: There exists a polynomial time algorithm to decide if a graph belongs to $\mathcal{E}_{\infty}^{MIS}$.

Theorem: Deciding \mathcal{E}_1^{MIS} is NP-hard.

Proof: Focus on the 2-connected case where is it equivalent to “perfect stable” (= every node not in the set is dominated at least twice).



Theorem [Croitoru-Suditu 83] Deciding if there exists a perfect stable is NP-hard.

Conclusion

- ▶ Universal question: Good understanding, but small classes.
- ▶ Existential question: Large classes but difficult to recognize and use.
- ▶ Maybe the right question is in between: graphs such that there exist robust solutions that are easy to find/maintain.
- ▶ Even better: new criteria to quickly measure the robustness of subgraphs, in order to guide algorithms.