

On the constructive power of monadic second-order logic

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GROW 2011

Monadic second-order (MSO) logic is interesting for numerous reasons and in particular because all MSO expressible graph properties have fixed parameter tractable algorithms for tree-width and clique-width as parameters. In most non-trivial cases (e.g. perfect or planar graphs, or graphs of tree-width at most k) the MSO expressibility depends on difficult combinatorial arguments. The expressive power of MSO logic is thus related with graph theoretical properties.

MSO formulas can also be used to add structure to a given graph of some particular class. Examples of such added structure are an orientation (acyclic or not), a linear ordering, a planar embedding, the (unique) modular decomposition. The case of defining a tree-decomposition of bounded width is still open.

We examine here some basic cases: the definitions of orientations and of linear orderings. There are different answers according to whether we use MSO formulas allowing edge set quantifications or not. For linear orderability with edge set quantifications, we give a necessary condition and we prove that it is sufficient for proper minor closed classes (joint work with A. Blumensath). But there are many open questions. Our results are tools that help to construct MSO formulas and to reduce model-checking problems between one another.

B.Courcelle and J. Engelfriet, *Graph structure and monadic second-order logic, a language theoretic approach*, 2011, book to be published by Cambridge University Press.

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