



INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

On the Time Complexity of the Sort Problem

Emmanuel Jeannot

INRIA

N° 6635

Sept. 2008

Thème NUM



*R*apport
de recherche

On the Time Complexity of the Sort Problem

Emmanuel Jeannot
INRIA

Thème NUM — Systèmes numériques
Équipes-Projets AlGorille

Rapport de recherche n° 6635 — Sept. 2008 — 3 pages

Abstract: We revisit the famous problem of sorting objects using thermodynamics concepts. This helps us to easily derive the well-known time complexity of $\Omega(n \ln n)$ without any use of the classical computer-science objects (graph, Turing machine, etc.).

Key-words: Sort, Boltzmann formula, time complexity

Sur la complexité temporelle du Tri

Résumé : Nous revisitons la problème consistant à trier des objets en utilisant des concepts issues de la thermodynamique. Grace à ces concepts nous sommes capable de facilement dériver la bien-connue complexité temporelle en $\Omega(n \ln n)$ sans l'aide des objets classiques de l'informatique (graph, Machine de Turing, etc.).

Mots-clés : Tri, Equation de Boltzmann, complexité temporelle

Let be a system with n objects initially randomly arranged. Using Boltzmann formula, the entropy of a system is: $S = k \ln(\Omega)$. Where Ω is the number of possible configurations of this system and $k = 1.381 \cdot 10^{-23} \text{ J.K}^{-1}$ (joule per kelvin) is the Boltzmann constant.

Let us assume we want to sort the n objects of our system. Then, before sorting the entropy of the system is:

$$S_1 = k \ln(n!) \text{ J.K}^{-1}$$

and after sorting, the entropy is:

$$S_2 = k \ln(m) \text{ J.K}^{-1}$$

Where m is the number of possible ways to order the objects.

According to thermodynamics, the energy required for a system for going from one state to an other state is $E = T\Delta S$ where T is the temperature (in kelvin) where occurs the change of state and $\Delta S = S_1 - S_2$ is the difference of entropy of the considered states. For sorting our n objects we therefore need the following energy (in joule):

$$E = Tk (\ln(n!) - \ln(m)) = Tk \ln \left(\frac{n!}{m} \right) \text{ J}$$

Let us suppose we use a machine to make this change of state (find one possible sorting among the m). Let us suppose that this machine consumes a power of xW (watt) or xJs^{-1} (joule per second) and works at a constant temperature TK . Therefore, a lower bound of the time to sort the n objects is (in seconds):

$$t \geq \frac{T}{x} k \ln \left(\frac{n!}{m} \right) \text{ s}$$

This means that the time complexity of the sort problem is (when m is assumed to be a constant independent of n):

$$\Omega(\ln(n!))$$

As $n! = \sqrt{2\pi n} \left(\frac{n}{e}\right)^n \left(1 + O\left(\frac{1}{n}\right)\right)$ (Stirling formula), we obtain the well-known time complexity for the sort problem:

$$\Omega(n \ln n)$$



Centre de recherche INRIA Nancy – Grand Est
LORIA, Technopôle de Nancy-Brabois - Campus scientifique
615, rue du Jardin Botanique - BP 101 - 54602 Villers-lès-Nancy Cedex (France)

Centre de recherche INRIA Bordeaux – Sud Ouest : Domaine Universitaire - 351, cours de la Libération - 33405 Talence Cedex
Centre de recherche INRIA Grenoble – Rhône-Alpes : 655, avenue de l'Europe - 38334 Montbonnot Saint-Ismier
Centre de recherche INRIA Lille – Nord Europe : Parc Scientifique de la Haute Borne - 40, avenue Halley - 59650 Villeneuve d'Ascq
Centre de recherche INRIA Paris – Rocquencourt : Domaine de Voluceau - Rocquencourt - BP 105 - 78153 Le Chesnay Cedex
Centre de recherche INRIA Rennes – Bretagne Atlantique : IRISA, Campus universitaire de Beaulieu - 35042 Rennes Cedex
Centre de recherche INRIA Saclay – Île-de-France : Parc Orsay Université - ZAC des Vignes : 4, rue Jacques Monod - 91893 Orsay Cedex
Centre de recherche INRIA Sophia Antipolis – Méditerranée : 2004, route des Lucioles - BP 93 - 06902 Sophia Antipolis Cedex

Éditeur
INRIA - Domaine de Voluceau - Rocquencourt, BP 105 - 78153 Le Chesnay Cedex (France)
<http://www.inria.fr>
ISSN 0249-6399