Ping Pong in Dangerous Graphs: Optimal Black Hole Search with Pebbles

Paola Flocchini[1] David Ilcinkas[2] Nicola Santoro[3]

[1]University of Ottawa, Canada

[2]CNRS and University of Bordeaux (LaBRI), France

[3]Carleton University, Canada

DISC '08
September 24, 2008

Problem

Black hole in a network

Node blocking and destroying any mobile agent entering it.
Motivations :

- Site which is destroyed or dangerous for a physical robot
- Node infected by a "killer" virus
- Mute or crashed node

Black Hole Search

From a safe node (homebase), a team of mobile agents has to locate the black hole(s).

Performance : nb of agents, nb of moves, time

Problem

Black hole in a network

Node blocking and destroying any mobile agent entering it.
Motivations :

- Site which is destroyed or dangerous for a physical robot
- Node infected by a "killer" virus
- Mute or crashed node

Black Hole Search

From a safe node (homebase), a team of mobile agents has to locate the black hole(s).

Performance : nb of agents, nb of moves, time

Problem

Black hole in a network

Node blocking and destroying any mobile agent entering it.
Motivations :

- Site which is destroyed or dangerous for a physical robot
- Node infected by a "killer" virus
- Mute or crashed node

Black Hole Search

From a safe node (homebase), a team of mobile agents has to locate the black hole(s).
Performance : nb of agents, nb of moves, time

(A)synchronous

Synchronous

Agents act simultaneously at every pulse of a global clock.

$\Longrightarrow$ Possible to wait for an agent gone exploring a potentially dangerous edge.

Asynchronous

Every action takes a finite but unbounded time.

$\Longrightarrow$ A priori impossible to distinguish a very slow link from a link leading to the black hole.

$\Longrightarrow$ Useless to wait for an agent.

$\Longrightarrow$ Pb equivalent to the exploration of dangerous graphs.

(A)synchronous

Synchronous

Agents act simultaneously at every pulse of a global clock.

$\Longrightarrow$ Possible to wait for an agent gone exploring a potentially dangerous edge.

Asynchronous

Every action takes a finite but unbounded time.

$\Longrightarrow$ A priori impossible to distinguish a very slow link from a link leading to the black hole.

$\Longrightarrow$ Useless to wait for an agent.

$\Longrightarrow$ Pb equivalent to the exploration of dangerous graphs.

Necessary hypotheses (in asynchronous)

Required knowledge

- Knowledge of the number of black holes
  $\longrightarrow$ Here, exactly one black hole
- Knowledge of the number of nodes n

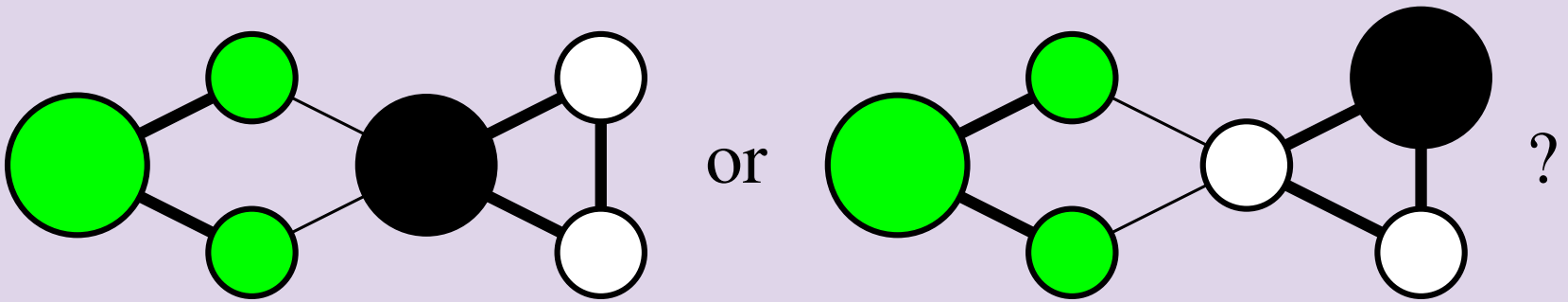
2-vertex-connected

Necessary hypotheses (in asynchronous)

Required knowledge

- Knowledge of the number of black holes
  $\longrightarrow$ Here, exactly one black hole
- Knowledge of the number of nodes n

2-vertex-connected



Model (still in asynchronous)

Communication/coordination

No direct communication but

Whiteboards (memory space on each node).

or Edge markers (mark a specific port).

or Pebbles (node markers).

Computational step

- Observation : sense for the presence of a pebble/message on the node.
- Interaction : put down or pick up a pebble/message.
- Action : move along an edge (or terminate).

This set of substeps (forming a computational step) is executed atomically in mutual exclusion.

Model (still in asynchronous)

Communication/coordination

No direct communication but

Whiteboards (memory space on each node).

or Edge markers (mark a specific port).

or Pebbles (node markers).

Computational step

- Observation : sense for the presence of a pebble/message on the node.
- Interaction : put down or pick up a pebble/message.
- Action : move along an edge (or terminate).

This set of substeps (forming a computational step) is executed atomically in mutual exclusion.

Related work (asynchronous, whiteboards)

Seminal results

[Dobrev, Flocchini, Prencipe, Santoro. Dist. Comp. 2006]

	nb of agents	nb of moves
No information (besides n)	$\Delta + 1$	$\Theta(n^2)$
Sense of direction	2	$\Theta(n^2)$
Complete knowledge	2	$\Theta(n \log n)$

Improvements (with complete knowledge)

[Dobrev, Flocchini, Santoro. SIROCCO 2004]

- Specific topologies (but not the ring)
- Nb of moves : $O(n)$

[Dobrev et al. Networks 2006]

- Nb of moves : $O(n + D \log D)$ (D = diameter)

Related work (asynchronous, whiteboards)

Seminal results

[Dobrev, Flocchini, Prencipe, Santoro. Dist. Comp. 2006]

	nb of agents	nb of moves
No information (besides n)	$\Delta + 1$	$\Theta(n^2)$
Sense of direction	2	$\Theta(n^2)$
Complete knowledge	2	$\Theta(n \log n)$

Improvements (with complete knowledge)

[Dobrev, Flocchini, Santoro. SIROCCO 2004]

- Specific topologies (but not the ring)
- Nb of moves : $O(n)$

[Dobrev et al. Networks 2006]

- Nb of moves : $O(n + D \log D)$ (D = diameter)

Related work (asynchronous)

Minimal conditions on agents coordination.

Edge markers, ring, FIFO links

[Dobrev, Santoro, Shi. CIAC 2006]

- $O(1)$ edge markers in total, up to 3 per node
- Nb of moves : $\Theta(n \log n)$

Edge markers, arbitrary graphs, FIFO links

[Dobrev, Flocchini, Kralovic, Santoro. IFIP TCS 2006]

- No information (besides n)
- Nb of agents : $\Delta + 1$
- Nb of moves : roughly $O(n^{10})$

Pb : Is coordination through pebbles sufficient ?

Related work (asynchronous)

Minimal conditions on agents coordination.

Edge markers, ring, FIFO links

[Dobrev, Santoro, Shi. CIAC 2006]

- $O(1)$ edge markers in total, up to 3 per node
- Nb of moves : $\Theta(n \log n)$

Edge markers, arbitrary graphs, FIFO links

[Dobrev, Flocchini, Kralovic, Santoro. IFIP TCS 2006]

- No information (besides n)
- Nb of agents : $\Delta + 1$
- Nb of moves : roughly $O(n^{10})$

Pb : Is coordination through pebbles sufficient ?

Related work (asynchronous)

Minimal conditions on agents coordination.

Edge markers, ring, FIFO links

[Dobrev, Santoro, Shi. CIAC 2006]

- $O(1)$ edge markers in total, up to 3 per node
- Nb of moves : $\Theta(n \log n)$

Edge markers, arbitrary graphs, FIFO links

[Dobrev, Flocchini, Kralovic, Santoro. IFIP TCS 2006]

- No information (besides n)
- Nb of agents : $\Delta + 1$
- Nb of moves : roughly $O(n^{10})$

Pb : Is coordination through pebbles sufficient ?

Related work (asynchronous)

Minimal conditions on agents coordination.

Edge markers, ring, FIFO links

[Dobrev, Santoro, Shi. CIAC 2006]

- $O(1)$ edge markers in total, up to 3 per node
- Nb of moves : $\Theta(n \log n)$

Edge markers, arbitrary graphs, FIFO links

[Dobrev, Flocchini, Kralovic, Santoro. IFIP TCS 2006]

- No information (besides n)
- Nb of agents : $\Delta + 1$
- Nb of moves : roughly $O(n^{10})$

Pb : Is coordination through pebbles sufficient ?

Our results

Model

- Complete knowledge (arbitrary graphs)
- Undistinguishable pebbles (at most one per node)
- Links do not need to be FIFO

Optimal result

- 1 pebble per agent
- 2 agents
- $\Theta(n \log n)$ moves

Our results

Model

- Complete knowledge (arbitrary graphs)
- Undistinguishable pebbles (at most one per node)
- Links do not need to be FIFO

Optimal result

- 1 pebble per agent
- 2 agents
- $\Theta(n \log n)$ moves

Case of the ring

Model

- Two identical agents equiped with one pebble each
- Identical pebbles
- At most one pebble per node / carried by an agent
- Asynchronous ring with exactly one black hole

Definition

- Edge of the homebase with the smaller port: right
- Other direction: left

Case of the ring

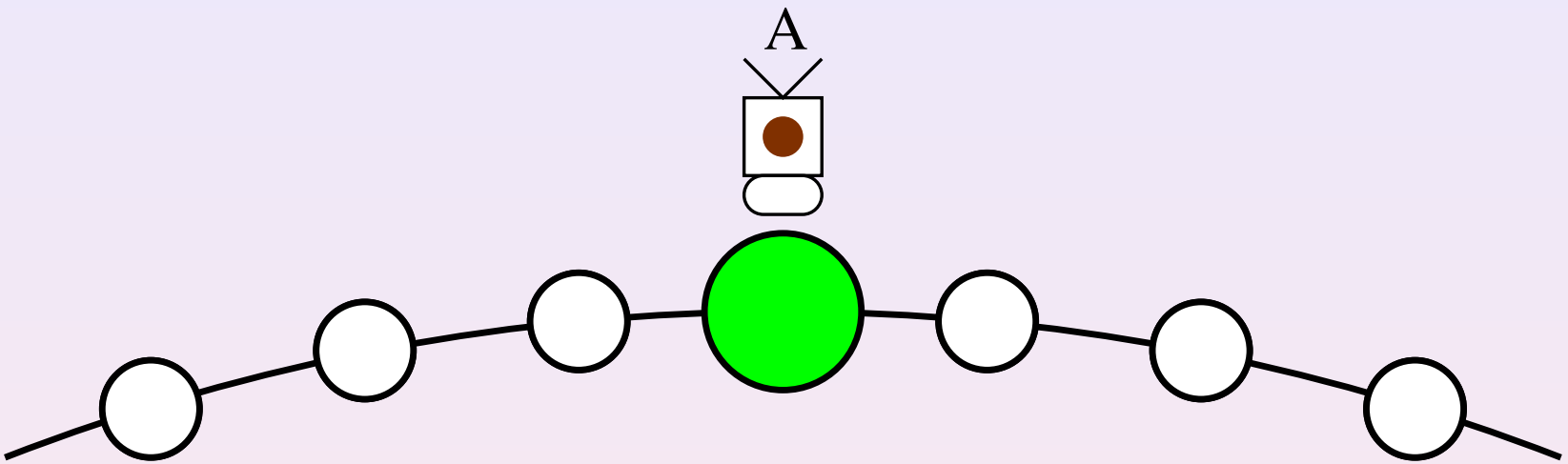
Model

- Two identical agents equiped with one pebble each
- Identical pebbles
- At most one pebble per node / carried by an agent
- Asynchronous ring with exactly one black hole

Definition

- Edge of the homebase with the smaller port: right
- Other direction: left

First steps

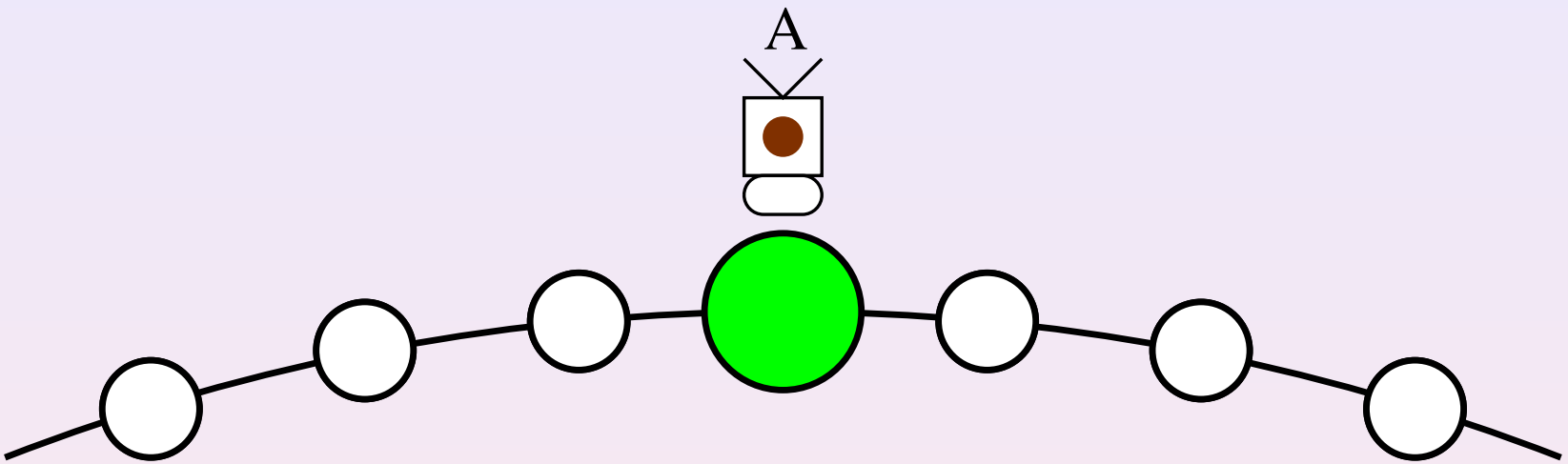


Beginning of the algorithm

If node is empty **then**

Go right

First steps



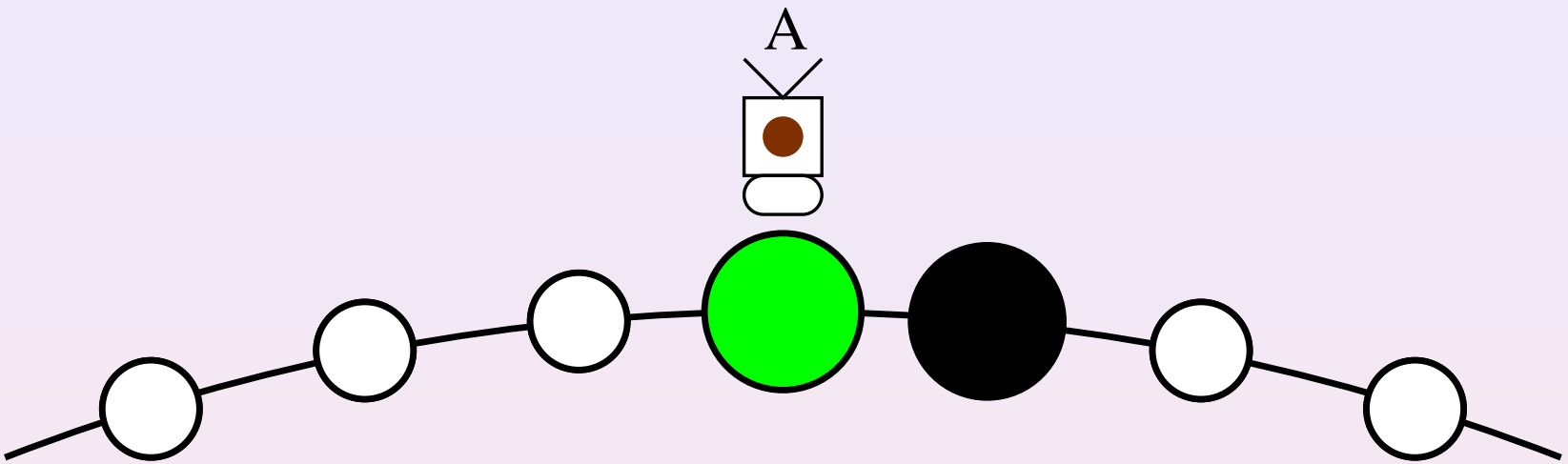
Beginning of the algorithm

If node is empty **then**

Do not put down the pebble

Go right

First steps



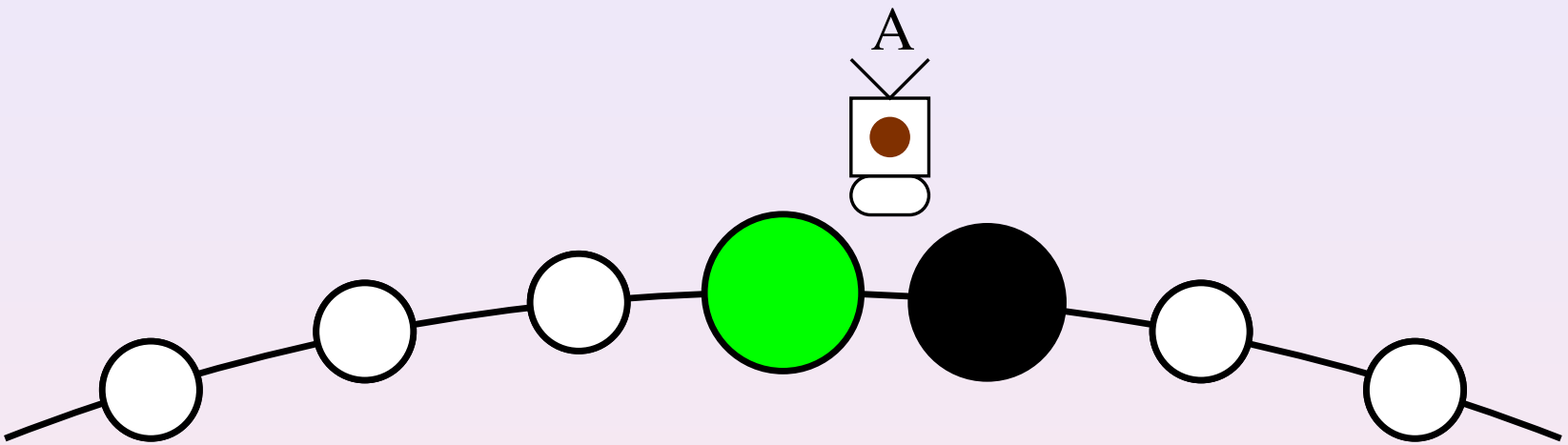
Beginning of the algorithm

If node is empty **then**

Do not put down the pebble

Go right

First steps



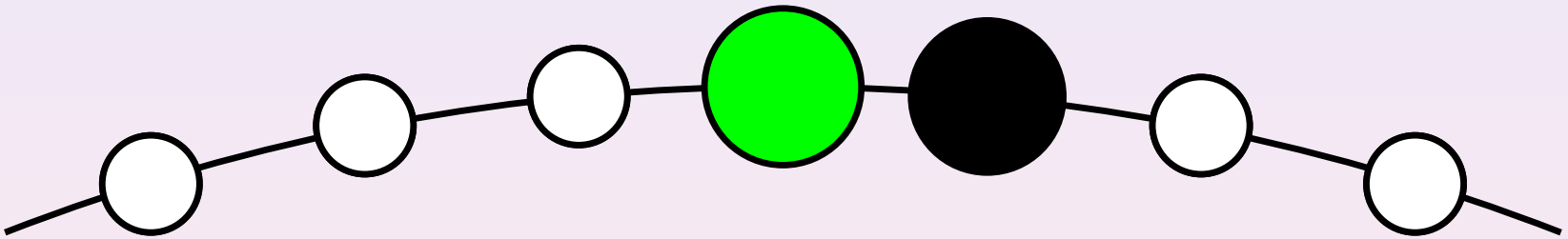
Beginning of the algorithm

If node is empty **then**

Do not put down the pebble

Go right

First steps



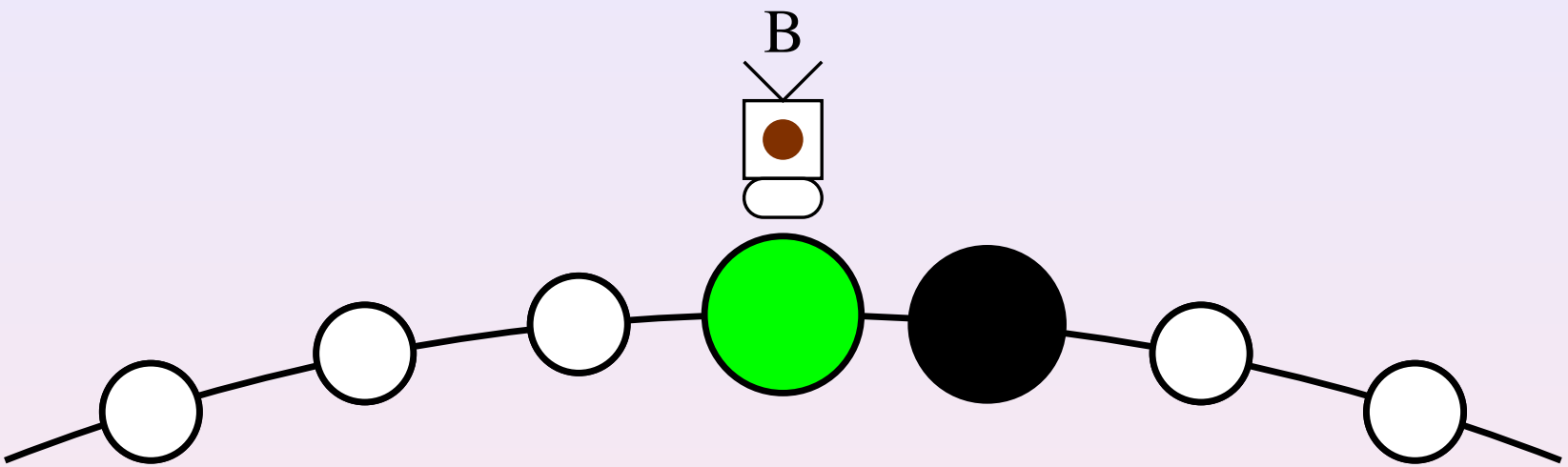
Beginning of the algorithm

If node is empty **then**

Do not put down the pebble

Go right

First steps



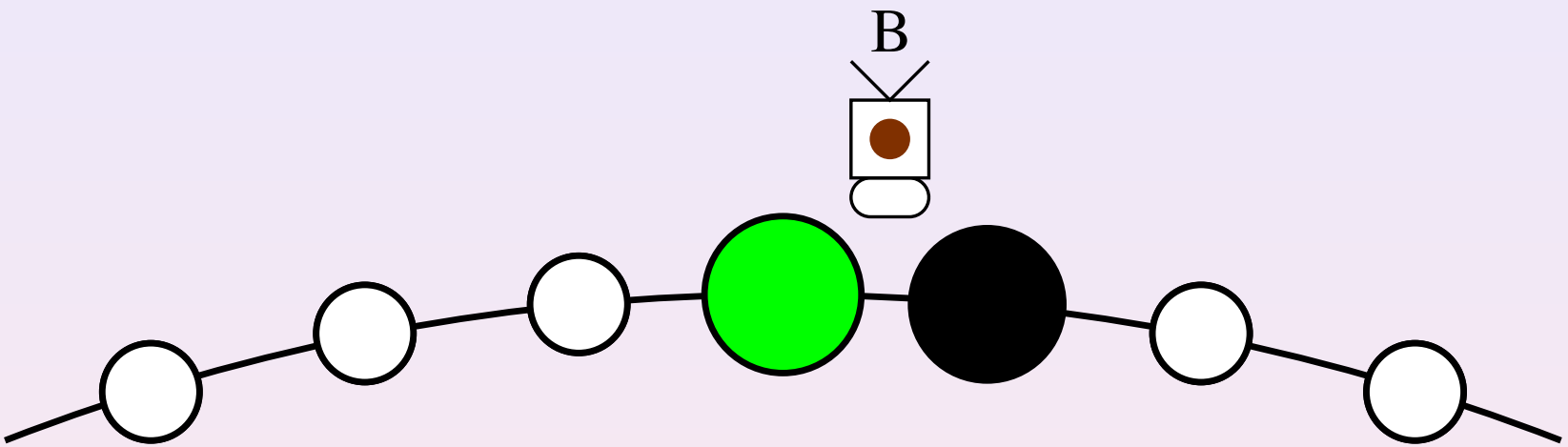
Beginning of the algorithm

If node is empty **then**

Do not put down the pebble

Go right

First steps



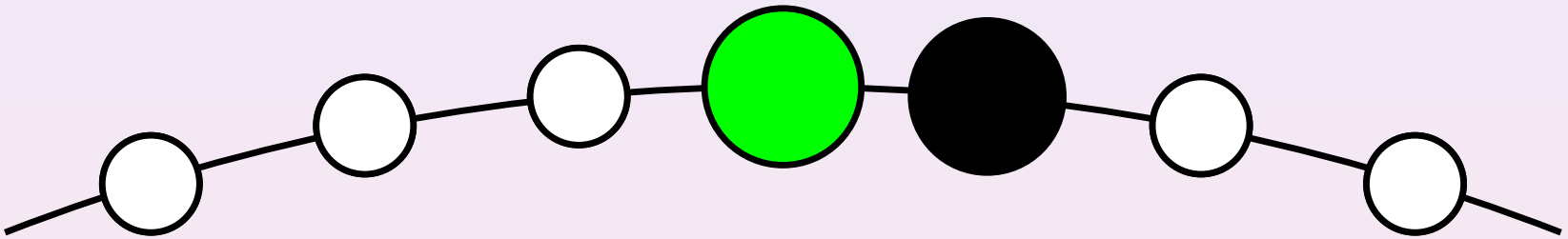
Beginning of the algorithm

If node is empty **then**

Do not put down the pebble

Go right

First steps



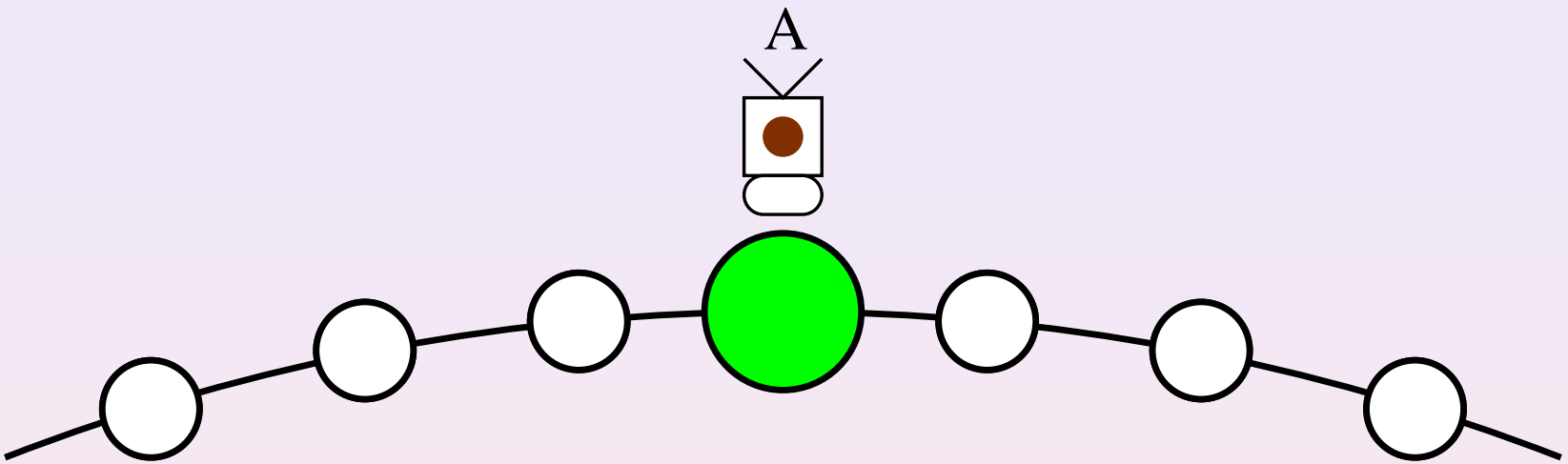
Beginning of the algorithm

If node is empty **then**

Do not put down the pebble

Go right

First steps



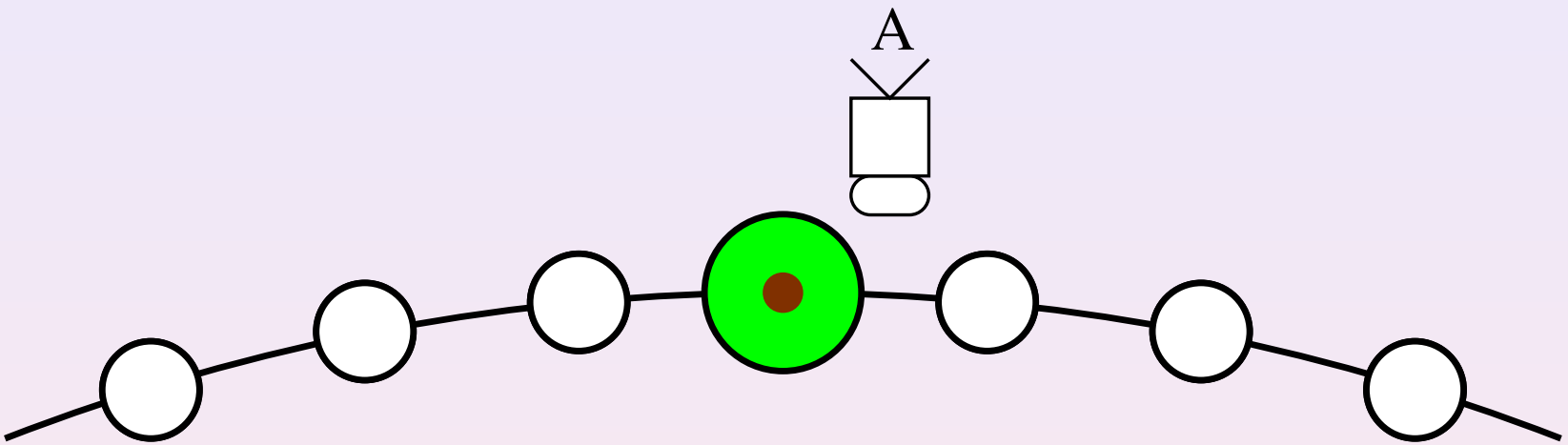
Beginning of the algorithm

If node is empty **then**

Put down the pebble

Go right

First steps



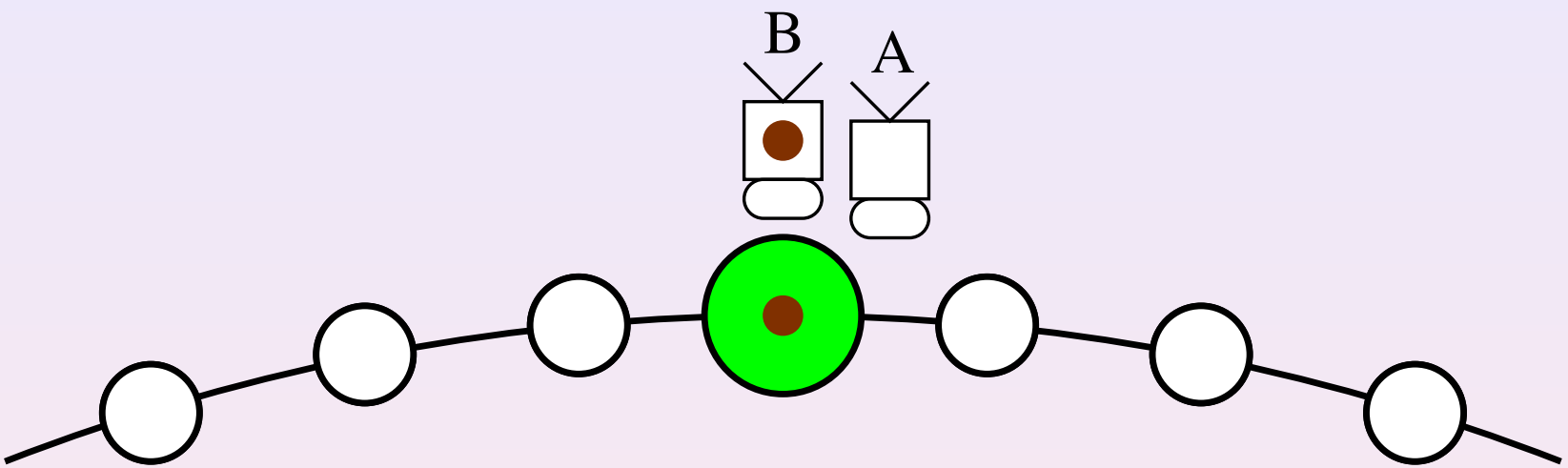
Beginning of the algorithm

If node is empty **then**

Put down the pebble

Go right

First steps



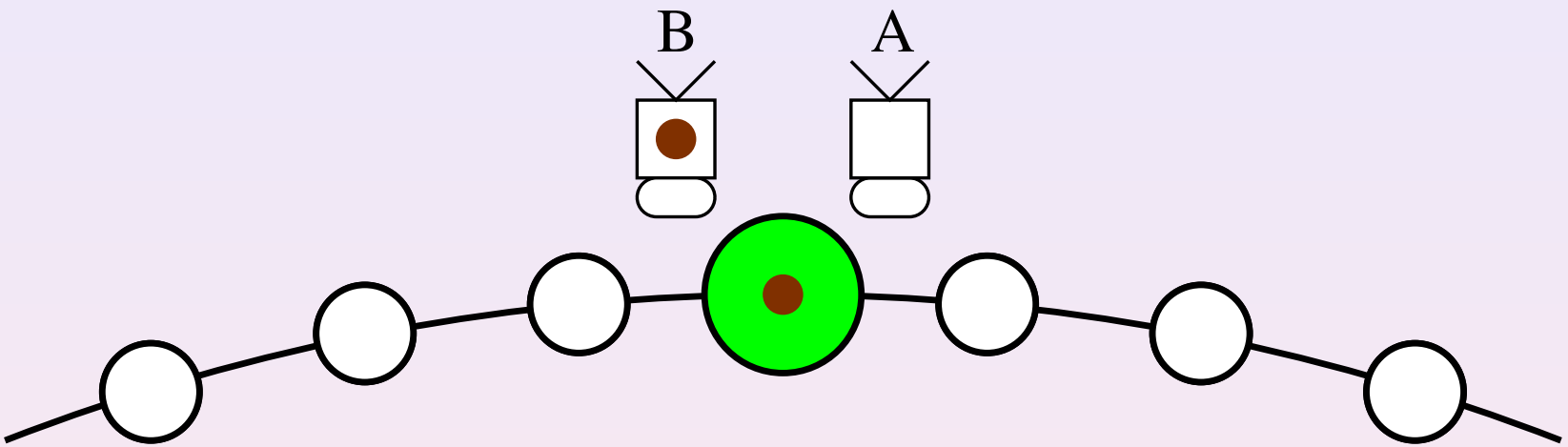
Beginning of the algorithm

If node is empty **then**

Put down the pebble

Go right

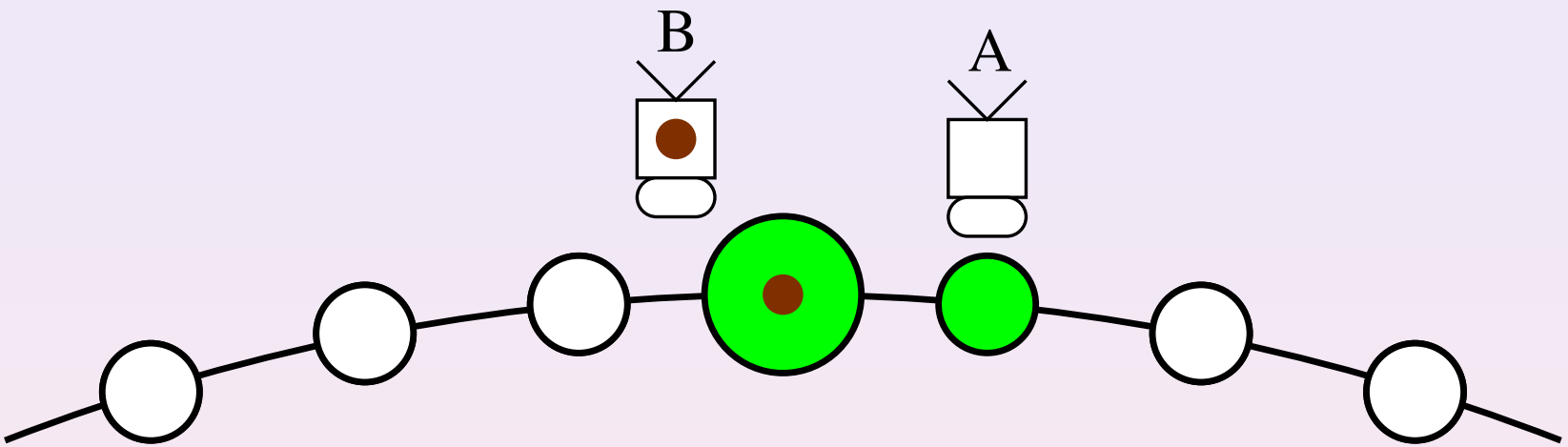
First steps



Beginning of the algorithm

If node is empty **then**
    Put down the pebble
    Go right
else Go left **endif**

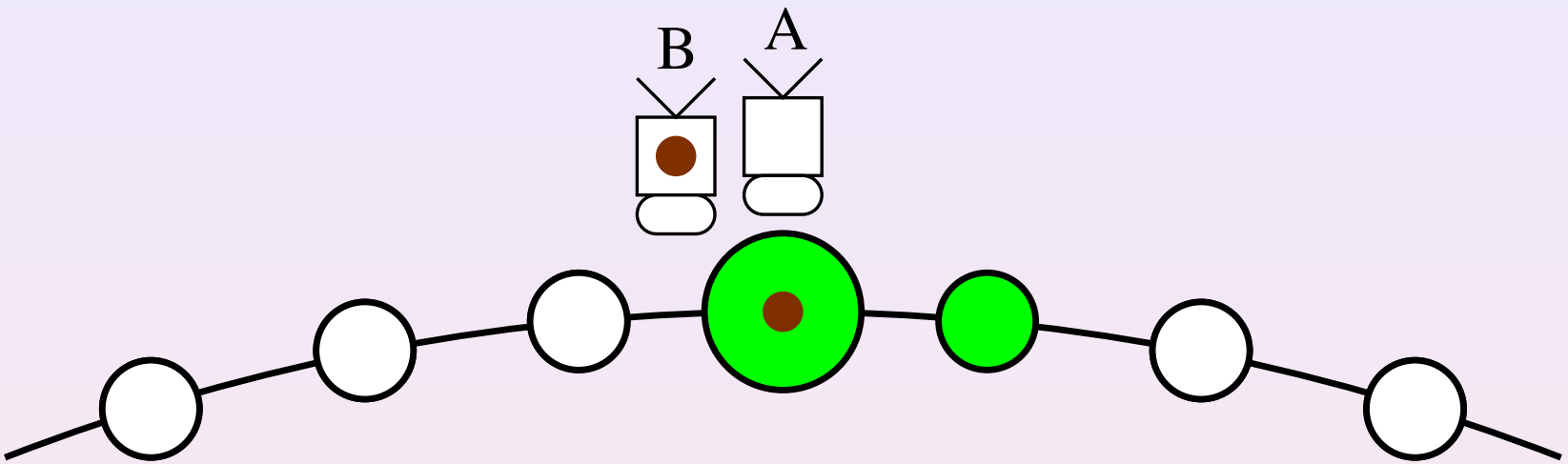
First steps



Beginning of the algorithm

If node is empty **then**
    Put down the pebble
    Go right
else Go left **endif**

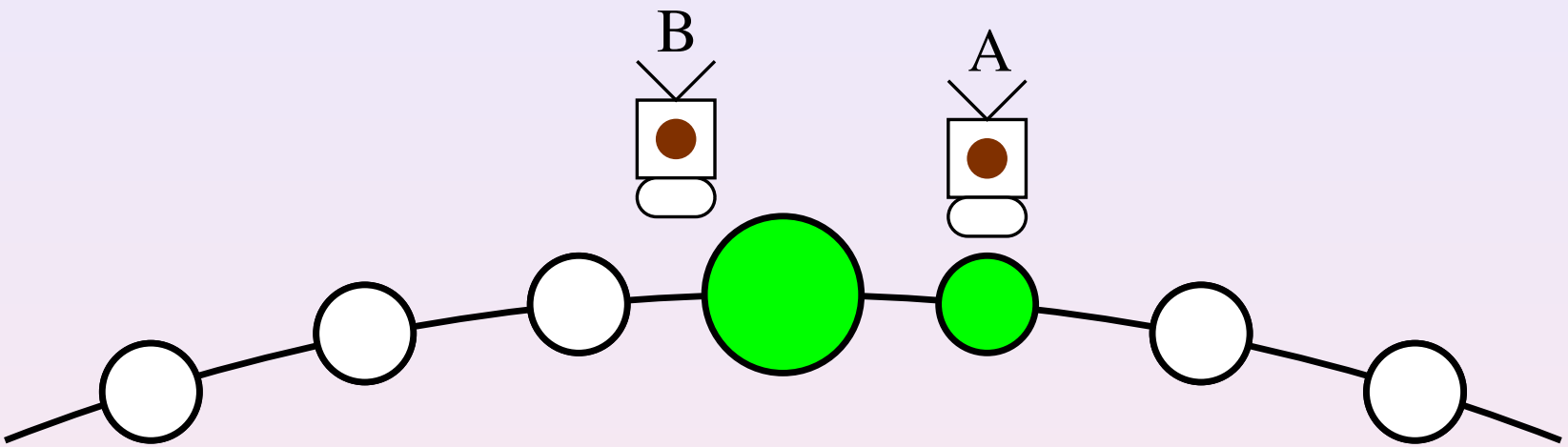
First steps



Beginning of the algorithm

If node is empty **then**
    Put down the pebble
    Go right
else Go left **endif**

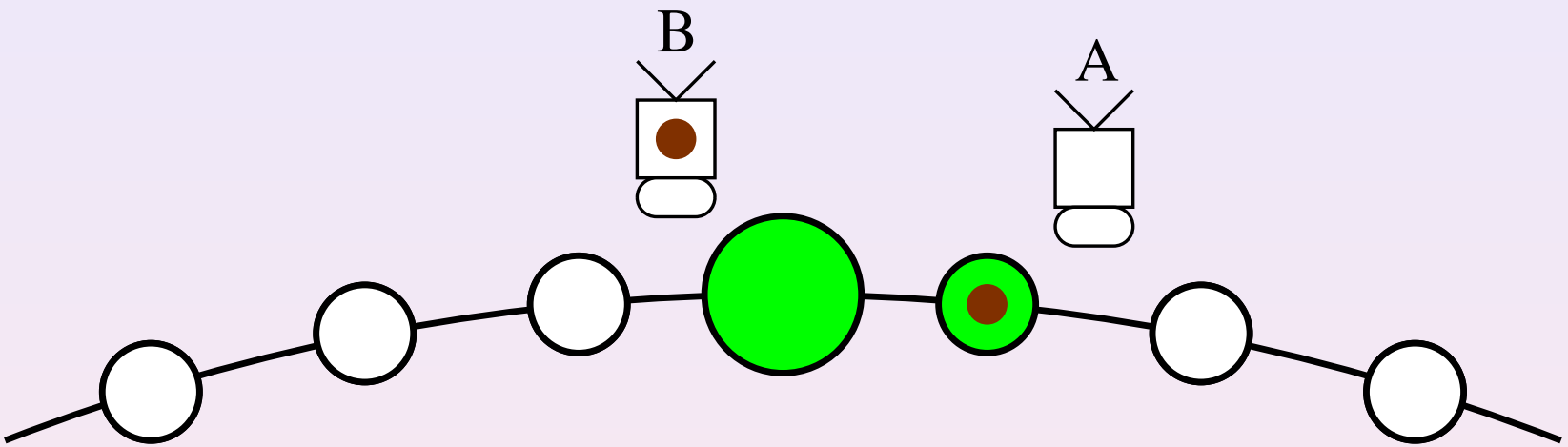
First steps



Beginning of the algorithm

If node is empty **then**
    Put down the pebble
    Go right
else Go left **endif**

First steps



Beginning of the algorithm

If node is empty **then**
    Put down the pebble
    Go right
else Go left **endif**

Cautious walk

Exploration of $e = \{u, v\}$ from u to v

Simple cautious walk

Initially: agent at u with a pebble, node without a pebble

- Put down the pebble (warn the other agent) and go to v
- Come back to u
- Retrieve the pebble and return at v

Double cautious walk

Initially: agent at u with a pebble, node u with a pebble

- Go to v
- Put down the pebble and go back to u
- Retrieve the other pebble and return at v

Cautious walk

Exploration of $e = \{u, v\}$ from u to v

Simple cautious walk

Initially: agent at u with a pebble, node without a pebble

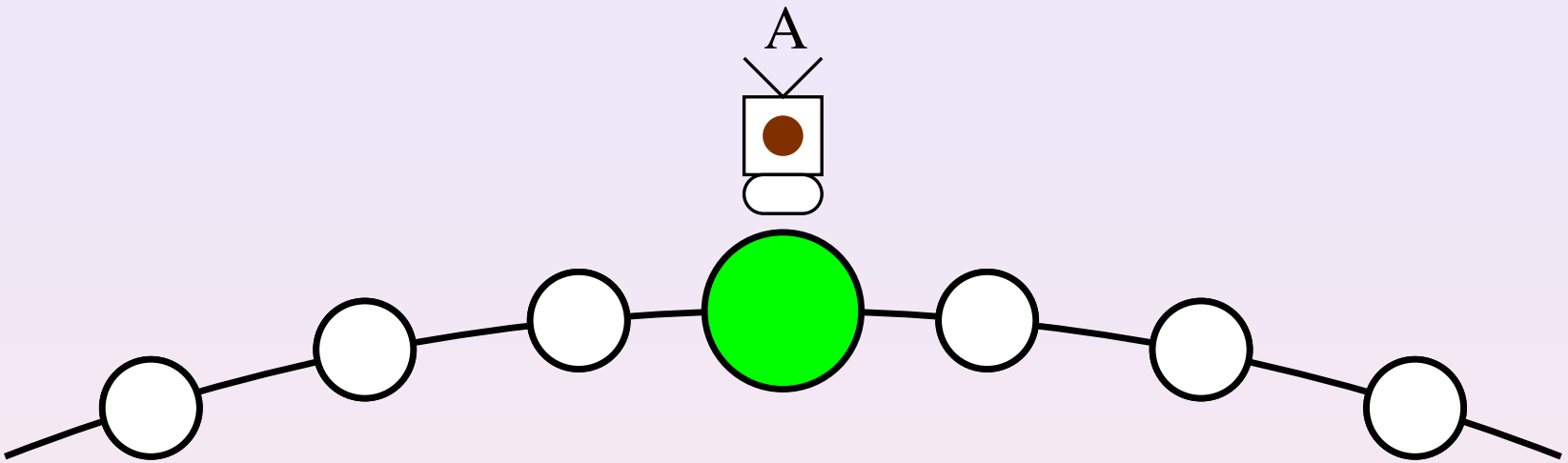
- Put down the pebble (warn the other agent) and go to v
- Come back to u
- Retrieve the pebble and return at v

Double cautious walk

Initially: agent at u with a pebble, node u with a pebble

- Go to v
- Put down the pebble and go back to u
- Retrieve the other pebble and return at v

Ping Pong algorithm

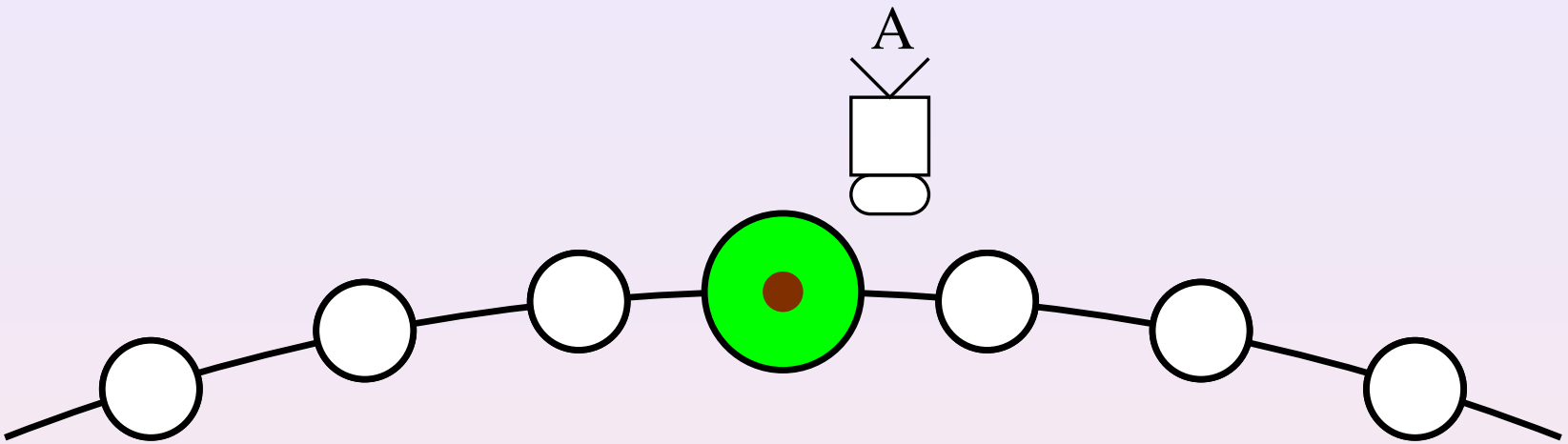


Quick description

If the agent "controls"

- 1 pebble $\longrightarrow$ Simple cautious walk to the right
- 2 pebbles $\longrightarrow$ Double cautious walk to the left
- 0 pebbles $\longrightarrow$ Non-cautious walk to the left

Ping Pong algorithm

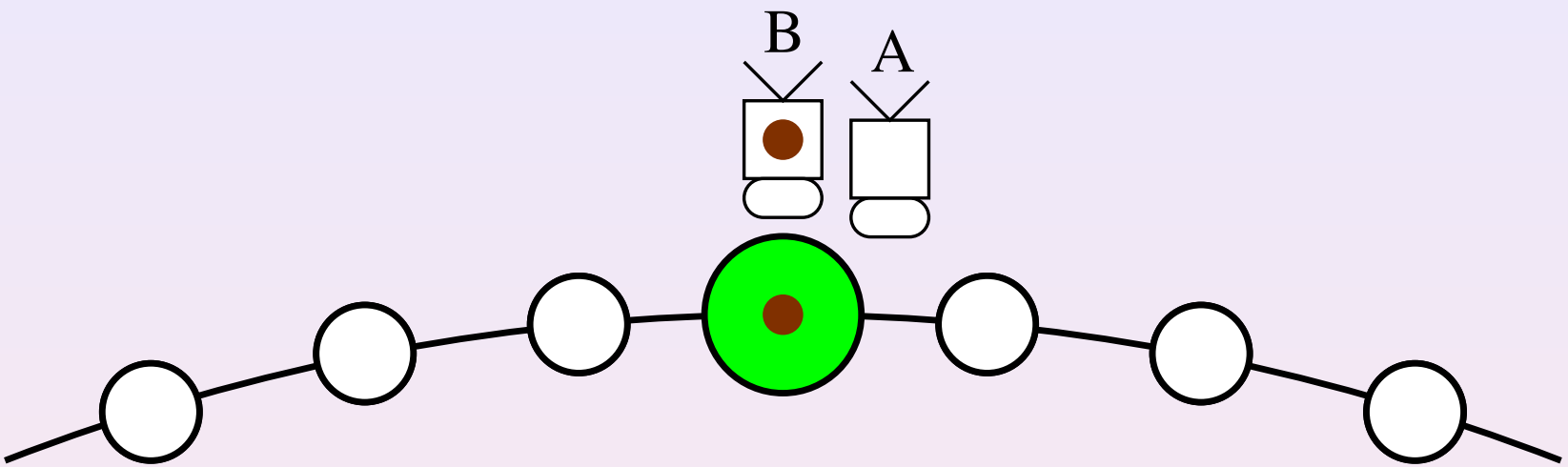


Quick description

If the agent "controls"

- 1 pebble $\longrightarrow$ Simple cautious walk to the right
- 2 pebbles $\longrightarrow$ Double cautious walk to the left
- 0 pebbles $\longrightarrow$ Non-cautious walk to the left

Ping Pong algorithm

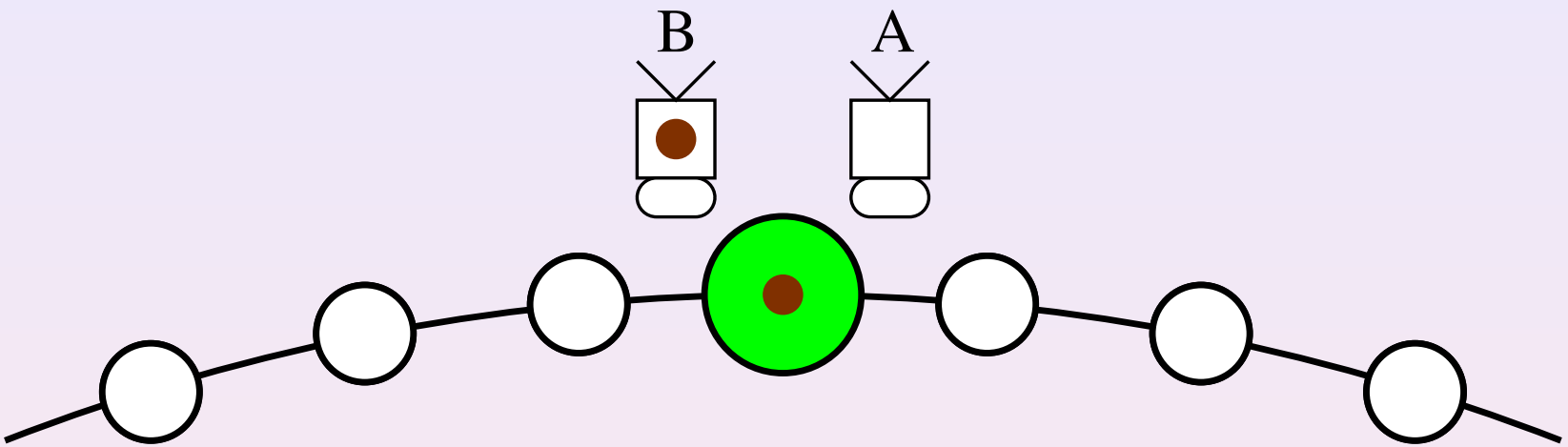


Quick description

If the agent "controls"

- 1 pebble $\longrightarrow$ Simple cautious walk to the right
- 2 pebbles $\longrightarrow$ Double cautious walk to the left
- 0 pebbles $\longrightarrow$ Non-cautious walk to the left

Ping Pong algorithm

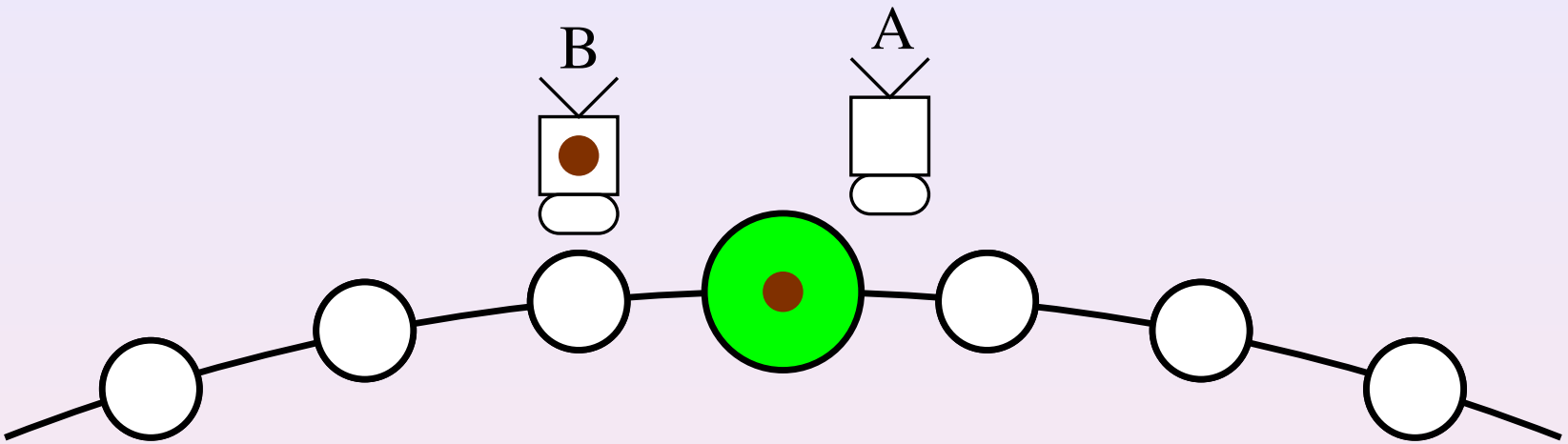


Quick description

If the agent "controls"

- 1 pebble $\longrightarrow$ Simple cautious walk to the right
- 2 pebbles $\longrightarrow$ Double cautious walk to the left
- 0 pebbles $\longrightarrow$ Non-cautious walk to the left

Ping Pong algorithm

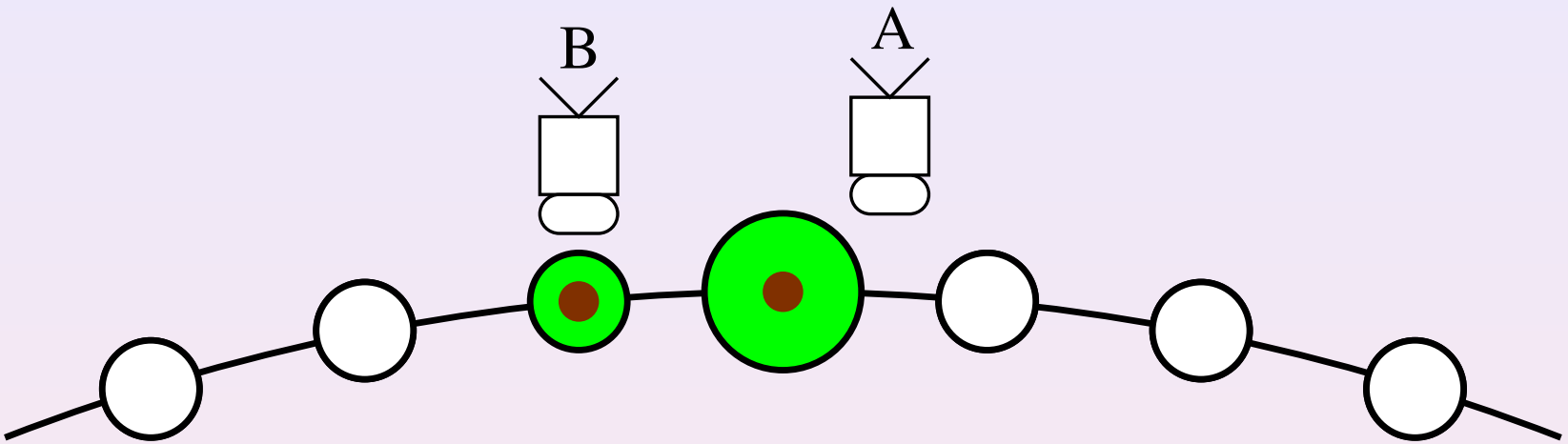


Quick description

If the agent "controls"

- 1 pebble $\longrightarrow$ Simple cautious walk to the right
- 2 pebbles $\longrightarrow$ Double cautious walk to the left
- 0 pebbles $\longrightarrow$ Non-cautious walk to the left

Ping Pong algorithm

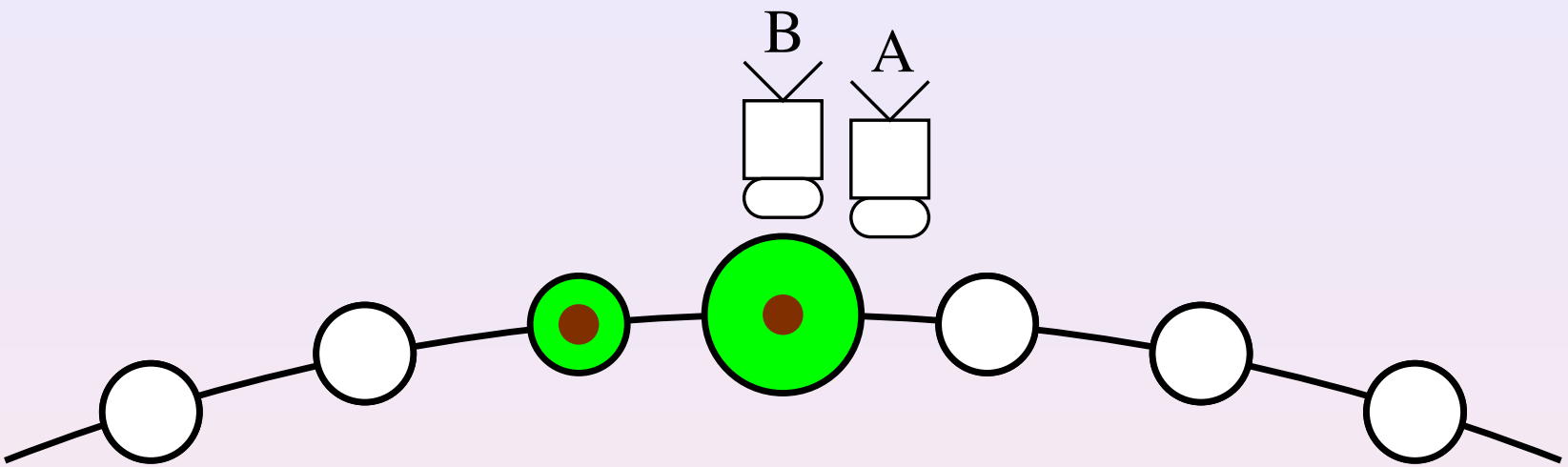


Quick description

If the agent "controls"

- 1 pebble $\longrightarrow$ Simple cautious walk to the right
- 2 pebbles $\longrightarrow$ Double cautious walk to the left
- 0 pebbles $\longrightarrow$ Non-cautious walk to the left

Ping Pong algorithm

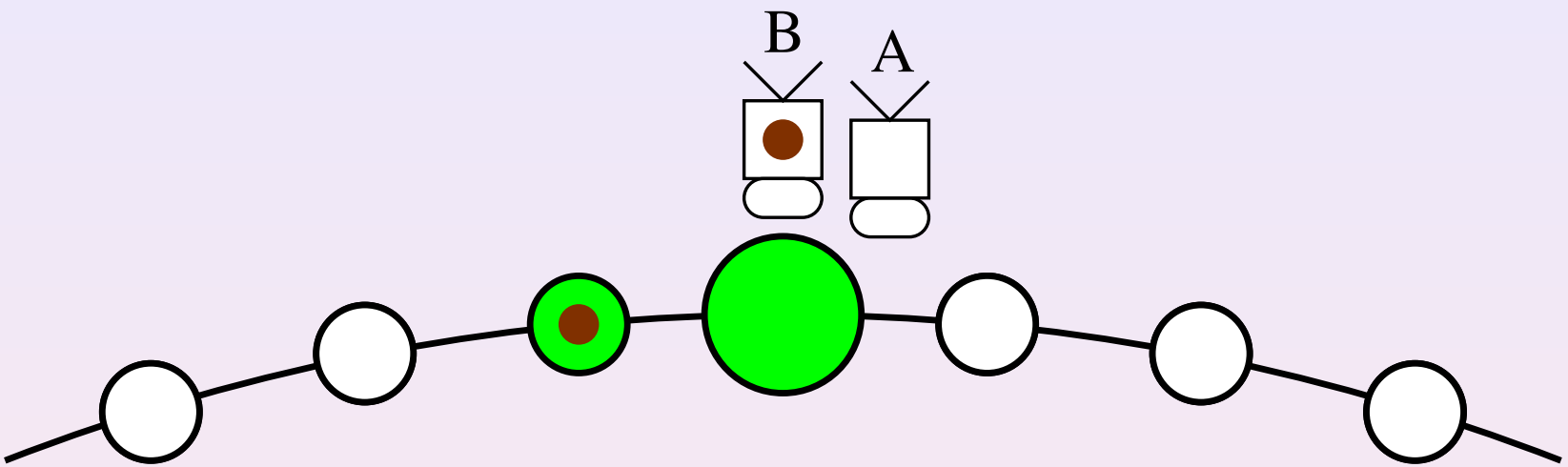


Quick description

If the agent "controls"

- 1 pebble $\longrightarrow$ Simple cautious walk to the right
- 2 pebbles $\longrightarrow$ Double cautious walk to the left
- 0 pebbles $\longrightarrow$ Non-cautious walk to the left

Ping Pong algorithm

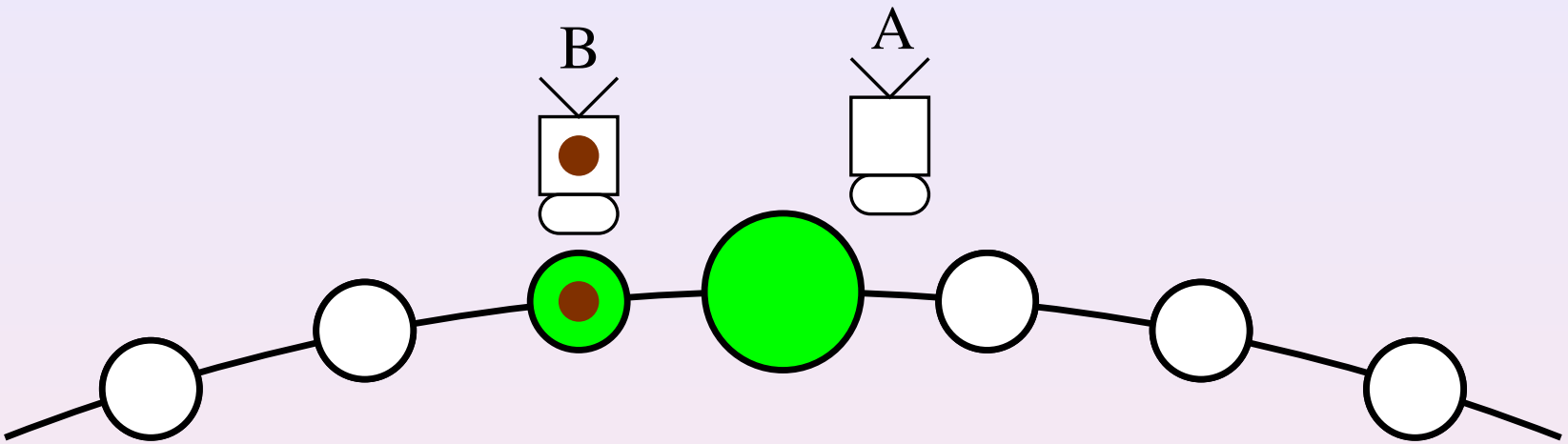


Quick description

If the agent "controls"

- 1 pebble $\longrightarrow$ Simple cautious walk to the right
- 2 pebbles $\longrightarrow$ Double cautious walk to the left
- 0 pebbles $\longrightarrow$ Non-cautious walk to the left

Ping Pong algorithm

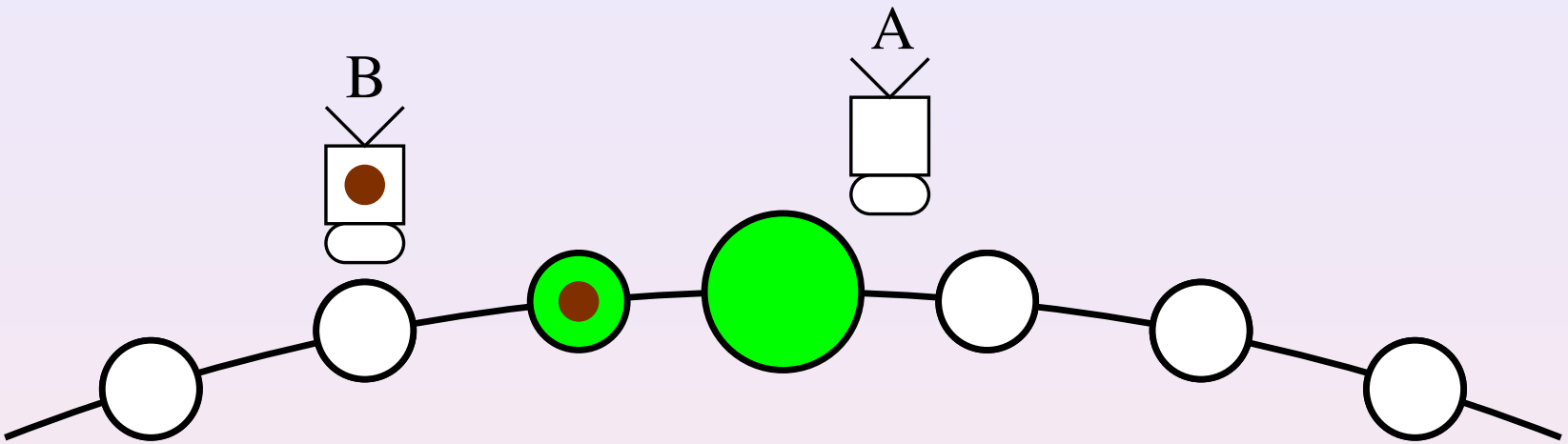


Quick description

If the agent "controls"

- 1 pebble $\longrightarrow$ Simple cautious walk to the right
- 2 pebbles $\longrightarrow$ Double cautious walk to the left
- 0 pebbles $\longrightarrow$ Non-cautious walk to the left

Ping Pong algorithm

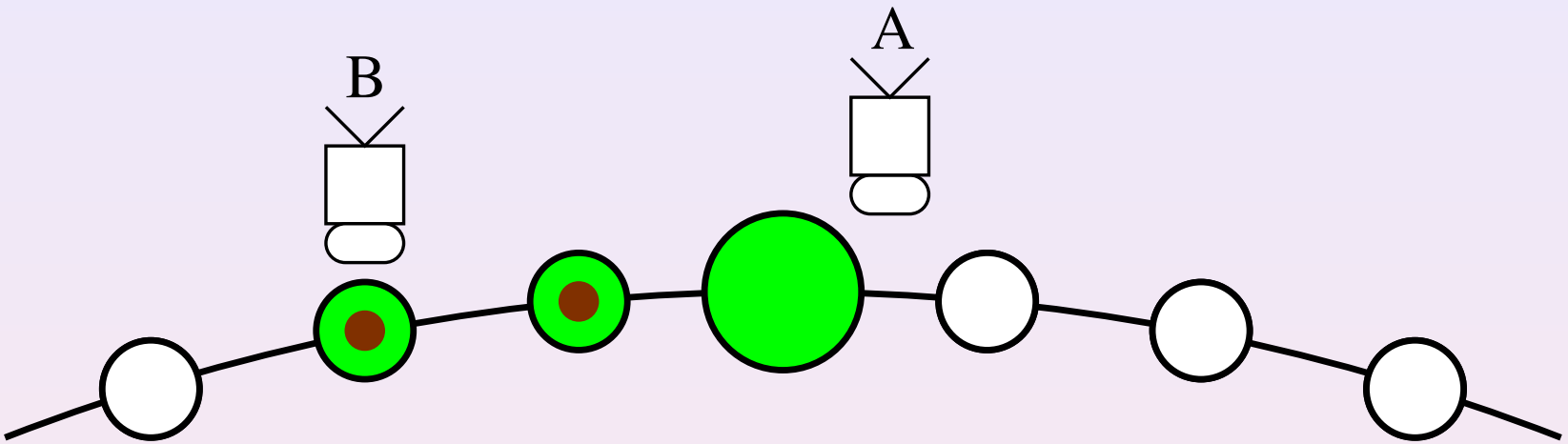


Quick description

If the agent "controls"

- 1 pebble $\longrightarrow$ Simple cautious walk to the right
- 2 pebbles $\longrightarrow$ Double cautious walk to the left
- 0 pebbles $\longrightarrow$ Non-cautious walk to the left

Ping Pong algorithm

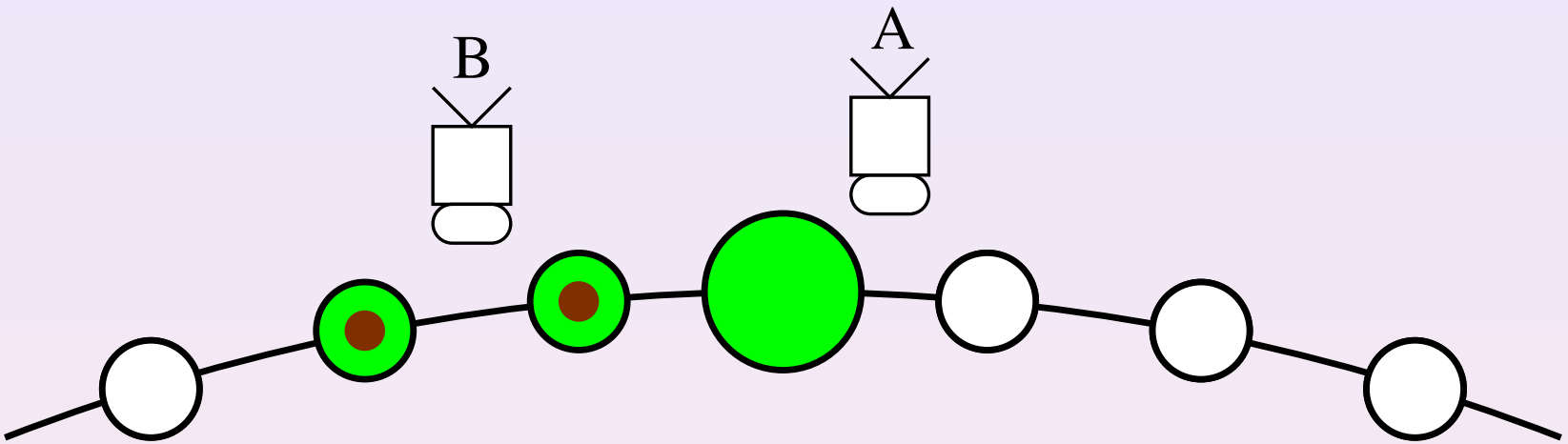


Quick description

If the agent "controls"

- 1 pebble $\longrightarrow$ Simple cautious walk to the right
- 2 pebbles $\longrightarrow$ Double cautious walk to the left
- 0 pebbles $\longrightarrow$ Non-cautious walk to the left

Ping Pong algorithm

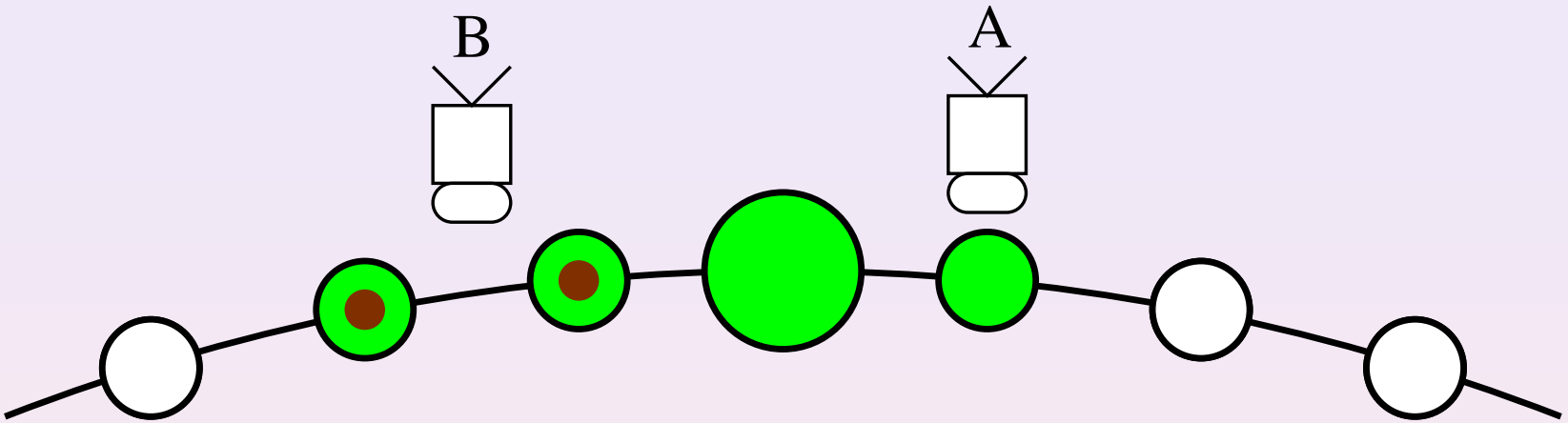


Quick description

If the agent "controls"

- 1 pebble $\longrightarrow$ Simple cautious walk to the right
- 2 pebbles $\longrightarrow$ Double cautious walk to the left
- 0 pebbles $\longrightarrow$ Non-cautious walk to the left

Ping Pong algorithm

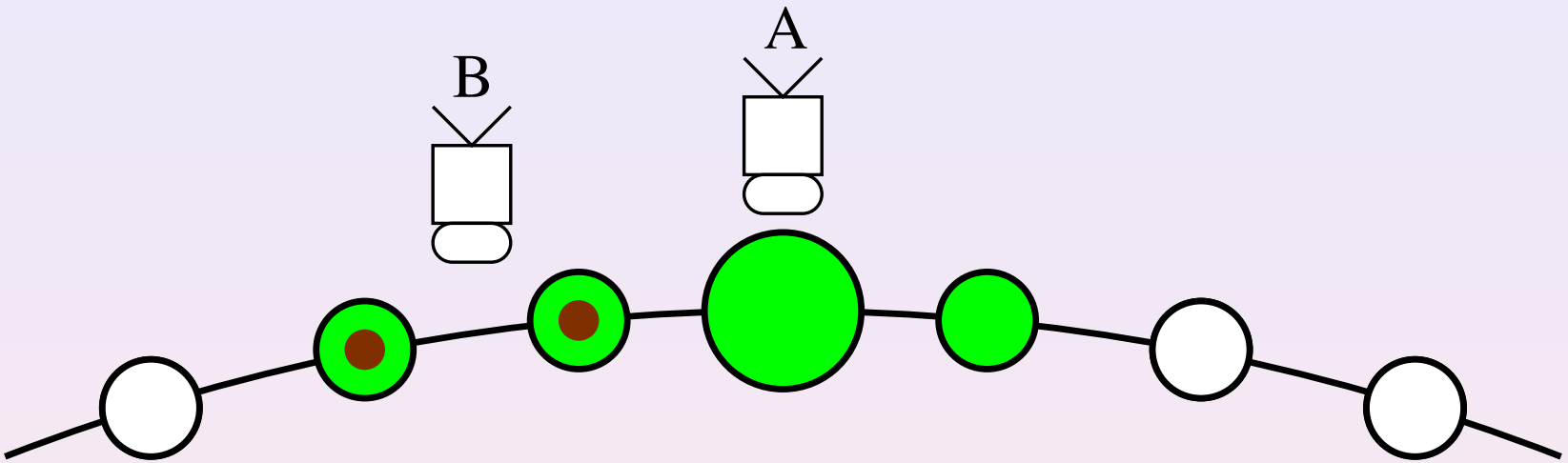


Quick description

If the agent "controls"

- 1 pebble $\longrightarrow$ Simple cautious walk to the right
- 2 pebbles $\longrightarrow$ Double cautious walk to the left
- 0 pebbles $\longrightarrow$ Non-cautious walk to the left

Ping Pong algorithm

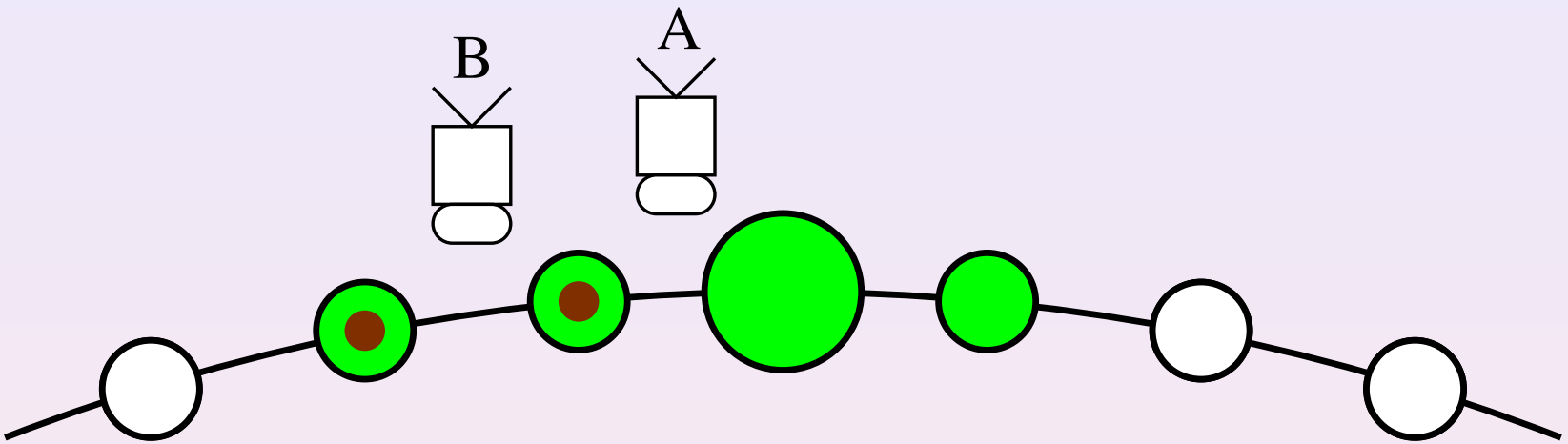


Quick description

If the agent "controls"

- 1 pebble $\longrightarrow$ Simple cautious walk to the right
- 2 pebbles $\longrightarrow$ Double cautious walk to the left
- 0 pebbles $\longrightarrow$ Non-cautious walk to the left

Ping Pong algorithm

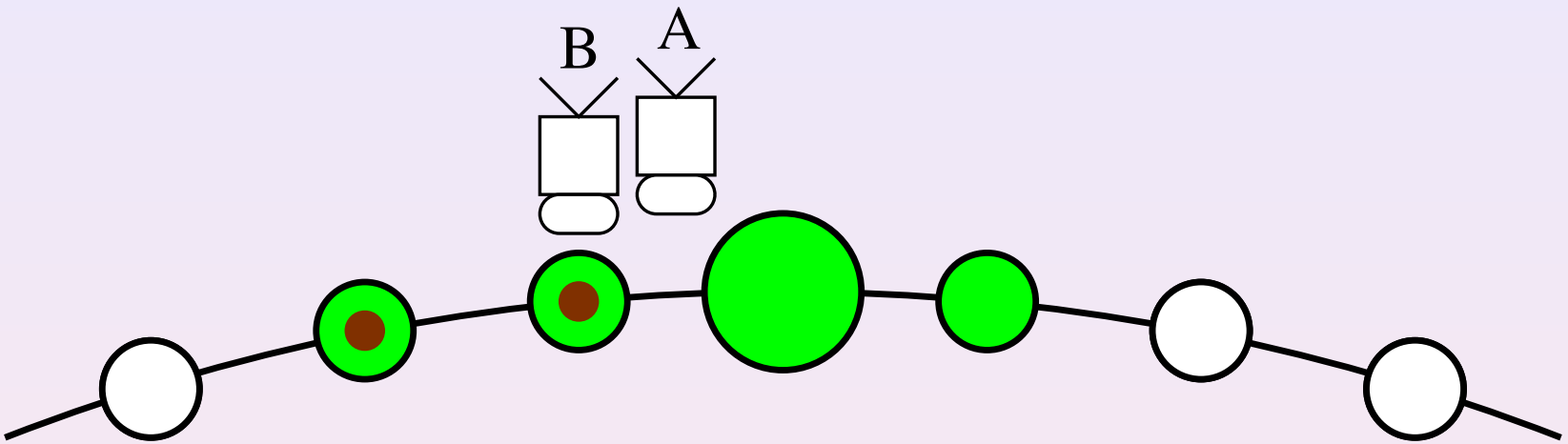


Quick description

If the agent "controls"

- 1 pebble $\longrightarrow$ Simple cautious walk to the right
- 2 pebbles $\longrightarrow$ Double cautious walk to the left
- 0 pebbles $\longrightarrow$ Non-cautious walk to the left

Ping Pong algorithm

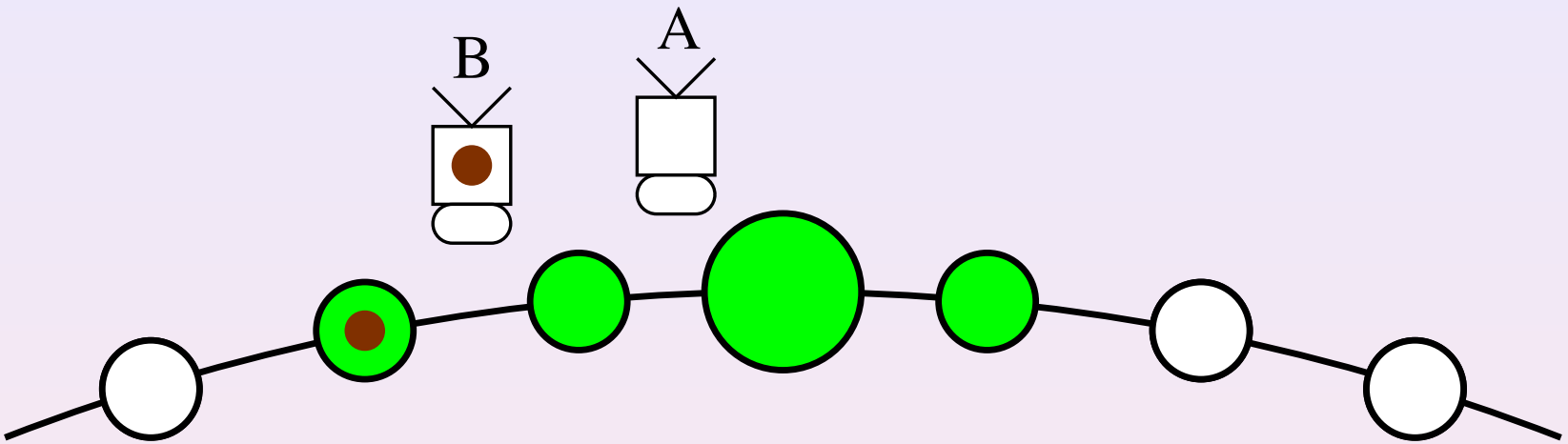


Quick description

If the agent "controls"

- 1 pebble $\longrightarrow$ Simple cautious walk to the right
- 2 pebbles $\longrightarrow$ Double cautious walk to the left
- 0 pebbles $\longrightarrow$ Non-cautious walk to the left

Ping Pong algorithm

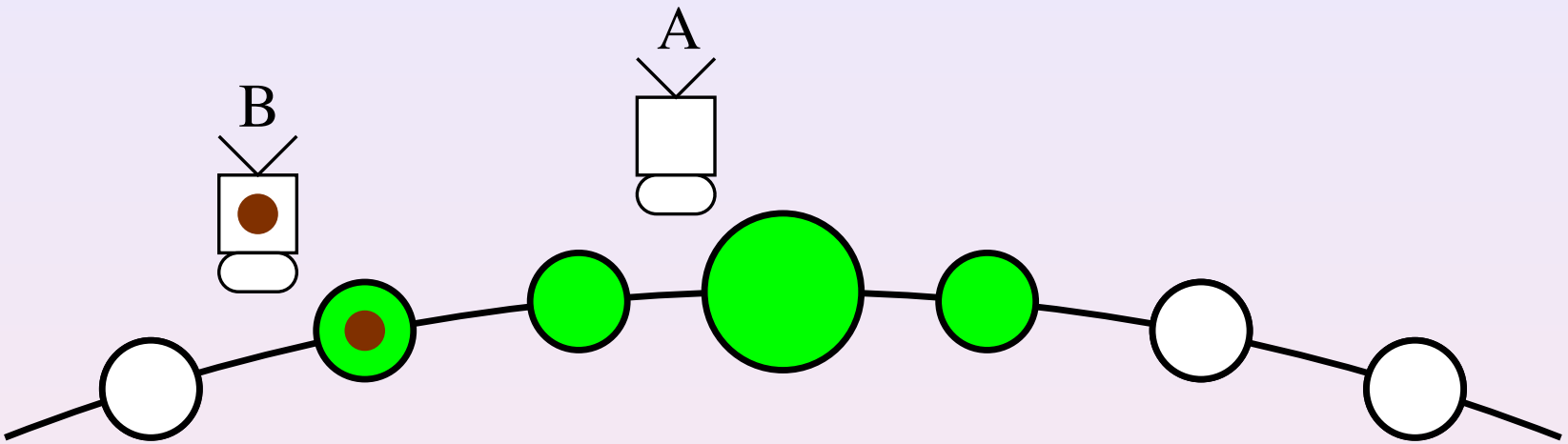


Quick description

If the agent "controls"

- 1 pebble $\longrightarrow$ Simple cautious walk to the right
- 2 pebbles $\longrightarrow$ Double cautious walk to the left
- 0 pebbles $\longrightarrow$ Non-cautious walk to the left

Ping Pong algorithm

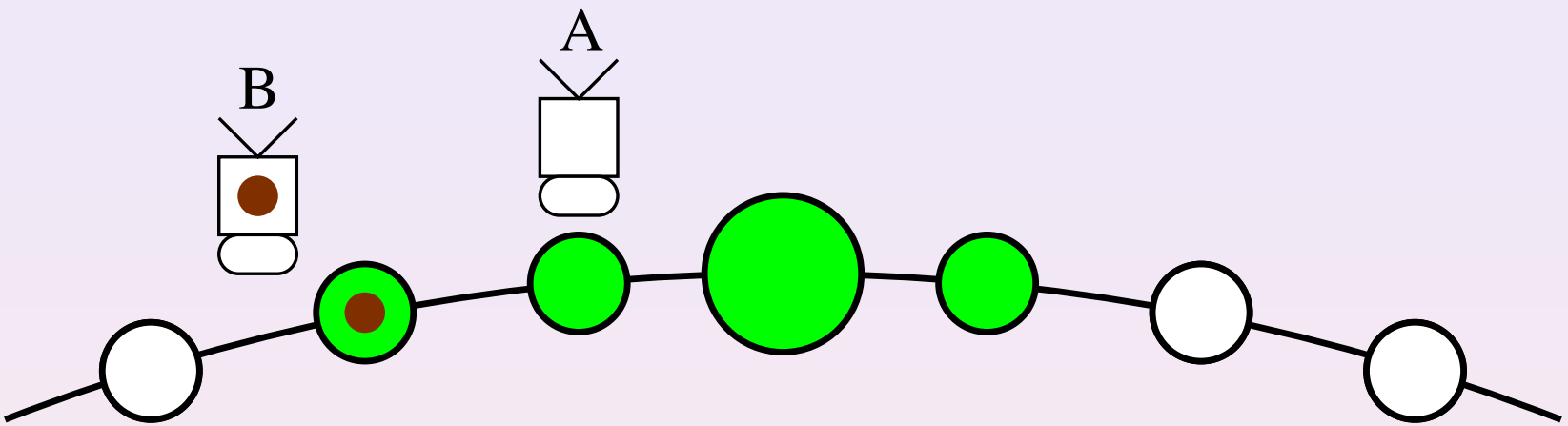


Quick description

If the agent "controls"

- 1 pebble $\longrightarrow$ Simple cautious walk to the right
- 2 pebbles $\longrightarrow$ Double cautious walk to the left
- 0 pebbles $\longrightarrow$ Non-cautious walk to the left

Ping Pong algorithm

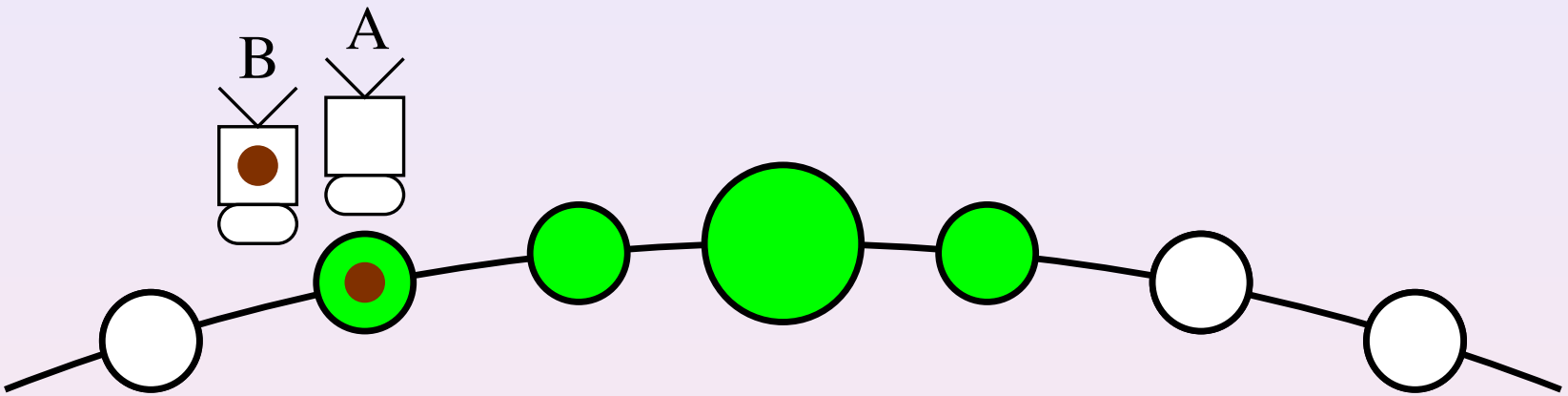


Quick description

If the agent "controls"

- 1 pebble $\longrightarrow$ Simple cautious walk to the right
- 2 pebbles $\longrightarrow$ Double cautious walk to the left
- 0 pebbles $\longrightarrow$ Non-cautious walk to the left

Ping Pong algorithm

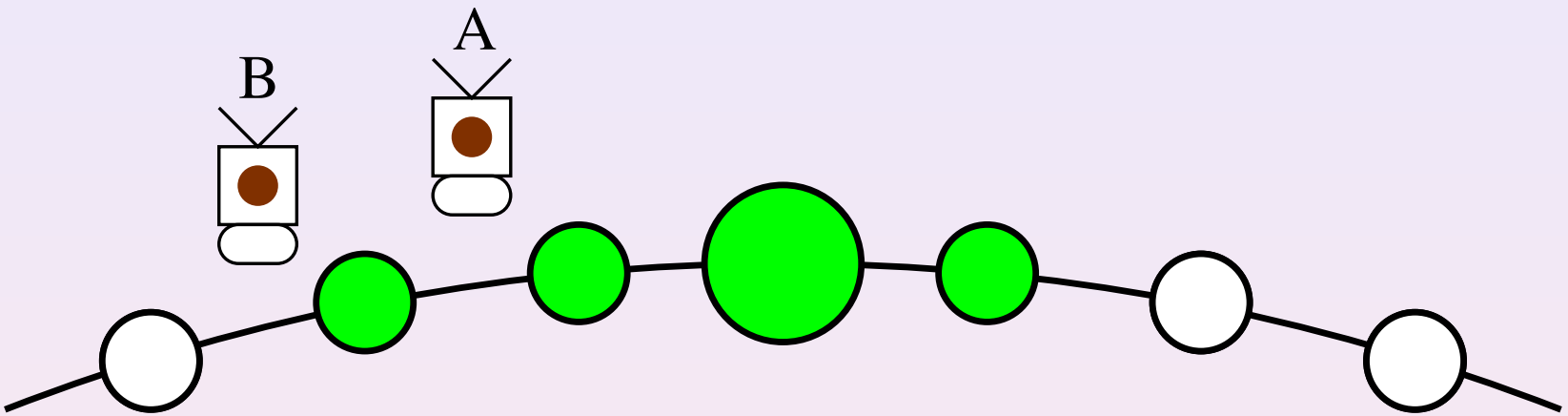


Quick description

If the agent "controls"

- 1 pebble $\longrightarrow$ Simple cautious walk to the right
- 2 pebbles $\longrightarrow$ Double cautious walk to the left
- 0 pebbles $\longrightarrow$ Non-cautious walk to the left

Ping Pong algorithm

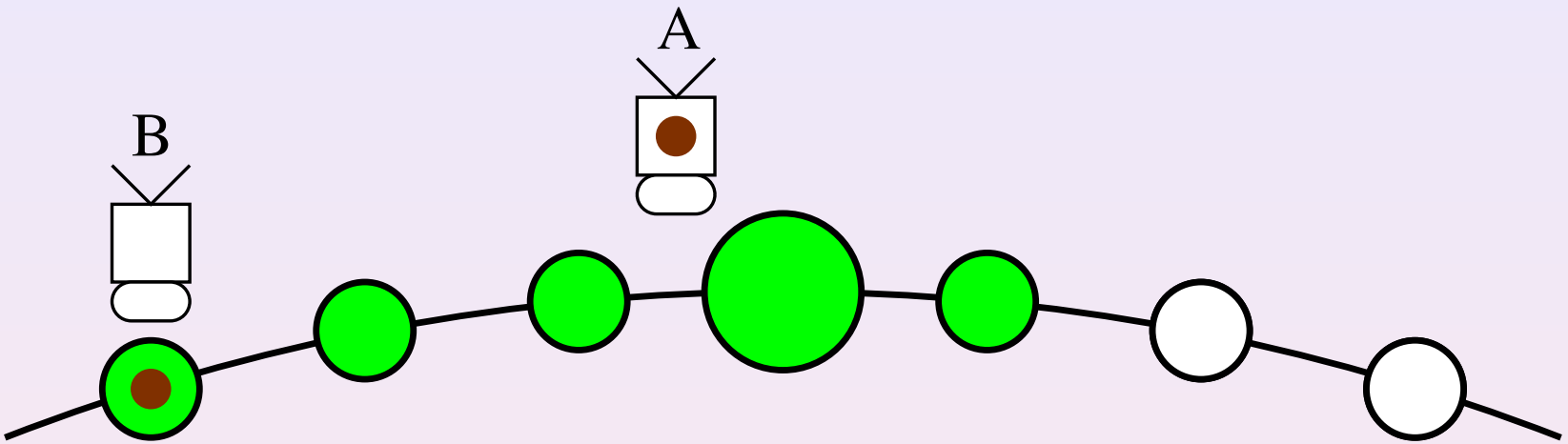


Quick description

If the agent "controls"

- 1 pebble $\longrightarrow$ Simple cautious walk to the right
- 2 pebbles $\longrightarrow$ Double cautious walk to the left
- 0 pebbles $\longrightarrow$ Non-cautious walk to the left

Ping Pong algorithm

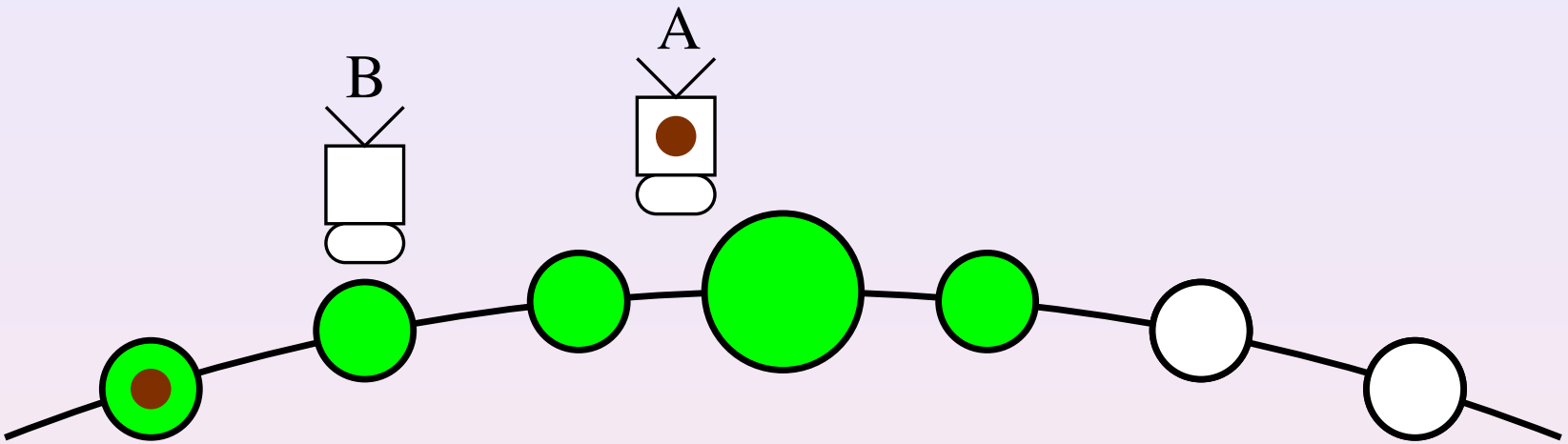


Quick description

If the agent "controls"

- 1 pebble $\longrightarrow$ Simple cautious walk to the right
- 2 pebbles $\longrightarrow$ Double cautious walk to the left
- 0 pebbles $\longrightarrow$ Non-cautious walk to the left

Ping Pong algorithm

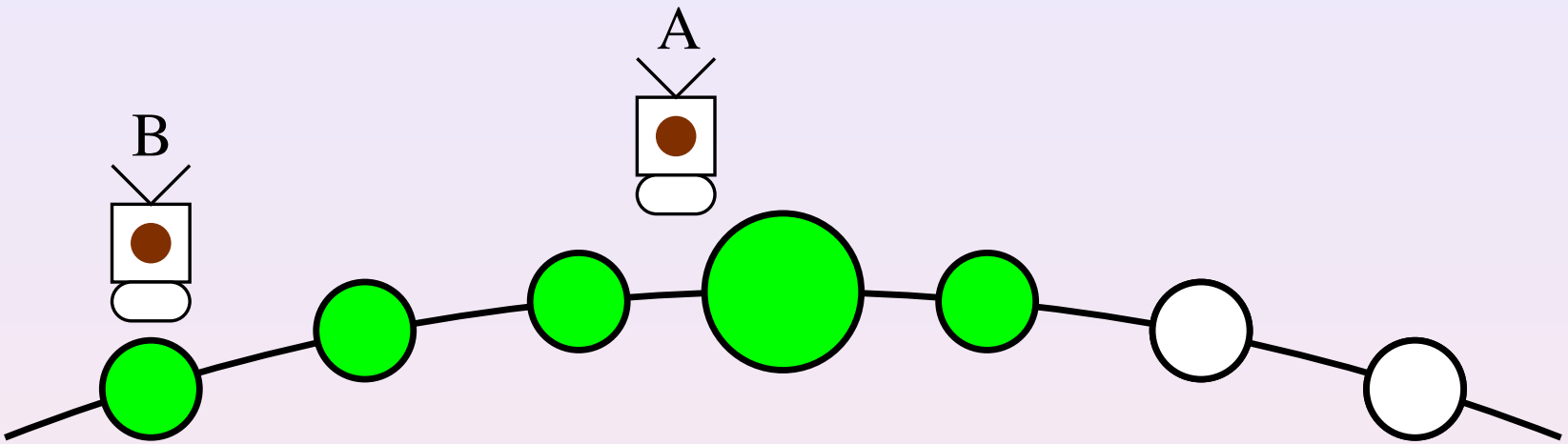


Quick description

If the agent "controls"

- 1 pebble $\longrightarrow$ Simple cautious walk to the right
- 2 pebbles $\longrightarrow$ Double cautious walk to the left
- 0 pebbles $\longrightarrow$ Non-cautious walk to the left

Ping Pong algorithm

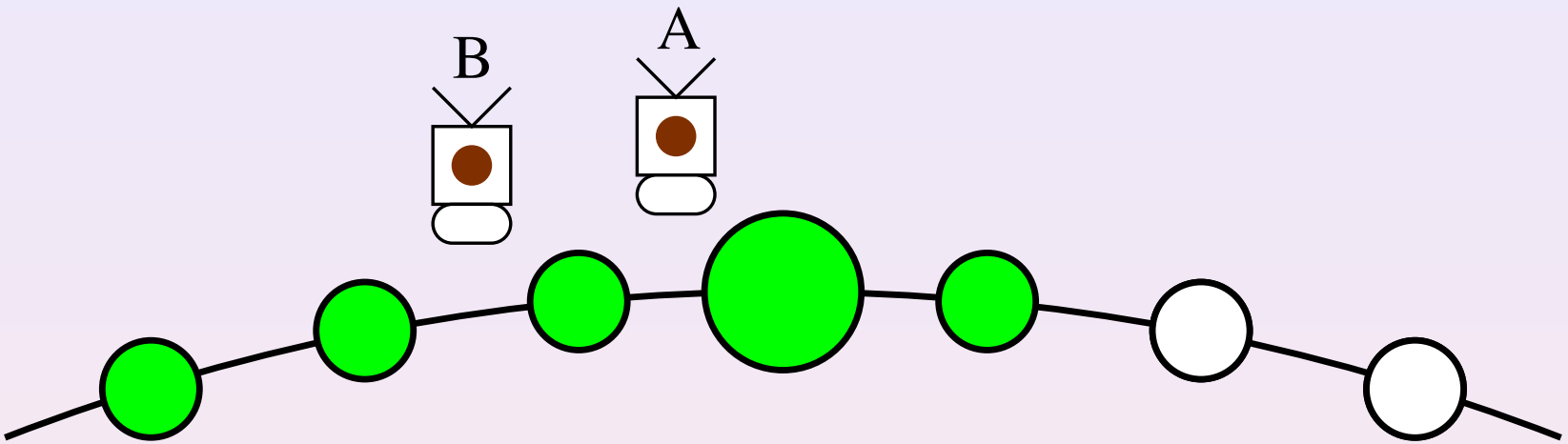


Quick description

If the agent "controls"

- 1 pebble $\longrightarrow$ Simple cautious walk to the right
- 2 pebbles $\longrightarrow$ Double cautious walk to the left
- 0 pebbles $\longrightarrow$ Non-cautious walk to the left

Ping Pong algorithm

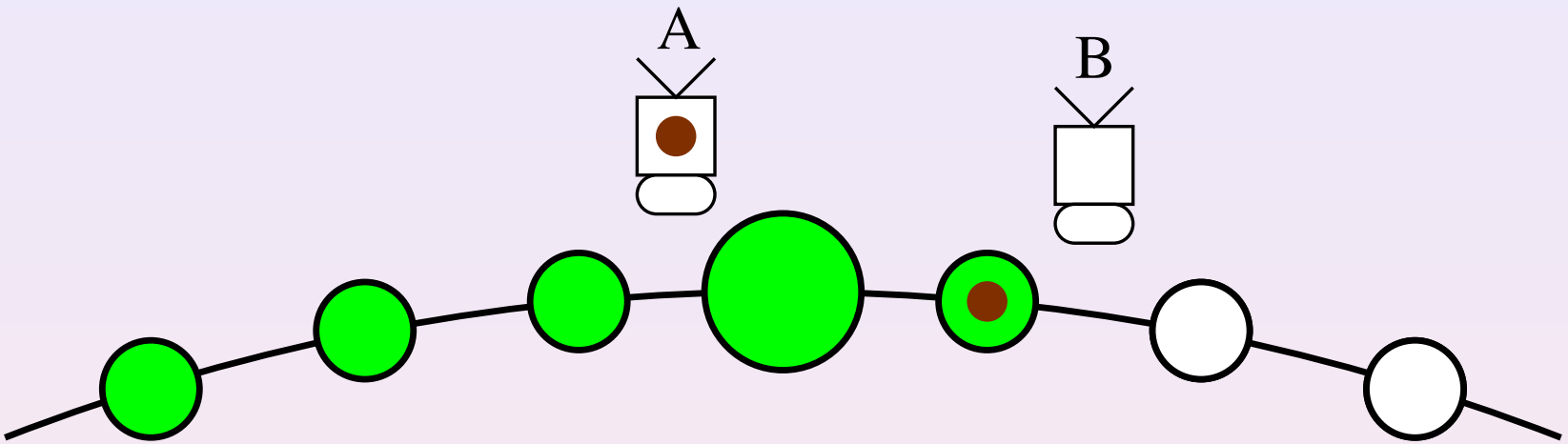


Quick description

If the agent "controls"

- 1 pebble $\longrightarrow$ Simple cautious walk to the right
- 2 pebbles $\longrightarrow$ Double cautious walk to the left
- 0 pebbles $\longrightarrow$ Non-cautious walk to the left

Ping Pong algorithm

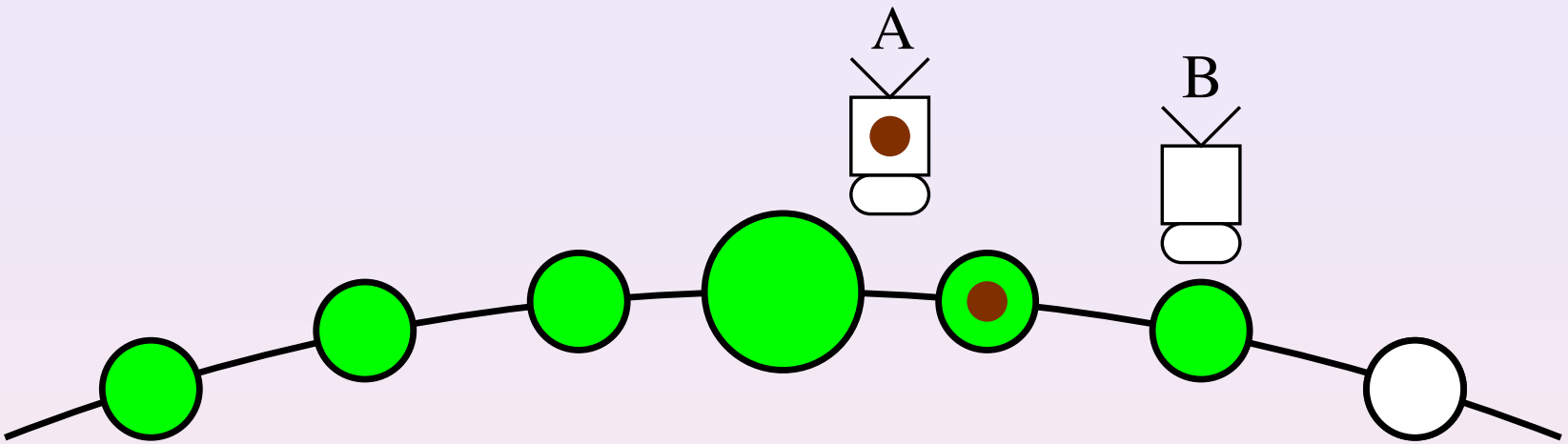


Quick description

If the agent "controls"

- 1 pebble $\longrightarrow$ Simple cautious walk to the right
- 2 pebbles $\longrightarrow$ Double cautious walk to the left
- 0 pebbles $\longrightarrow$ Non-cautious walk to the left

Ping Pong algorithm

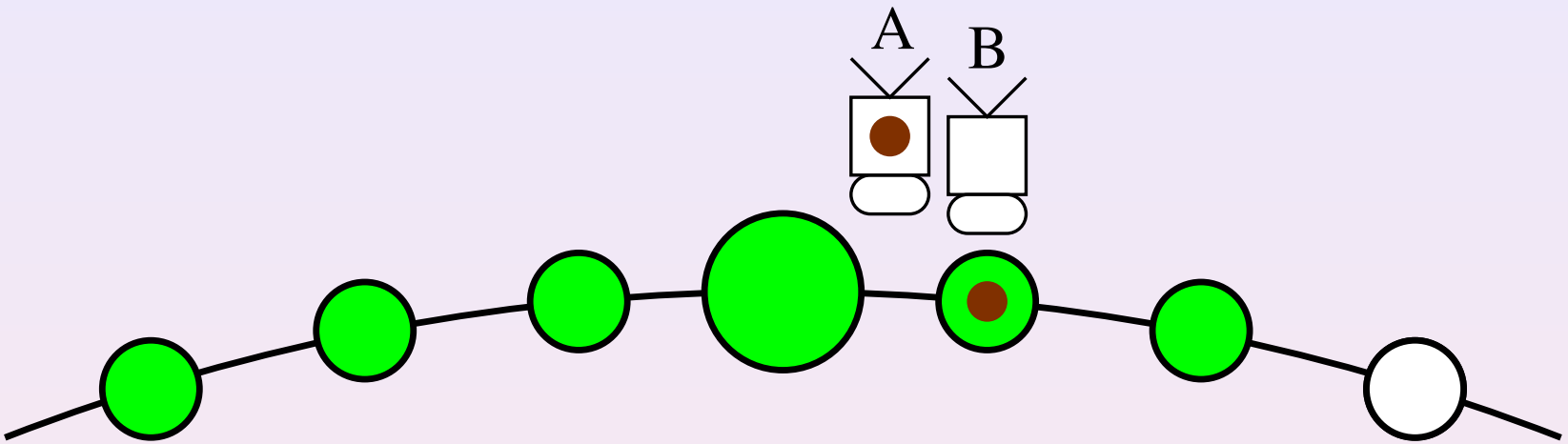


Quick description

If the agent "controls"

- 1 pebble $\longrightarrow$ Simple cautious walk to the right
- 2 pebbles $\longrightarrow$ Double cautious walk to the left
- 0 pebbles $\longrightarrow$ Non-cautious walk to the left

Ping Pong algorithm

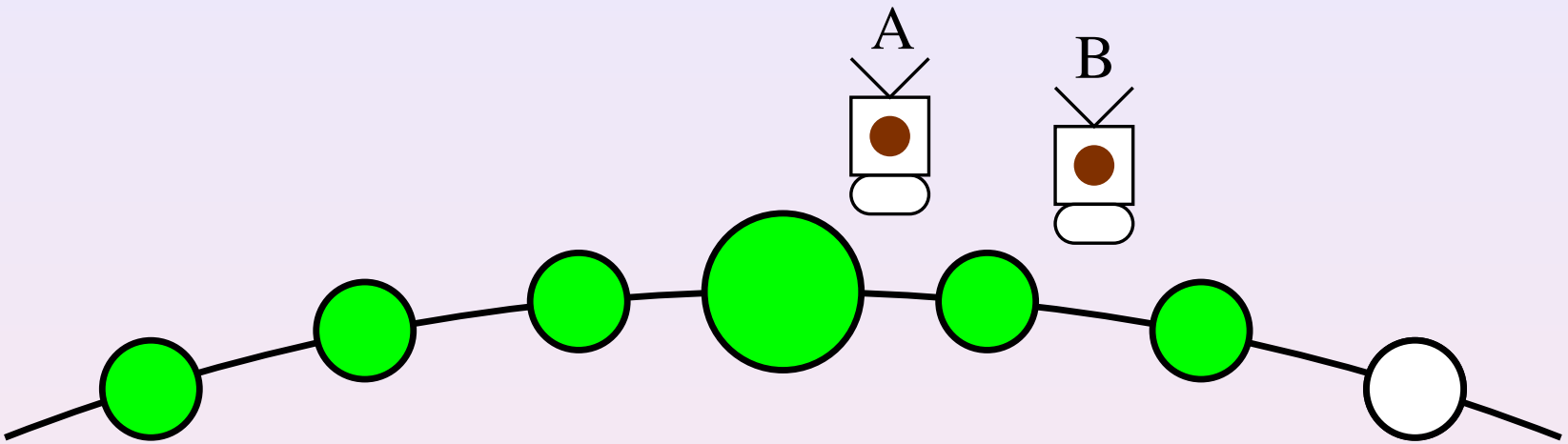


Quick description

If the agent "controls"

- 1 pebble $\longrightarrow$ Simple cautious walk to the right
- 2 pebbles $\longrightarrow$ Double cautious walk to the left
- 0 pebbles $\longrightarrow$ Non-cautious walk to the left

Ping Pong algorithm

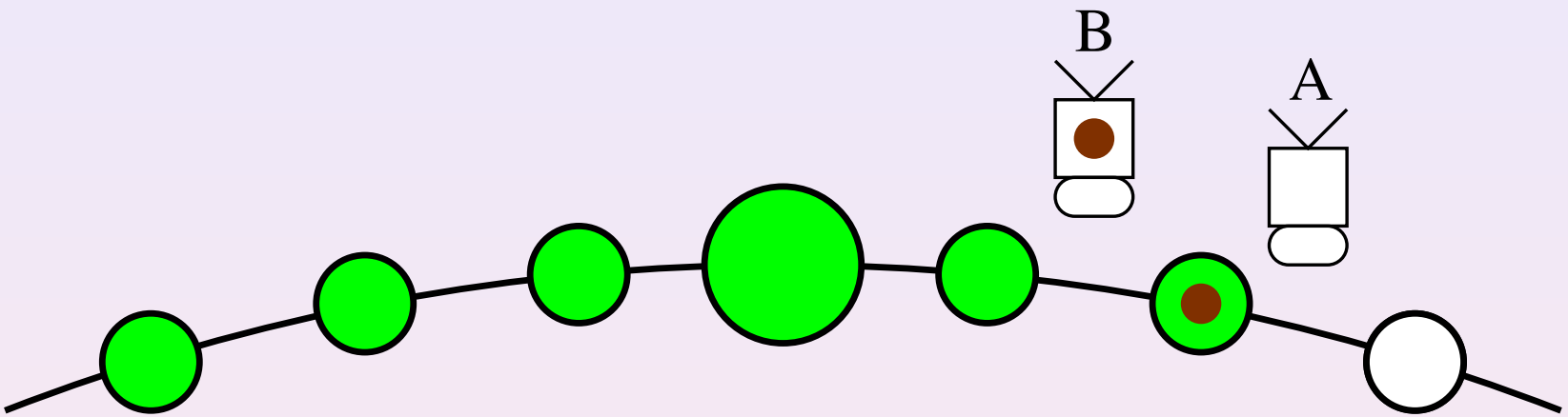


Quick description

If the agent "controls"

- 1 pebble $\longrightarrow$ Simple cautious walk to the right
- 2 pebbles $\longrightarrow$ Double cautious walk to the left
- 0 pebbles $\longrightarrow$ Non-cautious walk to the left

Ping Pong algorithm



Quick description

If the agent "controls"

- 1 pebble $\longrightarrow$ Simple cautious walk to the right
- 2 pebbles $\longrightarrow$ Double cautious walk to the left
- 0 pebbles $\longrightarrow$ Non-cautious walk to the left

Ping Pong algorithm

$\Rightarrow$ The algorithm is correct and uses $O(n^2)$ moves

Quick description

If the agent "controls"

- 1 pebble $\longrightarrow$ Simple cautious walk to the right
- 2 pebbles $\longrightarrow$ Double cautious walk to the left
- 0 pebbles $\longrightarrow$ Non-cautious walk to the left

Enhanced Ping Pong algorithm

Quick description

- Execute Ping Pong **until** agents "meet" at least two/three nodes from the homebase (until the safe zone becomes sufficiently large)
- **While** still an unexplored node **do**
- Divide the unexplored part in two
- Each agent explores an half
- The first agent to finish its part goes and helps the other agent to explore the other part
- **When** an agent finds the pebble of the other agent
  - it moves the pebble by one node toward the center
  - and restart the loop

Analysis of Enhanced Ping Pong

Analysis

Between two halving

- Roughly half of the unexplored nodes are explored
- Agents do at most $O(n)$ moves

Result

- 1 pebble per agent
- 2 agents
- $\Theta(n \log n)$ moves

Analysis of Enhanced Ping Pong

Analysis

Between two halving

- Roughly half of the unexplored nodes are explored
- Agents do at most $O(n)$ moves

Result

- 1 pebble per agent
- 2 agents
- $\Theta(n \log n)$ moves

Conclusion and perspectives

Conclusion

Equivalence, for Black Hole Search, between

- Coordination through whiteboards
- Coordination through pebbles

Perspectives

For which other mobile agents problems is this equivalence true?