

Distributed Computing with Advice: Information Sensitivity of Graph Coloring

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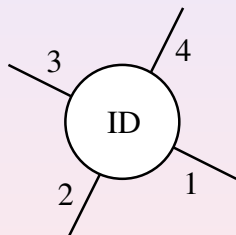
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Node's knowledge

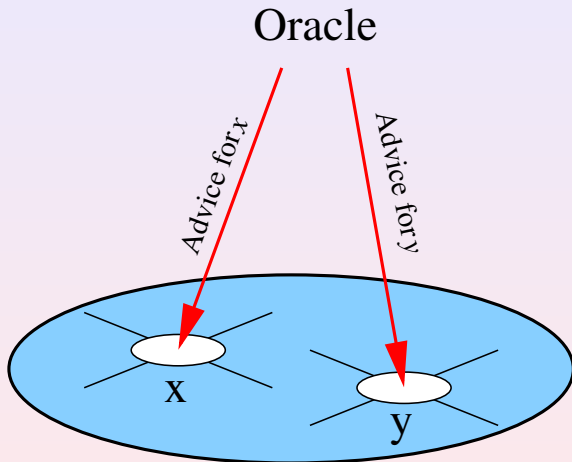
Local knowledge



Global knowledge

- number of nodes
- network diameter
- maximum degree
- genus, girth, treewidth, etc.

Global knowledge given by an oracle



Advice for node x of graph $G =$ a string of bits $\mathcal{O}(G, x)$.

Distributed computing with advice

Design of both

the oracle \mathcal{O}

- takes the whole graph G as input
- returns advice $\mathcal{O}(G, x)$ to every node x

the algorithm \mathcal{A}

- **executed locally** by every node
- **using** the pieces of **advice** given by the oracle

Size of advice

- Size of advice = **total amount** of advice given to the nodes
- Size of advice given by \mathcal{O} on graph G :

$$\sum_{x \in V(G)} |\mathcal{O}(G, x)|$$

- Worst case over all n -node graphs

Performance tradeoffs

Quantitative tradeoffs

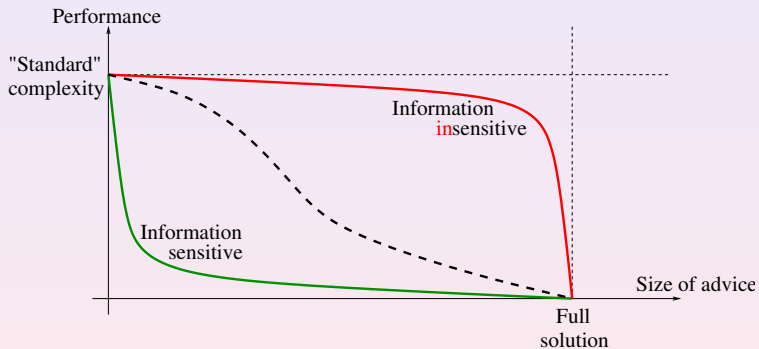
Size of advice versus Performance of the algorithm regardless of what kind of knowledge is supplied.

Observation

The quality of the algorithmic solutions often depends on the amount of knowledge that is given about the network.

- [Lynch, PODC 1989]: A hundred impossibility proofs for distributed computing.
- [Fich, Ruppert, Distributed Computing, 2003]: Hundreds of impossibility results for distributed computing.

Information sensitivity



Computing with advice project

Distributed computing

- Broadcast and wakeup [Fraigniaud, I., Pelc, PODC 2006]
- Minimum spanning tree [Fraigniaud, Korman, Lebhar, SPAA 2007]

Mobile computing

- Graph exploration [Fraigniaud, I., Pelc, MFCS 2006]
- Graph searching [Nisse, Soguet, SIROCCO 2007]

Related work: Information labelings

f -labeling (L, D)

Assign short labels to nodes so that f can be computed based solely on the labels:

$$f(x, y) = D(L(x), L(y))$$

Examples: routing, distance, adjacency, connectivity,...

Linial's model [Linial, SIAM J. Comp., 1992]
(a.k.a., *LOCAL* model [Peleg, 2000])

- At each round, a node
 - sends a message to each of its neighbors
 - receives a message from each of its neighbors
 - performs some local computations
- No restrictions on
 - the message size
 - the local computations

Designed to estimate **limitations of Local computing**.

Local computing

- What **can** be computed Locally?
[Naor, Stockmeyer, STOC 1993]
- What **cannot** be computed Locally?
[Kuhn, Moscibroda, Wattenhofer, PODC 2004]
- **If not**, what **amount of information (advice)** must be given so that Local computing becomes doable?

Distributed coloring

Goal

Obtain a **proper node-coloring** in a **distributed** way.

Known results [Linial, SIAM J. Comp., 1992]
(and [Goldberg, Plotkin, STOC 1987])

Time of 3-coloring

- of **rings**: $\Theta(\log^* n)$
- of **rooted trees**: $\Theta(\log^* n)$
- of **“unoriented” trees**: $\Theta(D)$

Our results (rings)

Theorem 1

3-coloring of all rings of size n in **time $o(\log^* n)$** requires **advice for $\Omega(n/\log^{(k)} n)$ nodes**, for any constant k .

Corollary

3-coloring of rings is **information insensitive**

Theorem 2

3-coloring of all rings of size n in **constant time** requires **advice for $\Omega(n)$ nodes**.

Our results (trees)

Theorem 3

3-coloring of all n -node unoriented trees in time $O(\log^* n)$ requires advice for $\Omega(n/\log^{(k)} n)$ nodes, for any constant k .

Theorem 4

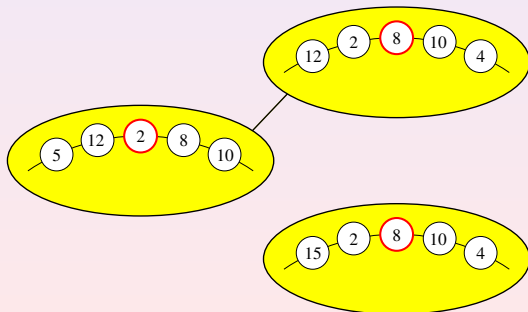
3-coloring of all n -node trees in constant time requires advice for $\Omega(n)$ nodes.

Neighborhood graph

Neighborhood graph $N_t(G)$ of G for t rounds

- **Vertices:** map gathered by some node of (a labeled version of) G in t rounds
- **Edges:** between two nodes with “compatible” maps

Example: $N_2(C_n)$



Property of the neighborhood graph

Property

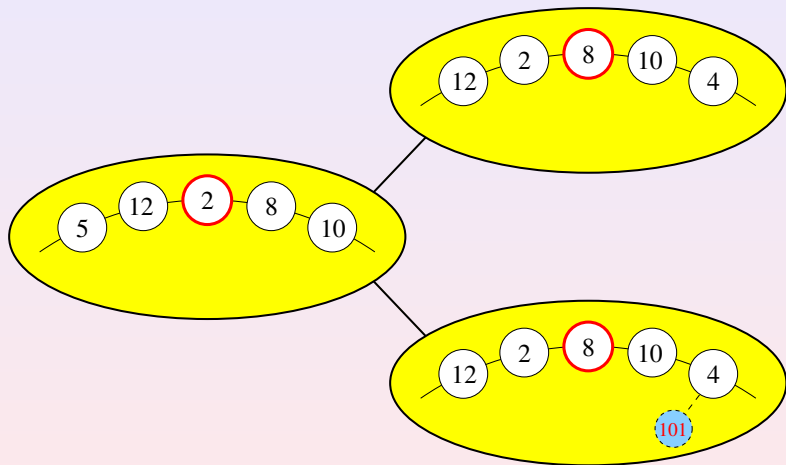
The minimum number c such that there exists a c -coloring algorithm for graph G working in t rounds is the **chromatic number of $N_t(G)$** .

Example of the ring [Linial, SIAM J. Comp., 1992]

$$\chi(N_t(C_n)) \geq \log^{(2t)}(n)$$

\implies time of 3-coloring rings is $\Omega(\log^* n)$

Neighborhood graph with advice



Overview of the proof

- Any uninformed segment of length $2t + 2$ contributes to a “classical” edge.
- t is small.
- If the number of informed nodes is small, then there are a lot of “classical” edges.
- Thus there exists a structured subgraph of high chromatic number.

What is the **information sensitivity** of distributed $(\Delta + 1)$ -coloring of **arbitrary graphs**?

- What is the size of advice for $\Theta(\log^* n)$ time ?
Conjecture: $\Theta(n)$ bits
- What is the size of advice for constant time ?
Conjecture: $\Theta(n \log(\Delta))$ bits