Setting Port Numbers for Fast Graph Exploration

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Graph exploration

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A mobile entity has to visit every node of an unknown anonymous graph.

Periodic exploration by a finite automaton

A finite automaton has to visit every node infinitely often.

Performance criterion

To minimize the period

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Motivations (1)

Exploration by mobile agents

- Physical robot: exploration of environments unreachable by humans
- Software agent: network maintenance

Equivalence between logic and automata [Engelfriet, Hoogeboom, STACS 2006]

Through characterization of string, tree or graph languages

- Automata with nested pebbles
- First-order logic with transitive closure

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Motivations (2)

USTCON (undirected st-connectivity)

- $G = \{V, E\}$ an undirected graph
- $s, t \in V$ two vertices of G

Are s and t in the same connected component of G?

- L = class of problems solvable by deterministic log-space computations
- SL ($\supseteq$ L) = class of problems solvable by symmetric non-deterministic log-space computations

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Reingold, STOC 2005
Undirected ST-Connectivity in Log-Space

USTCON $\in$ L $\Rightarrow$ SL=L

Unknown, anonymous

Unknown

- Unknown topology
- Unknown size (no upper bound)

Anonymous

- No node labeling
- Local edge labeling

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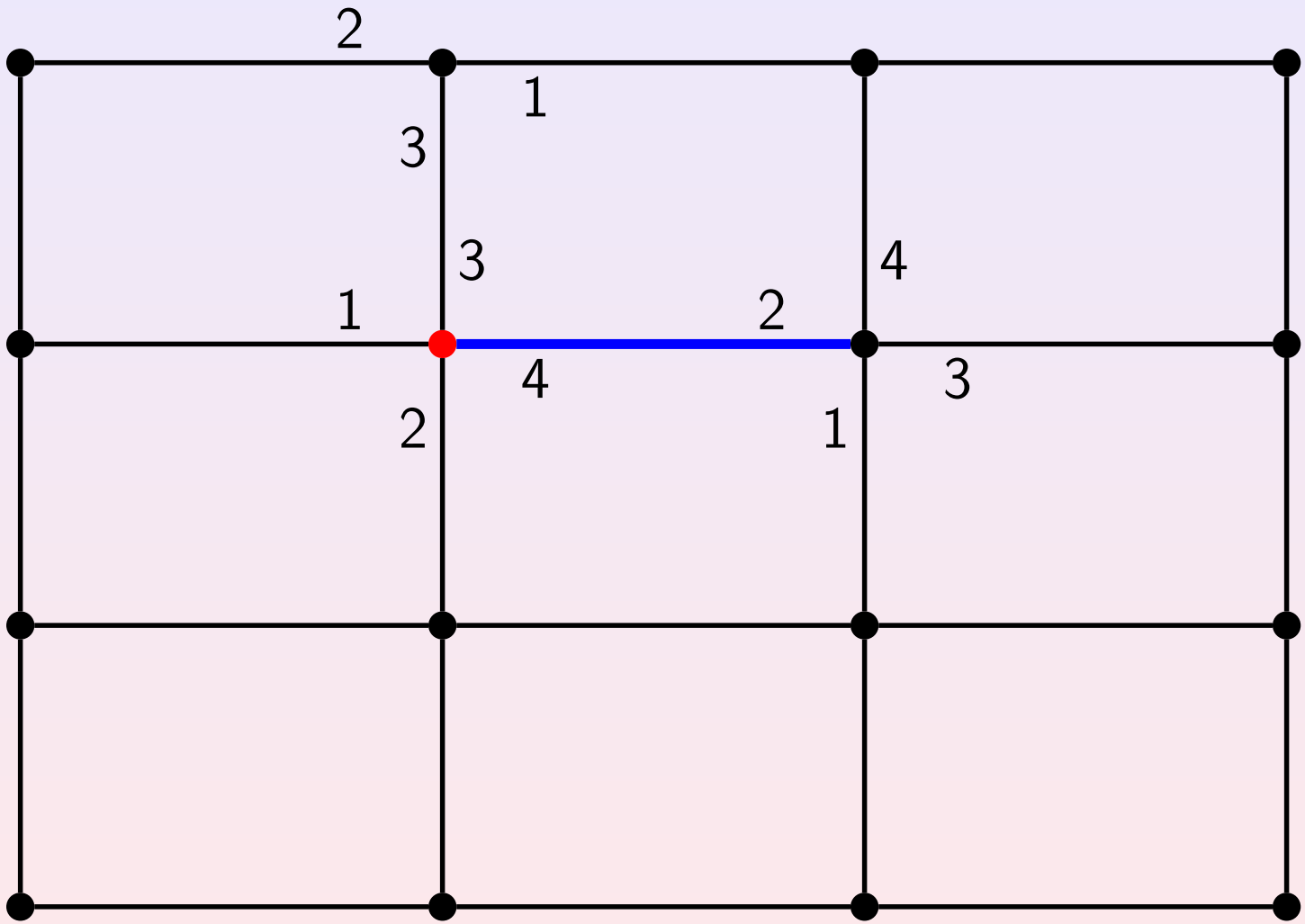
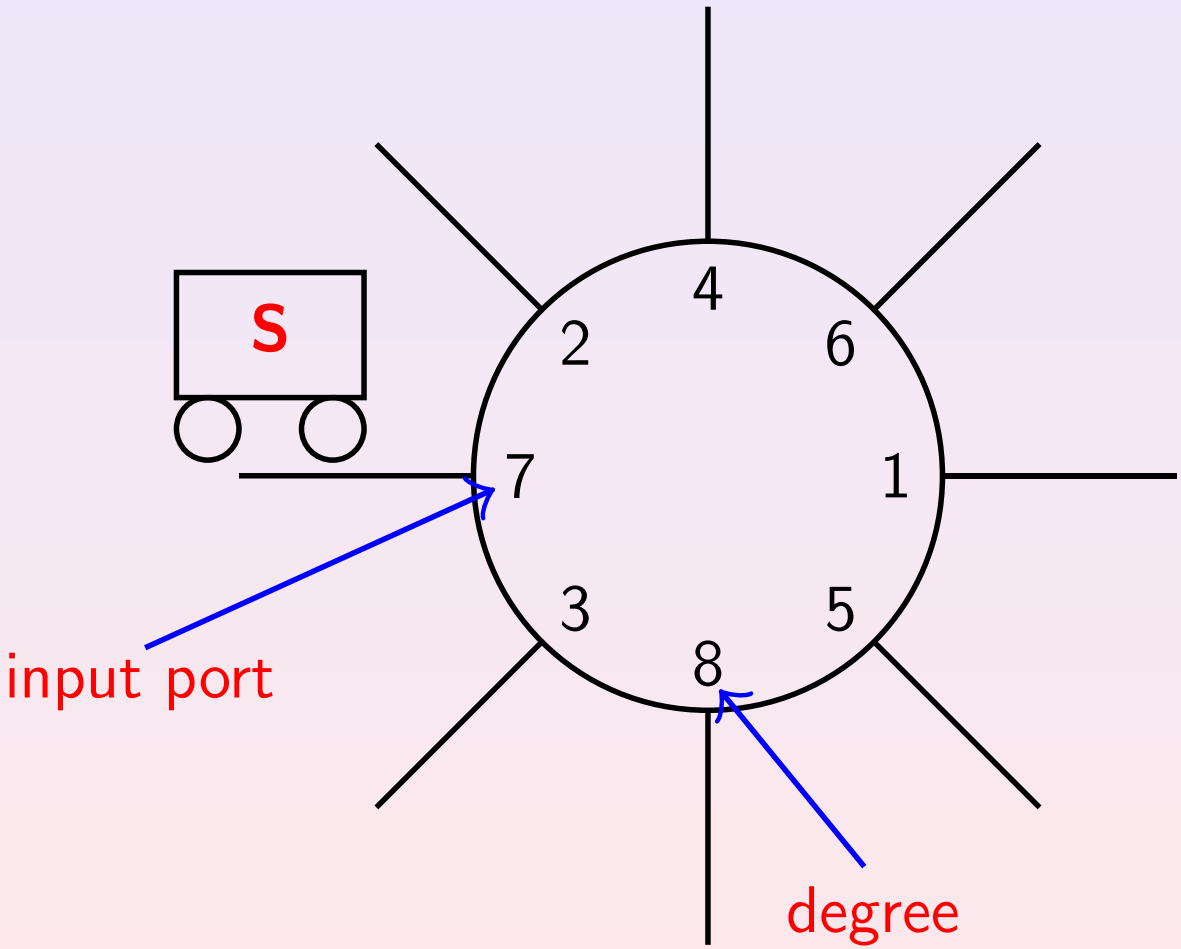
Example of an anonymous graph



Mealy automaton (1)



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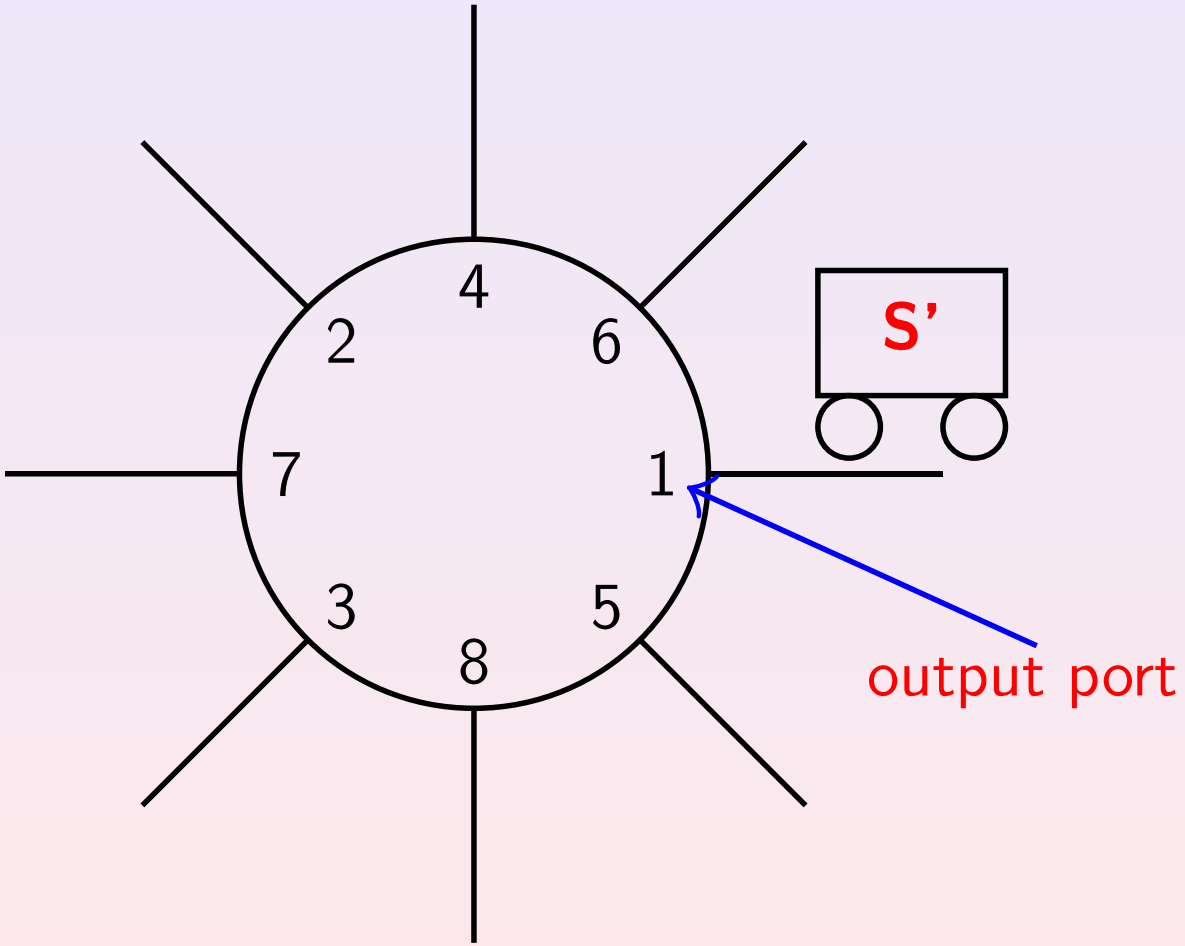

Mealy automaton (2)

Input

- S : current state
- i : input port number
- d : node's degree

Output

- S' : new state
- j : output port number

Transition function

- $f(S, i, d) = (S', j)$

Outline

Impossibility results (1)

Budach, Math. Nachrichten, 1978
Automata and Labyrinths

No finite automaton can explore all graphs.

A pebble is a node-marker that can be dropped at and removed from nodes.

Rabin, Seminar talk at Berkeley, 1967
Maze threading automata

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Automaten in planaren Graphen

No finite team of finite cooperative automata can explore all (cubic planar) graphs.

A JAG (Jumping Automaton for Graphs) is a team of finite automata that cooperate constantly. Moreover an automaton can jump to a vertex occupied by another automaton.

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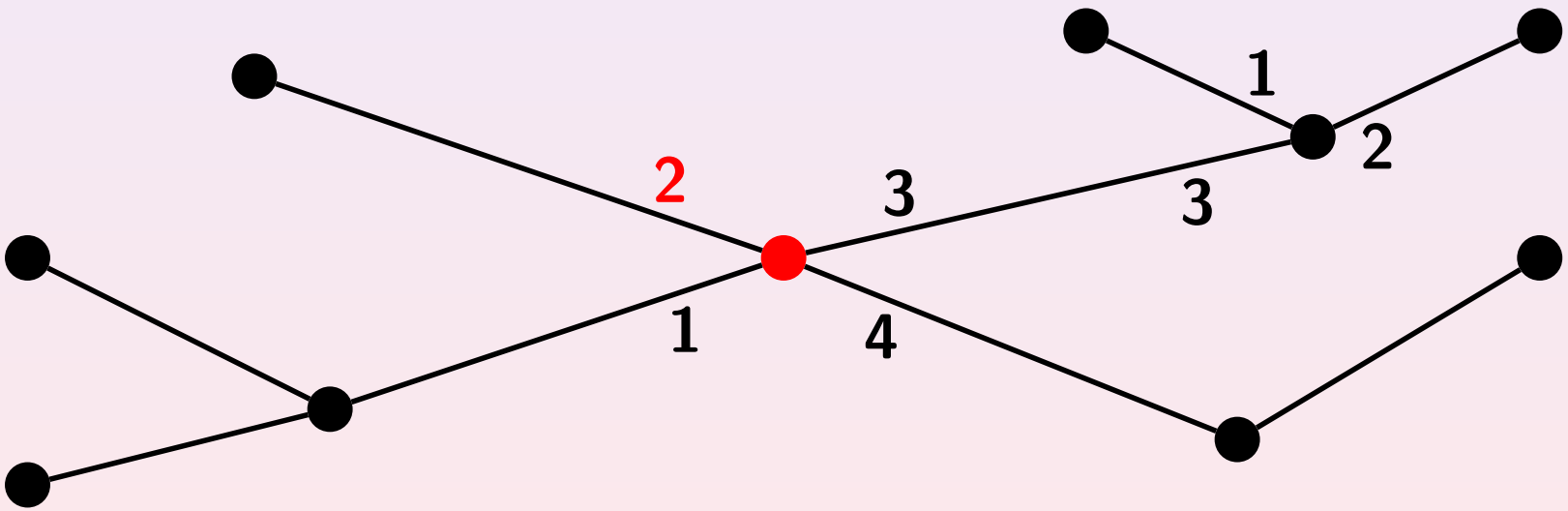
Universality for trees

An oblivious automaton (one single state) using the right-hand-on-the-wall rule ($i \mapsto i + 1$) explores all the trees.



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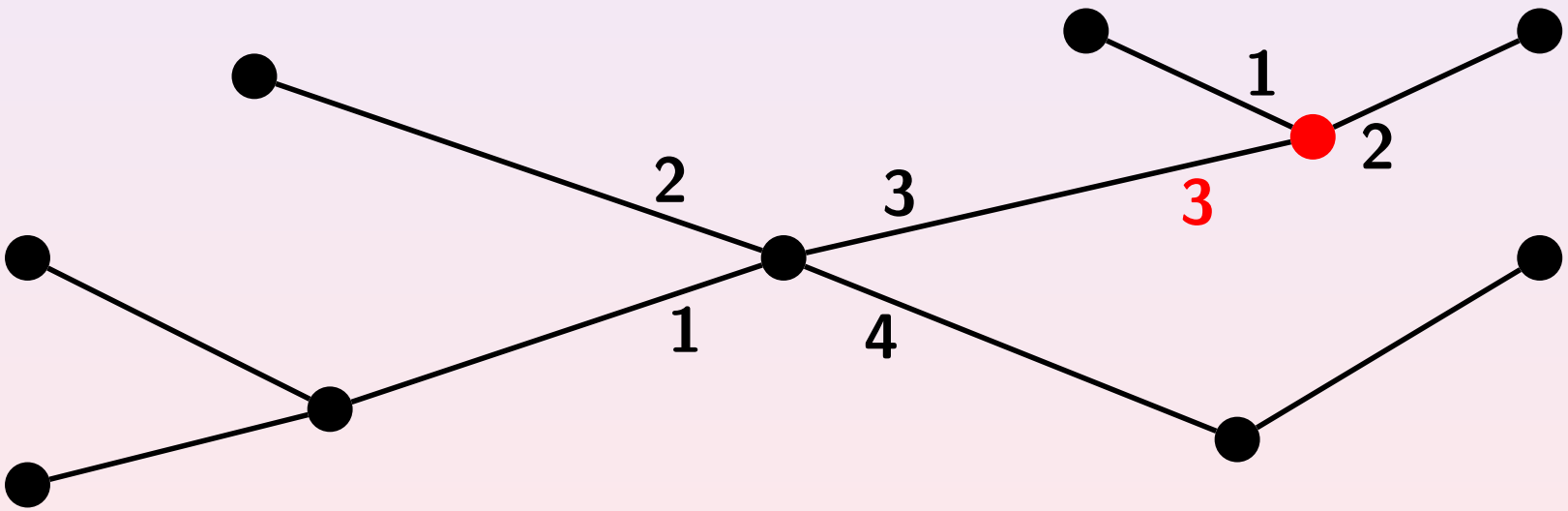

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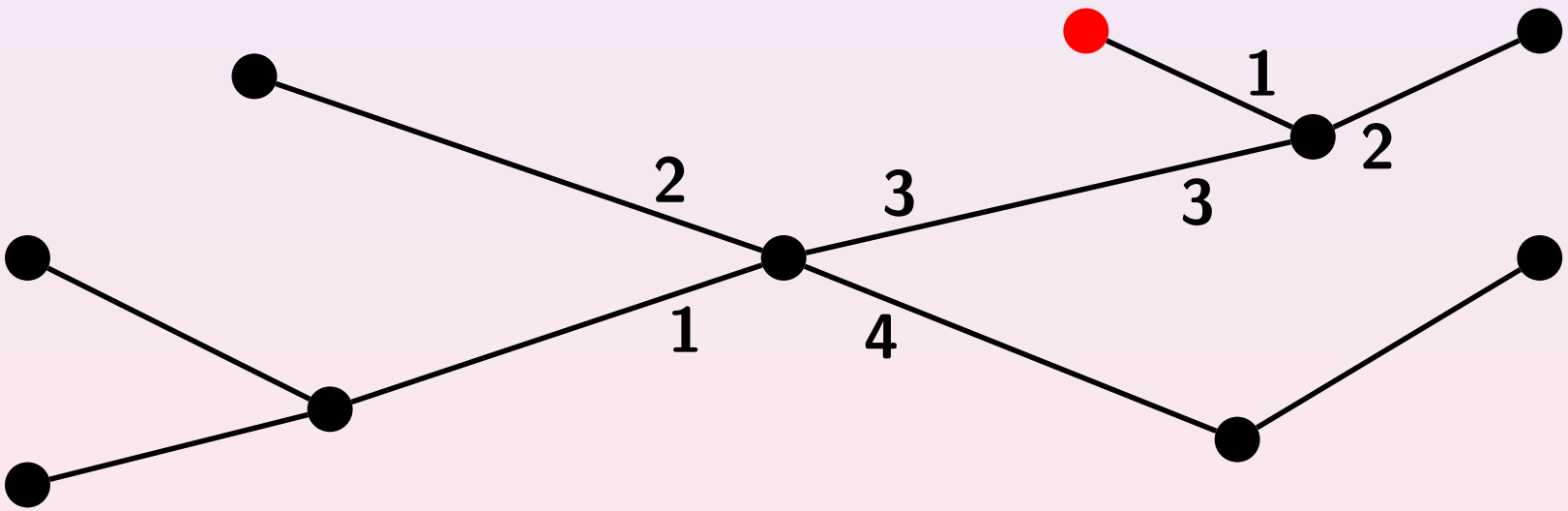

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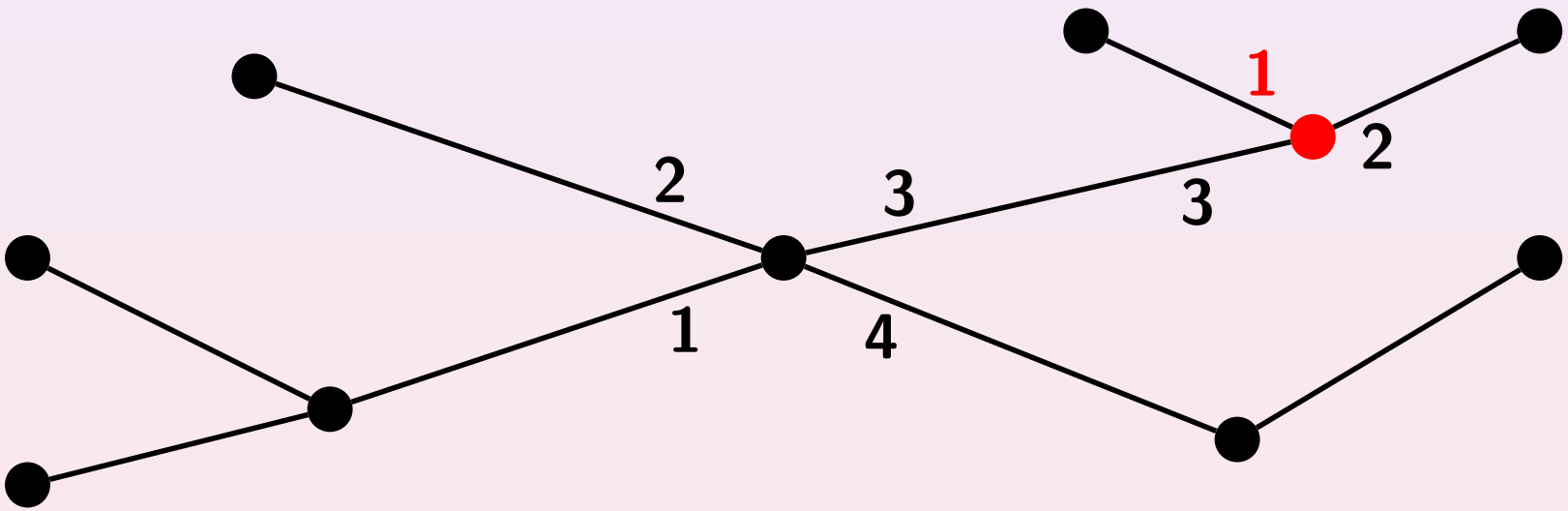

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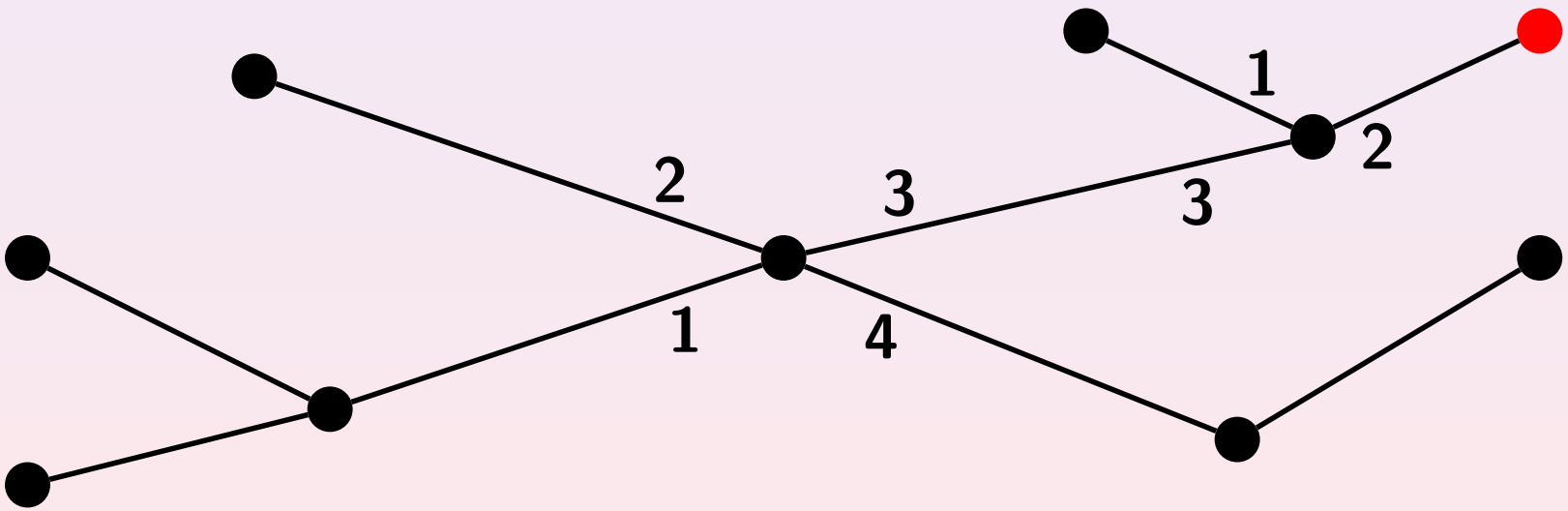

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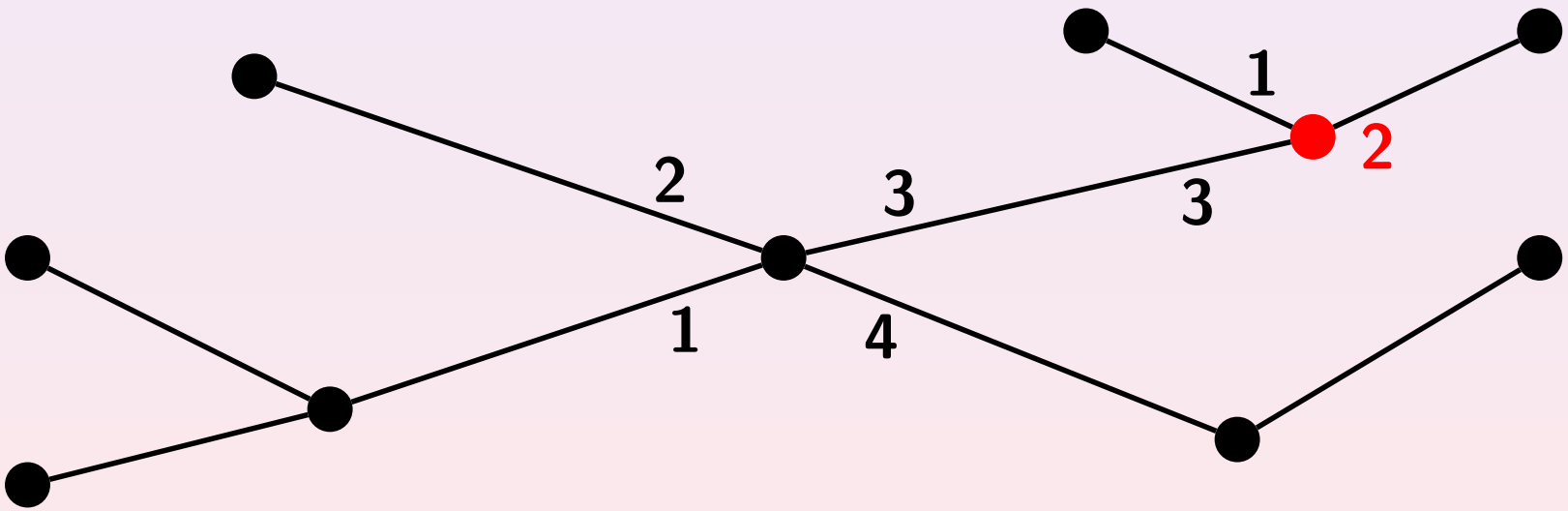

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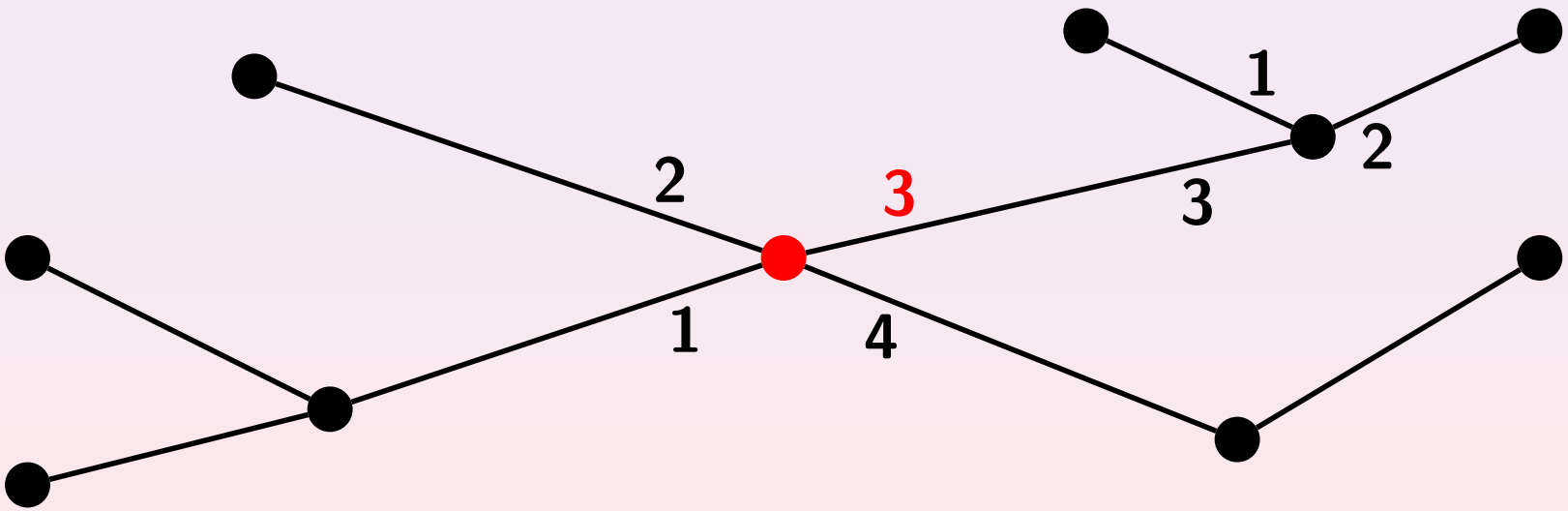

Coloring nodes

Model

- An oracle colors (labels) the graph to help the automaton.
- The finite automaton can read the color of the node as an input of its transition function.

Cohen, Fraigniaud, Ilcinkas, Korman, Peleg, ICALP, 2005
Label-Guided Graph Exploration by a Finite Automaton

- There exist a finite automaton and an algorithm coloring in three colors such that the automaton can explore all graphs.
- There exist a automaton of $O(log\,\Delta)$ memory bits and an algorithm coloring in only two colors such that the automaton can explore all graphs of maximum degree Δ.

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Setting port numbers

Model

- Port numbers are set to help the automaton.
- The automaton is ultimately simple : it is memoryless

Pelc, Panaite, Bui-Xuan, SIROCCO, 2005
Finding Short Right-Hand-on-the-Wall Walks in Graphs

There exist an algorithm for setting the port numbers, and an oblivious automaton using them, such that the automaton explores all graphs of size n within the period $10n$.

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Dobrev, Jansson, Sadakane, Sung, SIROCCO, 2005
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Outline

Our model

Model

- An algorithm sets the port numbers to help the automaton.
- The automaton is restricted to be finite (but not necessarily oblivious).

Question

What is the minimum function $\pi(n)$ such that there exist an algorithm for setting the local orientation, and a finite automaton using it, such that the automaton explores all graphs of size n within the period $\pi(n)$?

Dobrev et al.

$\pi(n) \leq 10n$

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Our results

Theorem

$\pi(n) \leq 4n - 2$

More precisely

- Very simple algorithm based on a spanning tree
- Three-state automaton
- Performance independent from the initial state and initial position of the automaton

Additional properties

- Distributed algorithm
- Dynamic environment

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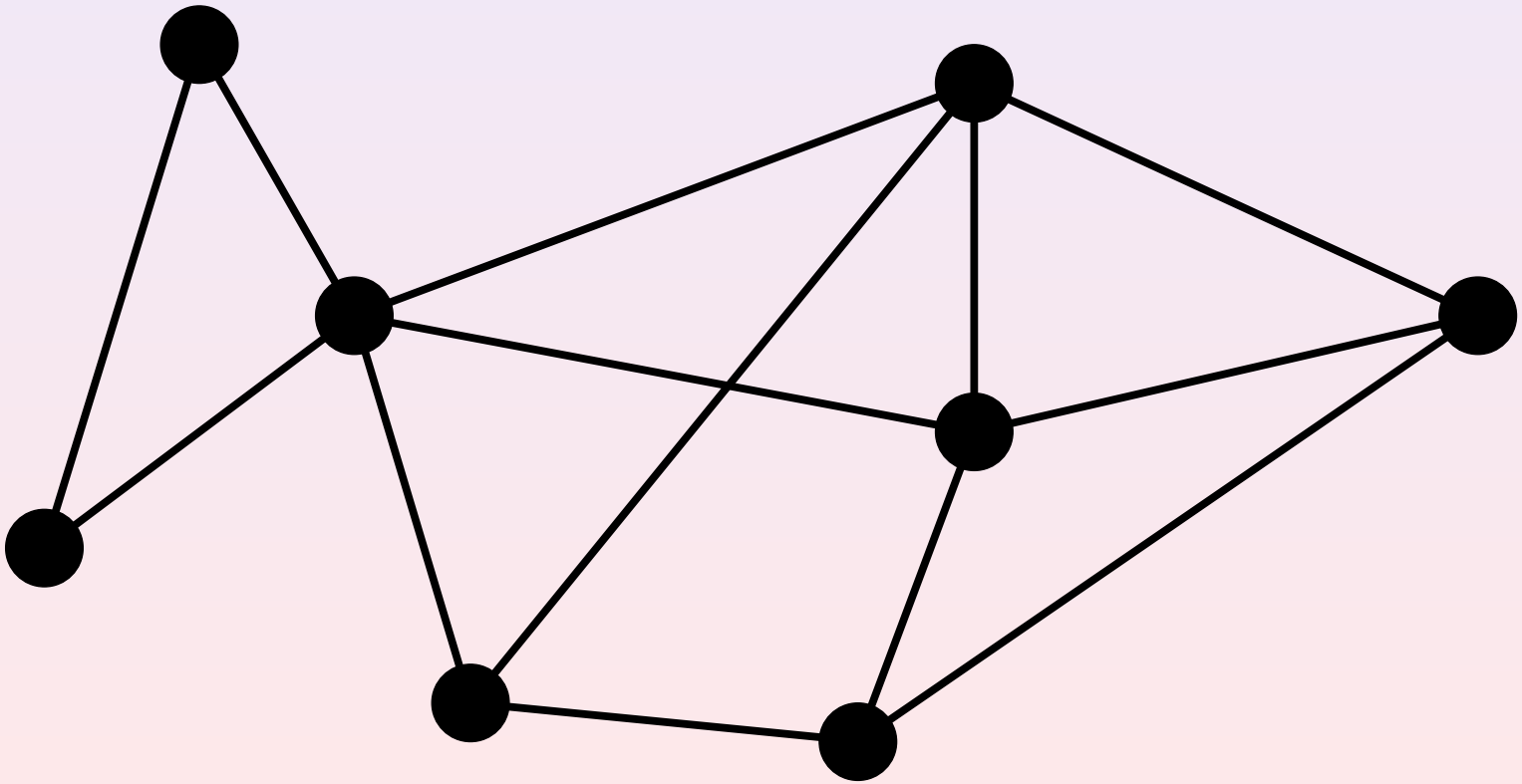
Port numbers compatible with a spanning tree T

- edge e is in T $\iff$ at least one of its port numbers is 1;
- the edges belonging to T have the smallest port numbers.



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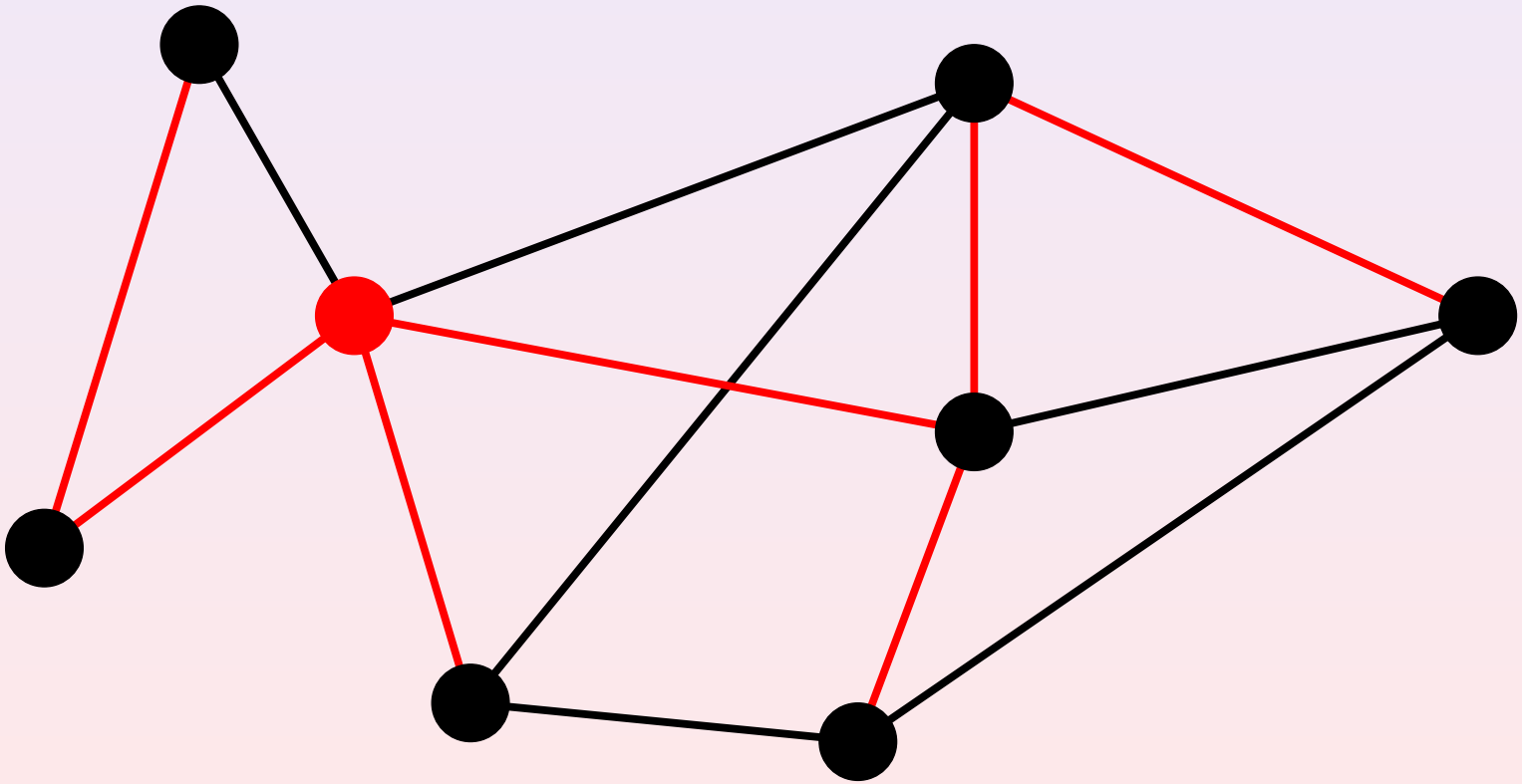

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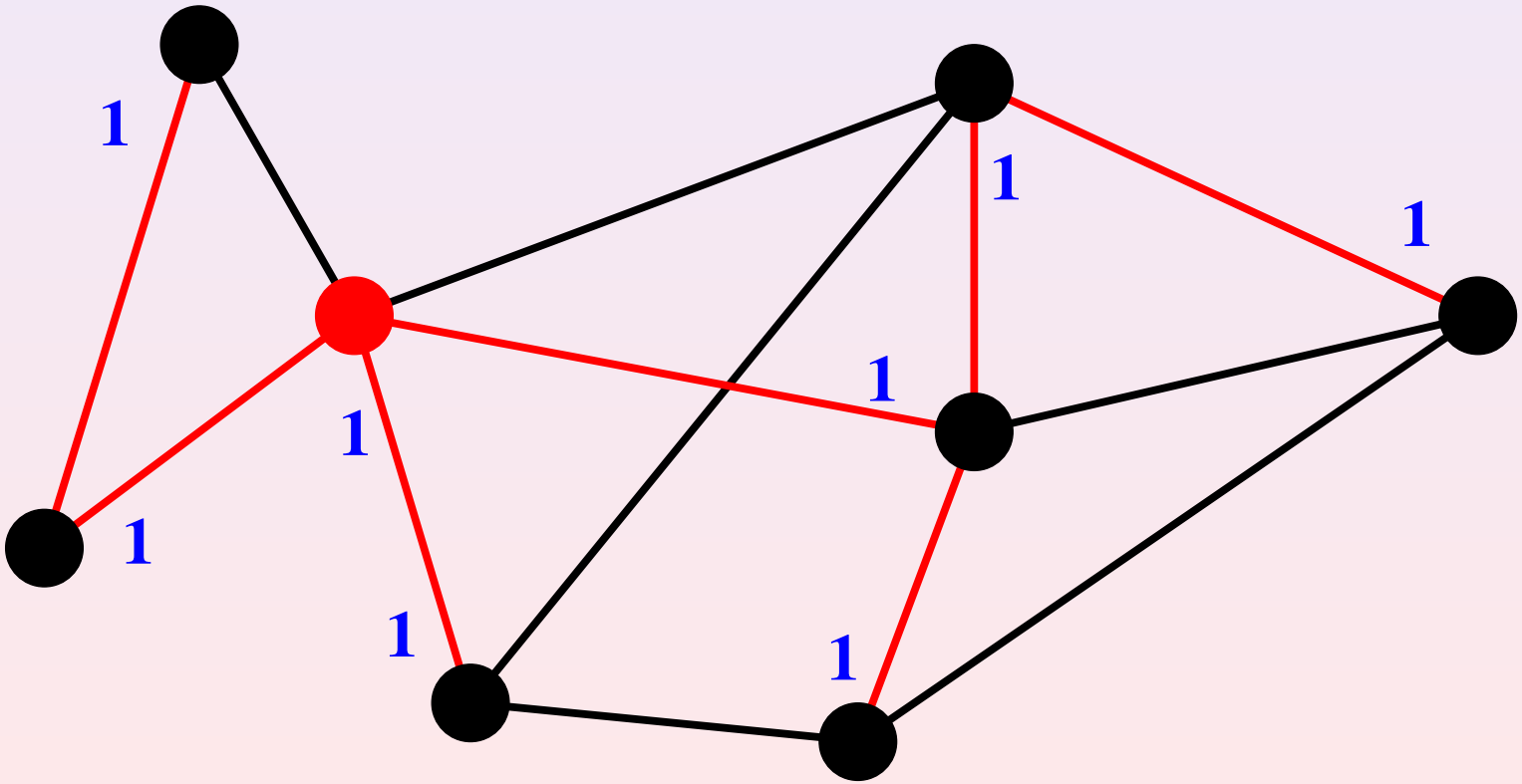

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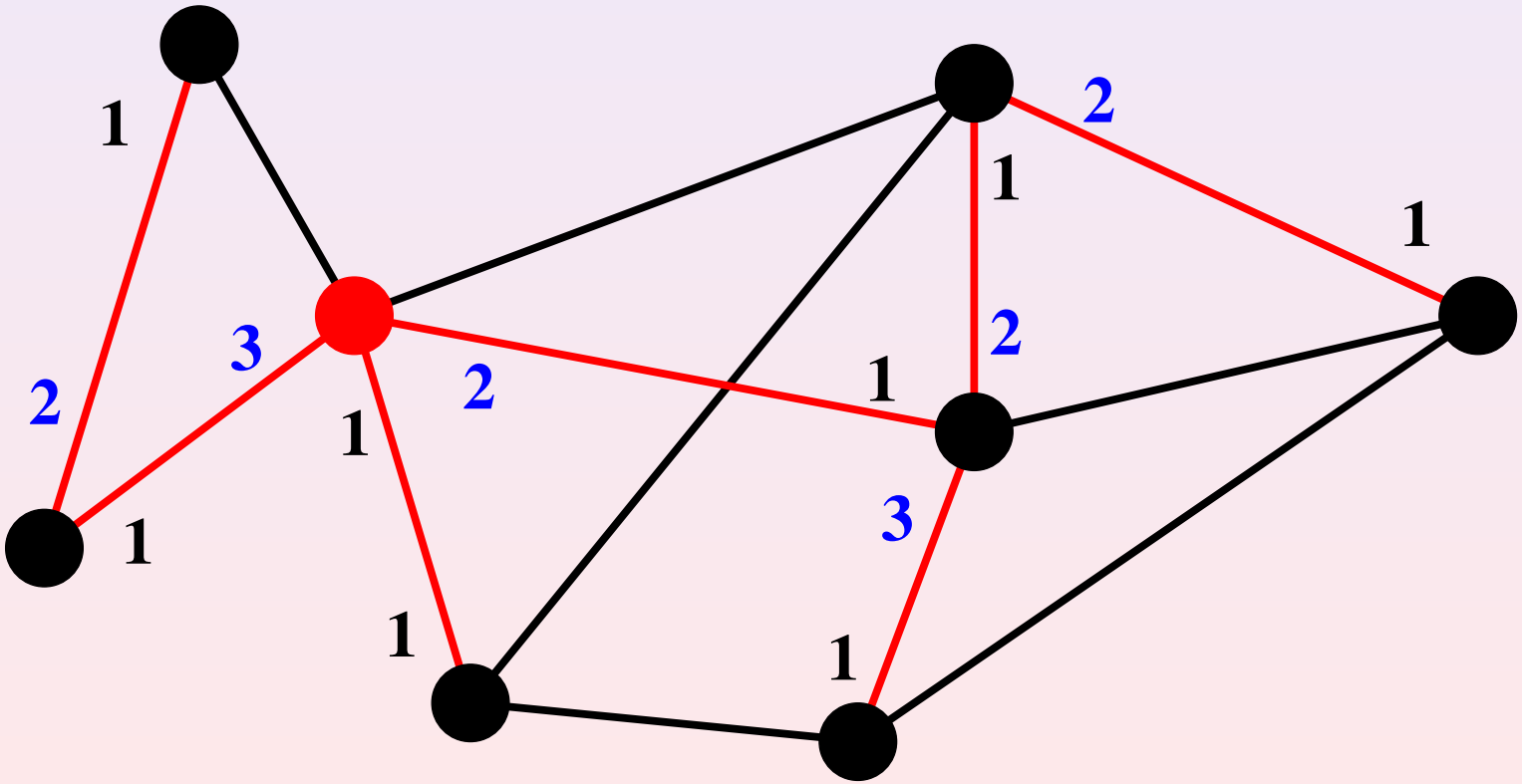

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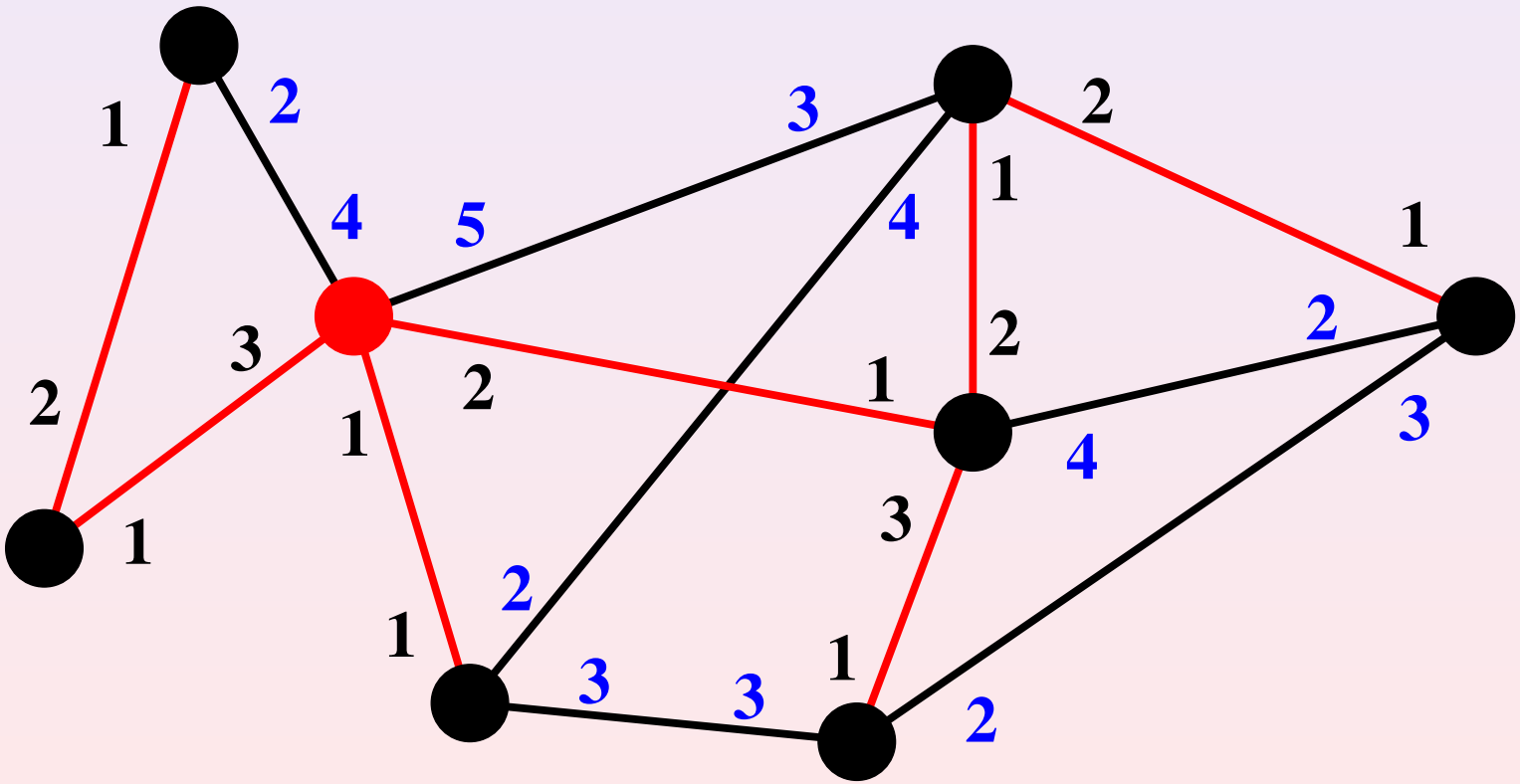

Automaton

States

- N: Normal
- T: Test
- B: Backtrack

Transition function

$$f(N,i,d) = \begin{cases} (N,1) & \text{if } i = d \\ (T,i+1) & \text{if } i \neq d \end{cases}$$

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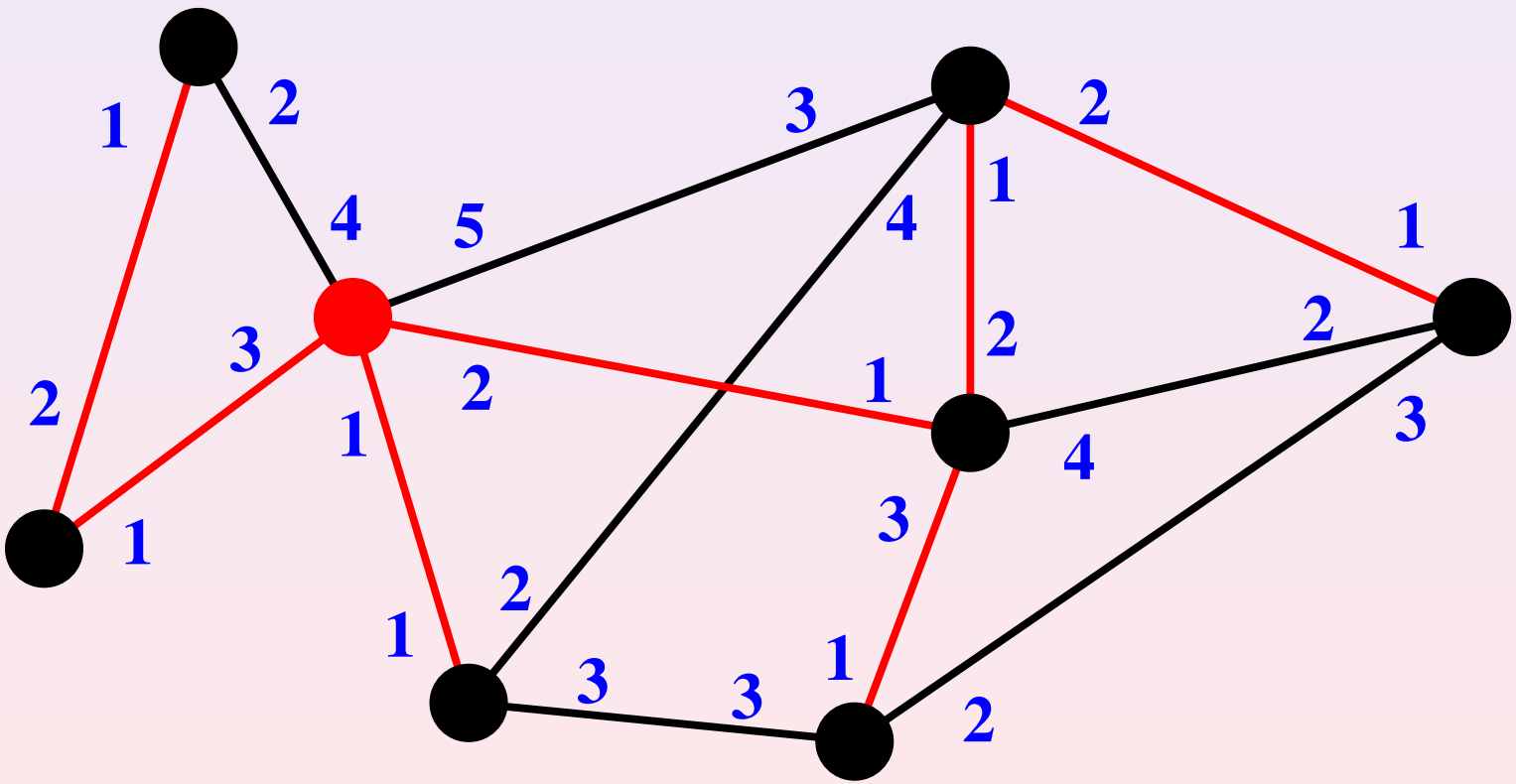
Example

- If on a tree-edge: right-hand-on-the-wall
- If not on a tree-edge: backtrack



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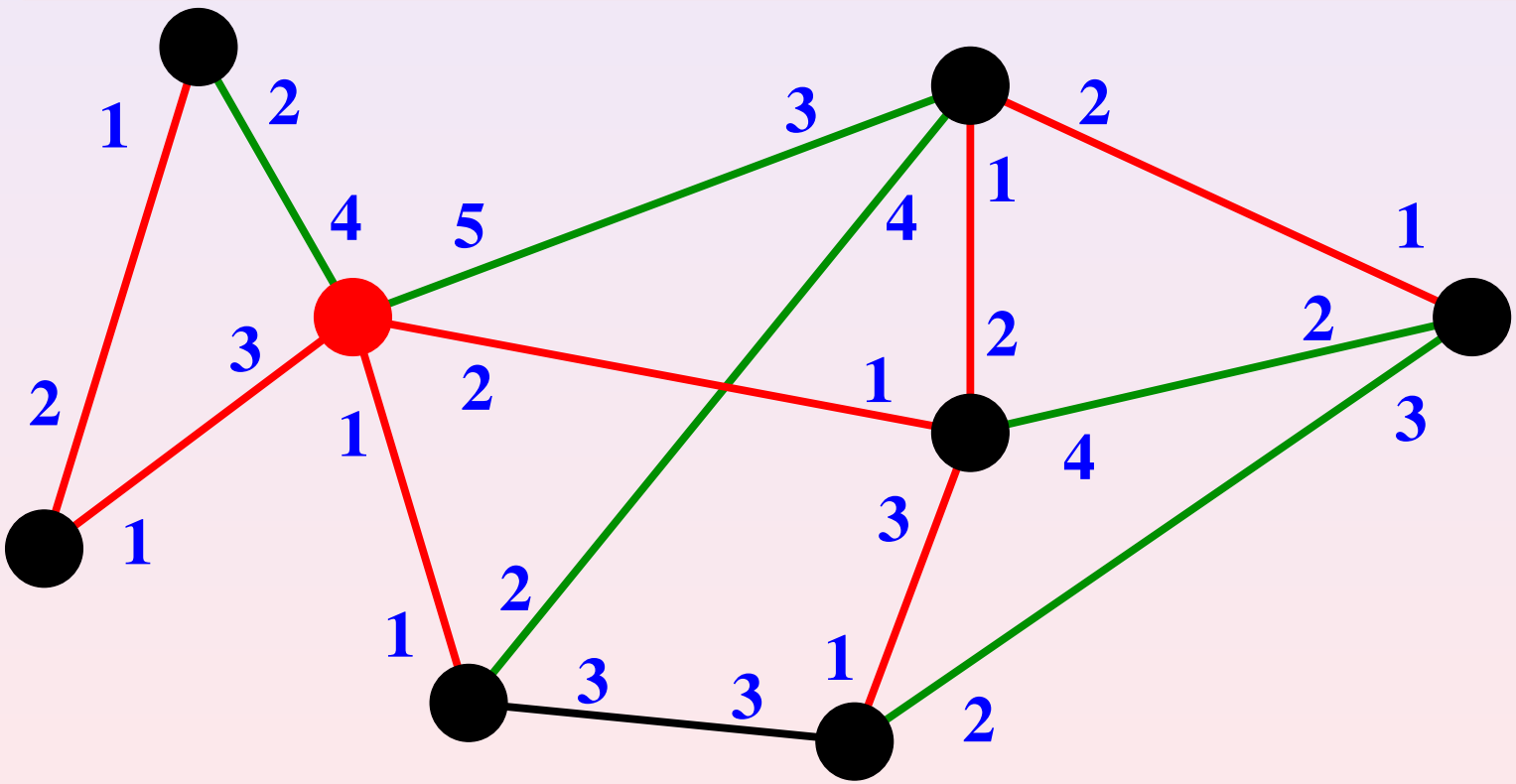

Outline

Open problem

Conclusion

$\pi(n) \leq 4n - 2$

Conjecture

$\pi(n) = 4n - O(1)$

Open problems

- $\pi(n)$ when the automaton is restricted to be oblivious?
- Find a fully self-stabilizing pair algorithm/automaton

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