

# Logic, Graph Transformations, Finite and Infinite Structures October 1-2, 2004

July 9, 2004

Like in Barcelona in 2002, a satellite workshop will be organized during one day and a half.

Its main topics are the *use of logic* for representing graph properties and graph transformations, and the definition and the study of *hierarchical structurings of graphs*, in order to obtain decidability results and polynomial time algorithms. The two main motivations for this type of study are the construction of *efficient algorithms for particular types of graphs* and the *verification of programs* based on infinite graphs representing all possible computations. In many cases, the properties to verify are specified in certain logical languages. It is thus important to understand the relationships between the *logical expression of properties* (graph properties or properties of programs) and the *complexity of their verification*. This is the aim of *Descriptive Complexity*. The possibility of constructing logical formulas in the powerful but nevertheless restricted language of *Monadic Second-Order Logic* is frequently based on deep combinatorial properties. The **Graph Minor Theorem** and its extensions to *Matroids* and related notions like *Isotropic Systems* have a prominent place among such properties. They are linked to *tree-decompositions* and similar hierarchical decompositions. Other notions of hierarchical structuring like *modular and split decompositions* are also recognized as important. Among the numerous notions of graph structuring proposed by graph theoreticians, some fit better with logic and algorithmics. It is thus interesting to revisit these notions in a logical and algorithmic perspective.

The term "graph" in the title is actually a generic and convenient wording, covering *hypergraphs, relational structures, and related combinatorial structures, like matroids and partial orders*.

The **Graph Minor Theorem** proved by Robertson and Seymour is certainly a major result in Graph Theory, with many consequences for the construction of graph algorithms and the understanding of graph structure in the

large. The notion of *tree-width* emerged from this work, and is the paradigmatic parameter in the theory of *Fixed Parameter Tractability*. *Clique-width* is another important parameter which has arisen from the theory of *Context-Free Graph Grammars*. This parameter is important for the construction of polynomial time algorithms and for the understanding of the structure of graphs obtained from trees by certain *graph transductions* (graph transformations) formalized in *Monadic Second-Order Logic*.

A natural question is whether the notion of *minor inclusion* (closely linked to tree-width) has some counter-part for “bounded clique-width”. The answer is **Yes**, as the two lectures by S. Oum of Princeton University (who is working with Paul Seymour) will show. Two key notions are that of a *vertex-minor*, the desired alternative to that of a *minor*, and the notion of *rank-width*. The same sets of graphs have bounded rank-width and bounded clique-width, but the set of graphs of rank-width at most any fixed  $k$  is characterized by *finitely many excluded vertex-minors*.

The 15 year long standing conjecture made by D. Seese that the *sets of graphs for which Monadic Second-Order Logic is decidable have a "tree structure"* equivalent to *bounded clique-width* can be “almost” proved as a consequence of these new results. “Almost” because the proved result uses the extension of Monadic Second-Order Logic with a set predicate expressing *even cardinality*, so that the hypothesis is stronger. This conjecture has been presented in detail in the previous workshop held in Barcelona in 2002.

These new results owe much to the notions of *matroid* and of *isotropic system*, two combinatorial and algebraic concepts associated with graphs. Together with their representations in Monadic Second-Order Logic, they will be presented in the talks by P. Hliněný, D. Seese, S. Oum and B. Courcelle.

Several talks will be devoted to the definitions of *hierarchical decompositions of graphs, hypergraphs and matroids*, from which one can define parameters like tree-width, branch-width, clique-width and now *rank-width*. Decompositions of hypergraphs can be characterized in terms of *games*, extending a characterization of tree-width in terms of the *Robber and Cops game* (P. Seymour *et al.*). Furthermore the notions of tree-width and clique-width can also be used for *logical formulas*, and yield polynomial algorithms in situations where the general case (like for **SAT**, the Satisfiability problem) is not (known to be) solvable in polynomial time. These topics will be presented by F. Scarcello and J. Makowsky. Other decompositions like the *modular* and the *split decompositions* have interesting algorithmic properties, to be presented by D. Kratsch, M. Rao and A. Brandstädt. E. Fisher will consider the complexity of testing a graph property satisfied by few graphs, by using tools from the theory of random graphs.

In order to be able to formulate algorithmic questions about *infinite structures* one needs to describe them by *finite tools* like automata, equation systems, logical formulas, or logically described transformations from fundamental structures like trees. Infinite graphs and trees can be seen as representations of the *sets of all possible computations of processes*. The notion of a *winning strategy* for games on the infinite objects that represent their behaviours is a convenient

way to express their correctness in an interactive context. There are intimate relations between games, automata and certain logical languages. They will be presented in the talks by D. Janin and D. Caucal.

The following talks have been confirmed at the date of July 10th.

B. Courcelle, Bordeaux (10 min.) : Wellcome and presentation of the program.

P. Hliněny, Ostrava (45 min.) : Are matroids interesting combinatorial structures ?

D. Seese, Karlsruhe (30 min.) : Decidability and computability for matroids of bounded branch-width. (Joint work with P. Hliněny).

S. Oum, Princeton (45 min.) : Rank-width, clique-width and vertex-minors.

B. Courcelle (30 min.) : Isotropic systems, monadic second-order definition of vertex-minors and Seese's Conjecture. (Joint work with S. Oum).

S. Oum (45 min.) : An algorithm for recognizing rank-width  $< k$  and the well-quasi-ordering of the vertex-minor relation.

D. Kratsch, Metz (30 min.) : Algorithms on  $(P_5, \text{gem})$ -free graphs

M. Rao, Metz (30 min.) : On 2-join decomposition.

A. Brandstädt, Rostock (30 min.) : Efficient algorithms for the Maximum Weight Stable Set problem: Graph decompositions and clique-width.

D. Janin, Bordeaux (45 min.) : Automata for distributed games.

D. Caucal, Rennes (30 min.) : Games on equational graphs.

F. Scarcello, Calabria (30 min.) : Hypertree decompositions of hypergraphs: game-theoretic and logical representations.

J. Makowsky, Haifa (45 min.) SAT related problems for clause sets of bounded clique-width. (Joint work with E. Fischer, A. Magid and E. Ravve)

E. Fisher, Haifa (30 min.) : The difficulty of testing for isomorphism against a graph that is given in advance

Original articles related to this presentations will be invited for publication (after the usual refereeing process) to a special issue a journal, most likely the new electronic free access journal : **Logical Methods in Computer Science** (Web site <http://www.lmcs-online.org>)

The workshop is organized by the **International Conference on Graph Transformations** and sponsored by the European network **GAMES**, coordinated by E. Graedel (Aachen, Germany). The program committee consists of B. Courcelle, D. Janin (Bordeaux, France), H. Ehrig (Berlin, Germany) and E. Graedel (Aachen, Germany).