Discretizations of timed automata behaviors

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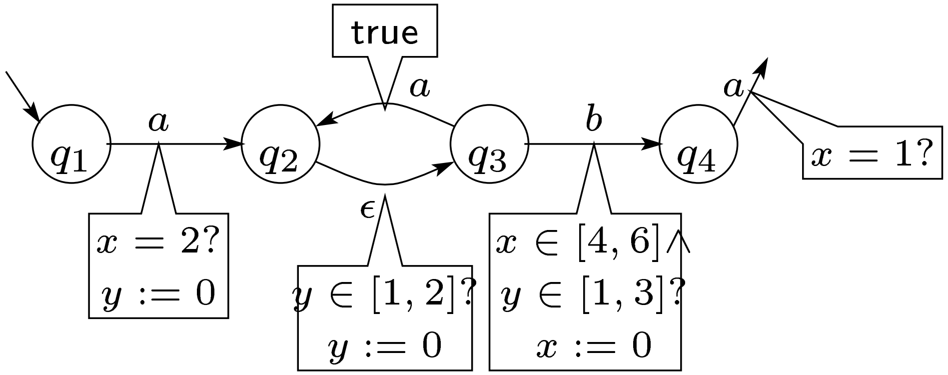
LABRI

Summary of the presentation

- Timed automata and clock relations
- Representing clock relations with n-automata
- Discretizations of timed languages
- Stack automata for discretizations of timed languages

Timed automata

Finite automata with clocks for measuring time passage

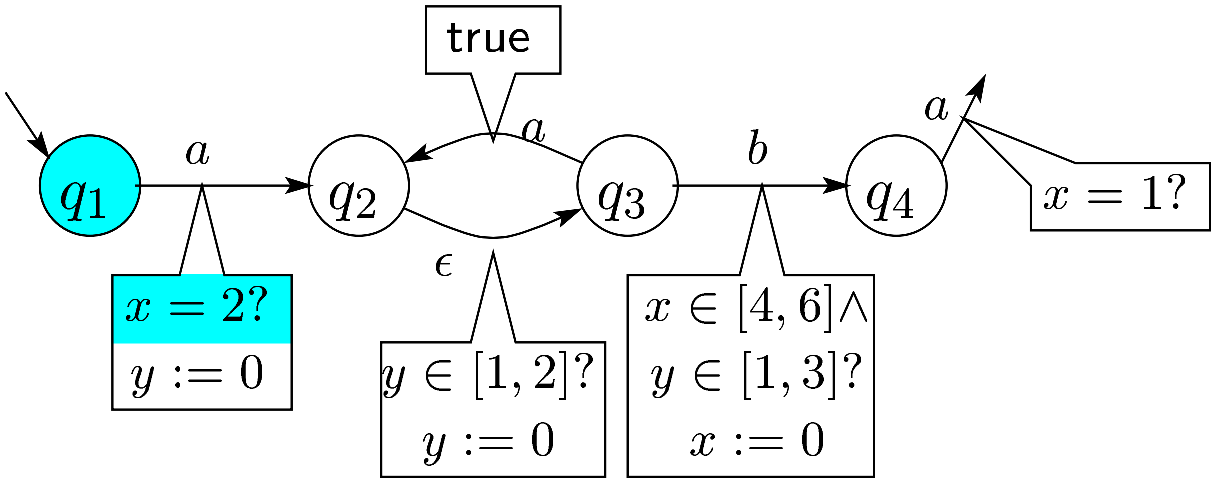


- Time passage in the states.
- Instantaneous transitions.
- Clocks evolve synchronously, at rate 1.

Reset time semantics for timed automata

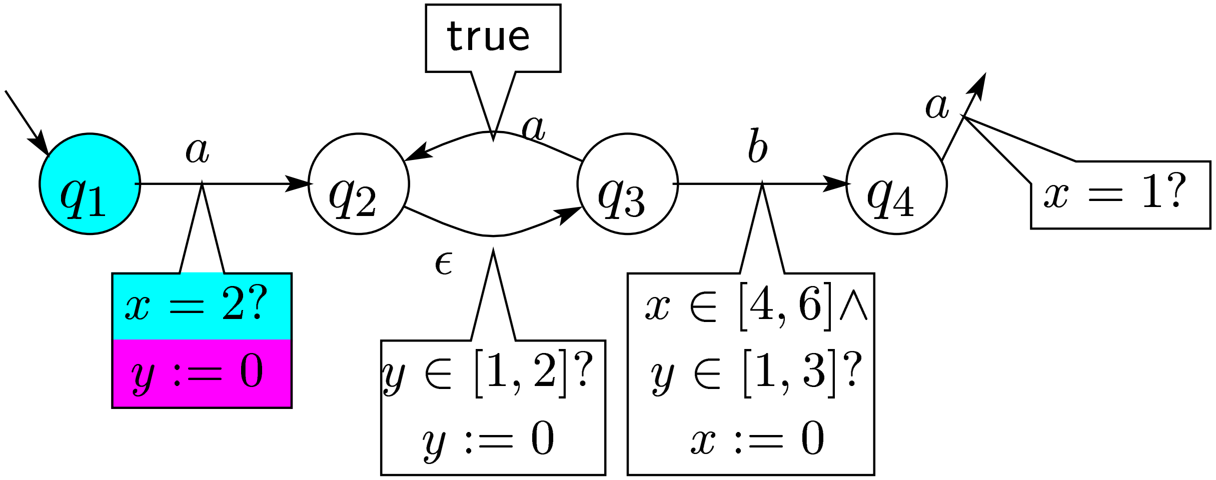
- *time* $=$ the current time.
- For each clock x, $r(x) \;= $ the latest moment before the current time when the clock x was reset.
- Time passage $=$ increment the *time* variable.
- Clock value for $x =$ *time* $- \, r(x)$.
- Reset clock $x \mapsto$ put $r(x) =$ *time*.

Reset time semantics



$r(x):$	0	0
$r(y):$	0	0
time :	0	2

Reset time semantics



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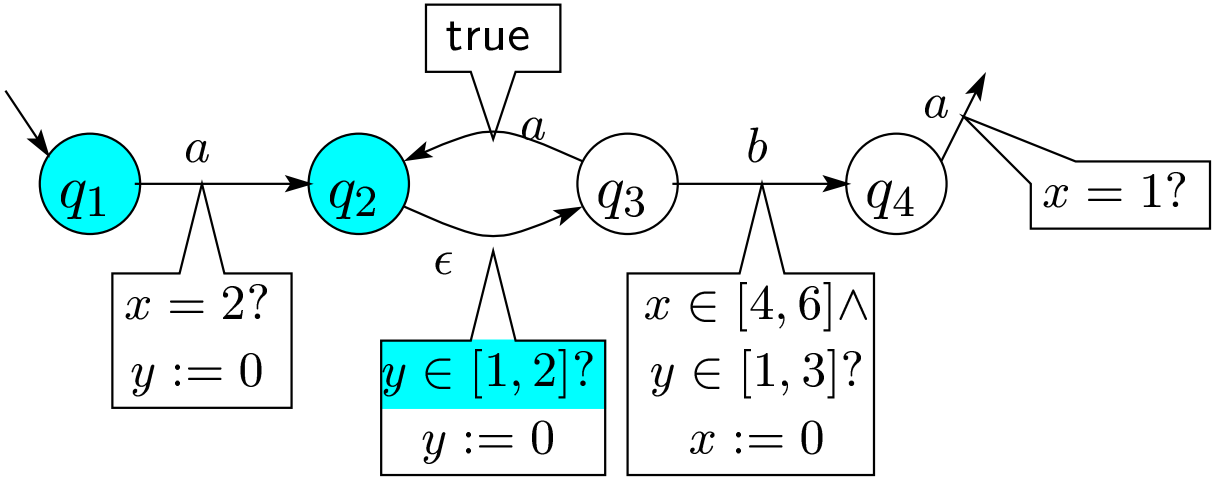
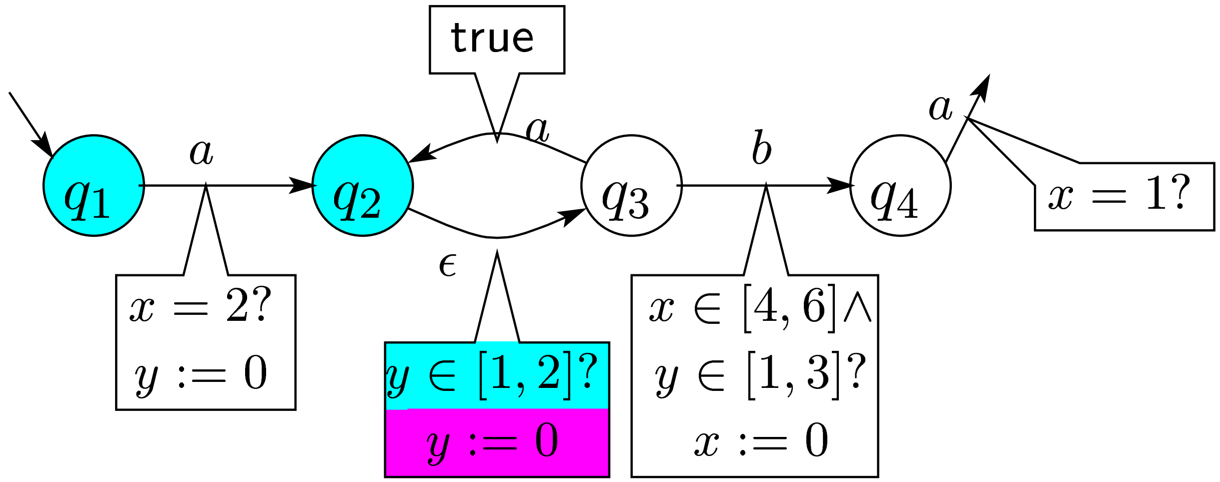


$r(x)$:	0	0	0
$r(y)$:	0	2	2
time :	0	2	3.5

Reset time semantics



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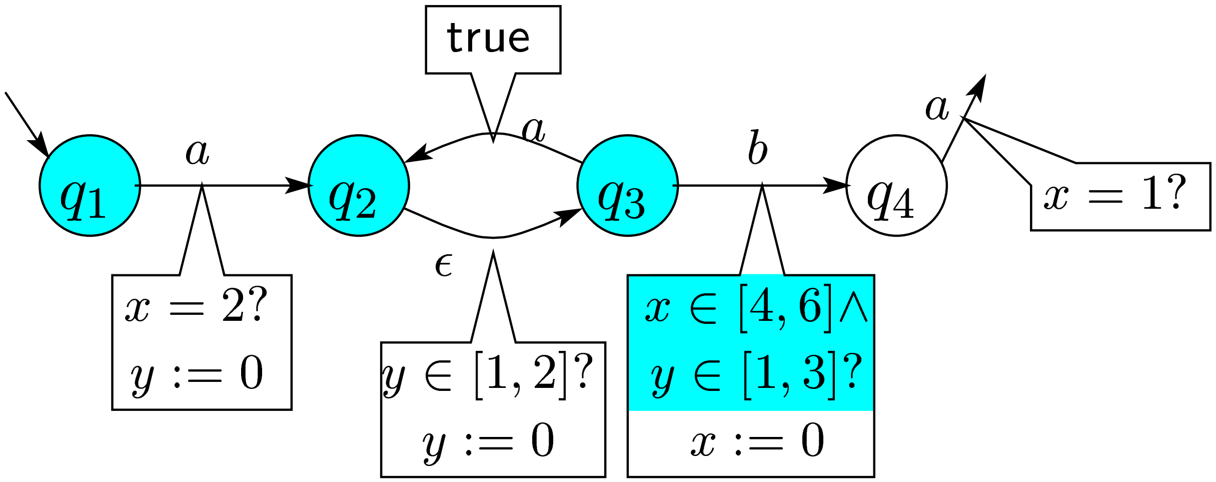
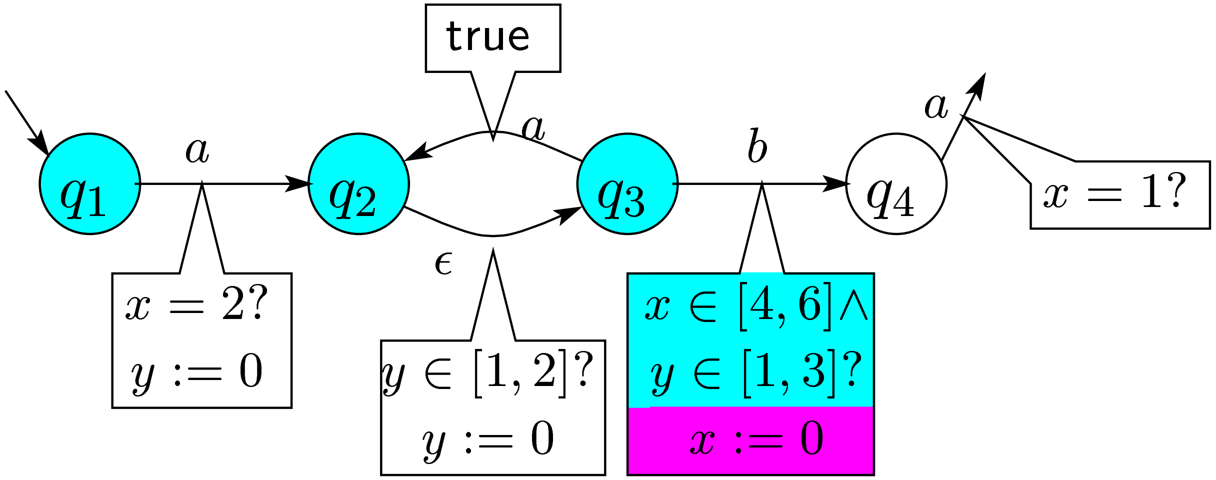


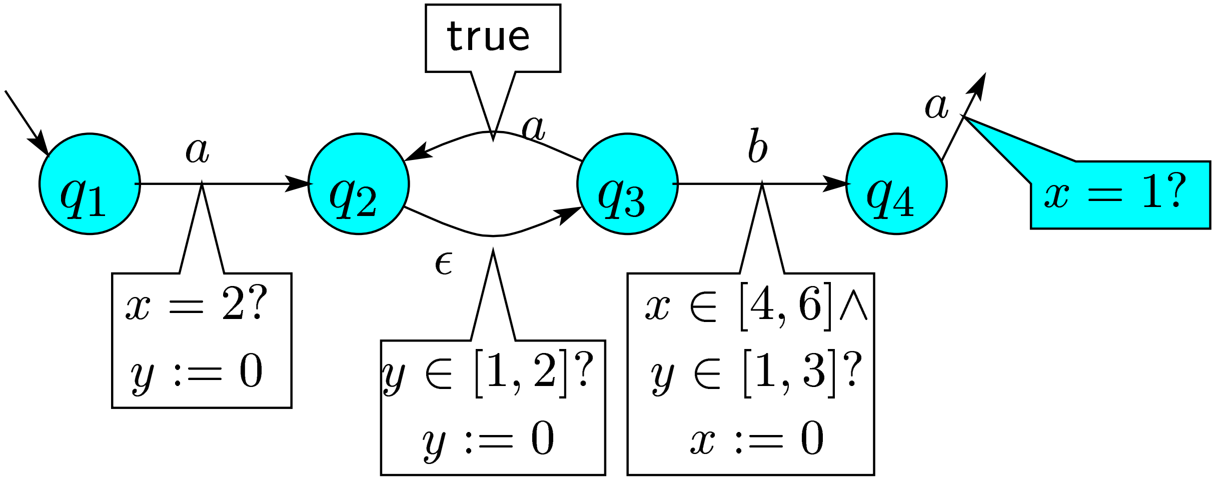
$r(x)$:	0	0	0	0
$r(y)$:	0	2	3.5	3.5
time :	0	2	3.5	5

Reset time semantics



$r(x)$:	0	0	0	5
$r(y)$:	0	2	3.5	3.5
time :	0	2	3.5	5

Reset time semantics



$r(x)$:	0	0	0	5	5
$r(y)$:	0	2	3.5	3.5	3.5
time :	0	2	3.5	5	6

The reachability relation

- One relation associated to each transition.
- Chains of transitions = $\circ$.
- Choices in taking transitions = $\cup$.
- Circuits in the automaton = *.

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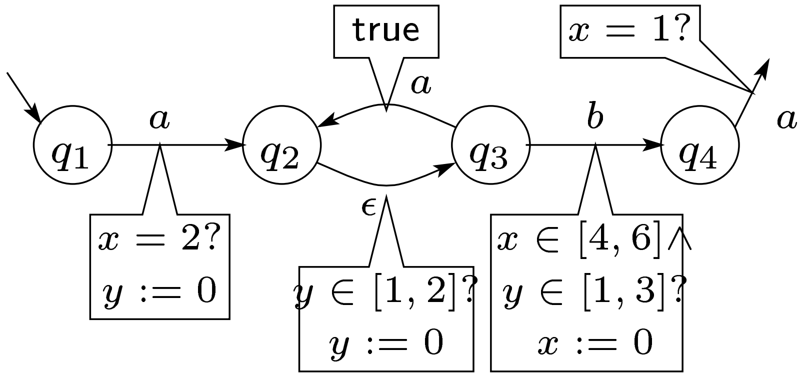

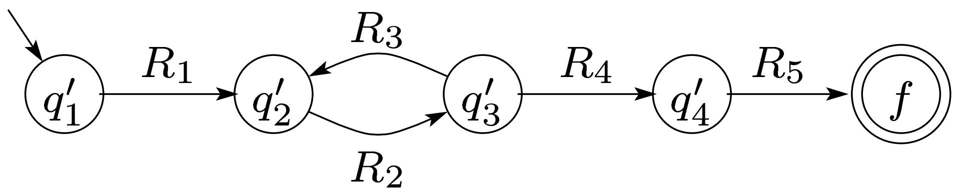

$$R_A = R_1 \circ (R_2 \circ R_3)^* \circ R_2 \circ R_4 \circ R_5$$

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$\overline{x}$ = variable representing the latest reset time for x

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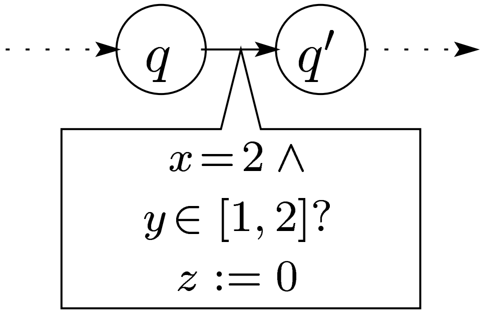

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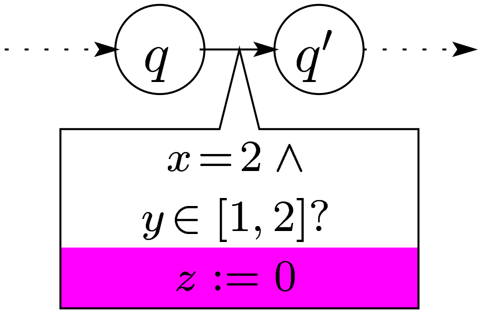

$\textit{time}, \overline{x}, \overline{y}, \overline{z} \leq \textit{time'} \wedge$

$\overline{x}' = \overline{x} \wedge$

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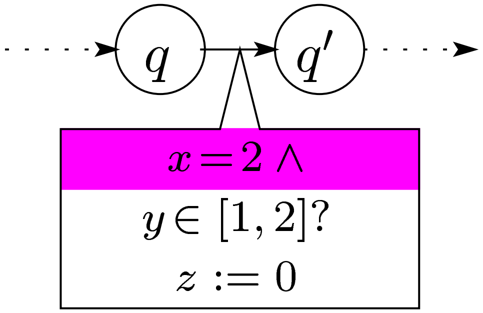

time, $\overline{x}, \overline{y}, \overline{z} \leq$ *time'* $\wedge$

$\overline{x}' = \overline{x} \wedge$

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$\mathit{time}' - \overline{x} = 2 \wedge$

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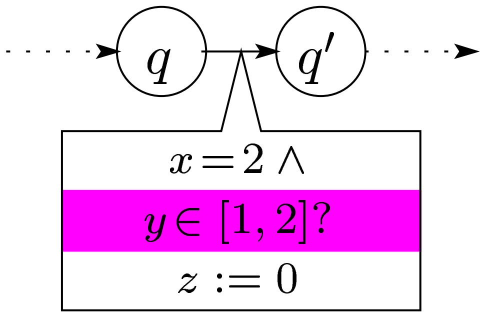

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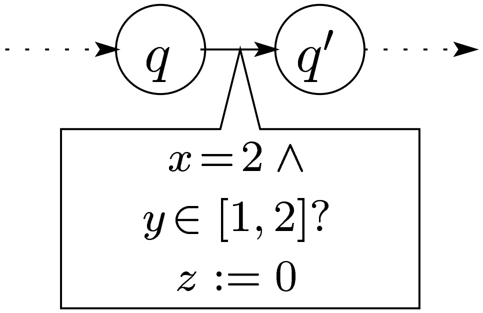

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By composition, difference constraints yield only difference constraints!

Modular construction of the reachability relation

- Unions = $\vee$.

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$$R_1 \mapsto f_1(x, y, z, x', y', z') \ , \qquad R_2 \mapsto f_2(x, y, z, x', y', z')$$

$$R_1 \circ R_2 \mapsto \exists x', y', z'.\ f_1(x, y, z, x', y', z') \wedge f_2(x', y', z', x'', y'', z'')$$

- Reflexive-transitive closure:
  Second-order quantification – verification is a problem.

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Required:

- Discretizations of clock relations;
- Automata accepting such discretizations.

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- Timed automata and clock relations
- Representing clock relations with n-automata
- Discretizations of timed languages
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Representing clock relations

(Discrete time case – resets occur at integer time points)

$$R : x' - x \in [2,3] \wedge x' - y \in]-1,2[\wedge y' - x' \in [0,\infty[$$

$$R \ni (x = 1, y = 3, x' = 4, y' = 4)$$

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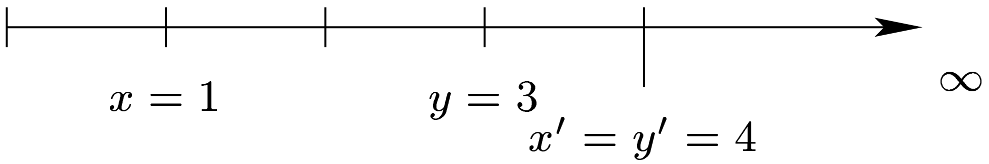

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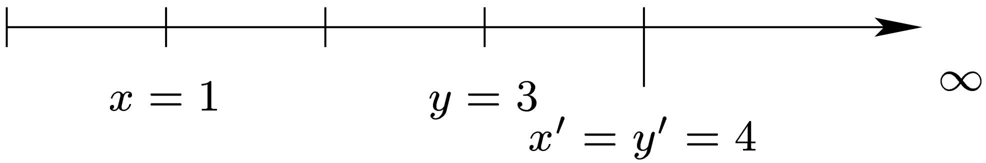
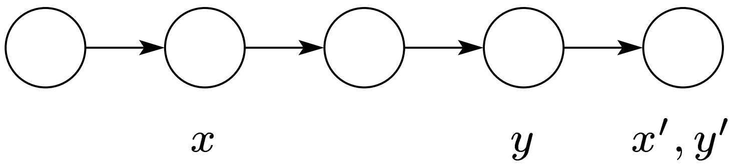
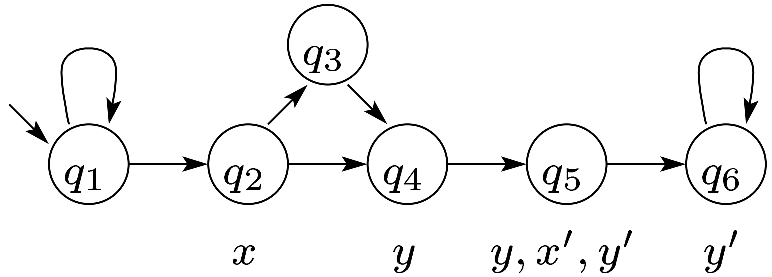




$2n$-automata

$$R : x'-x \in [2,3] \wedge x'-y \in \,]-1,2[\, \wedge y'-x' \in [0,\infty[$$

4-automaton for all the tuples in R:



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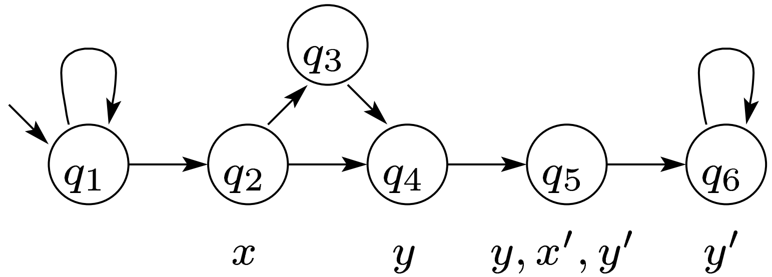
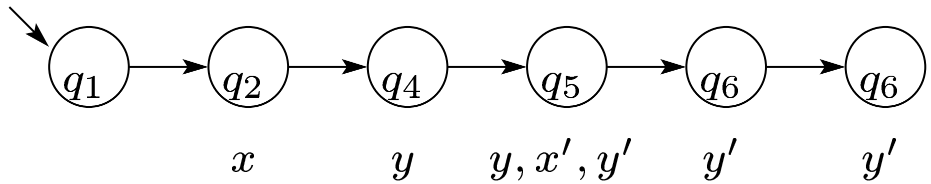


Accepting run:



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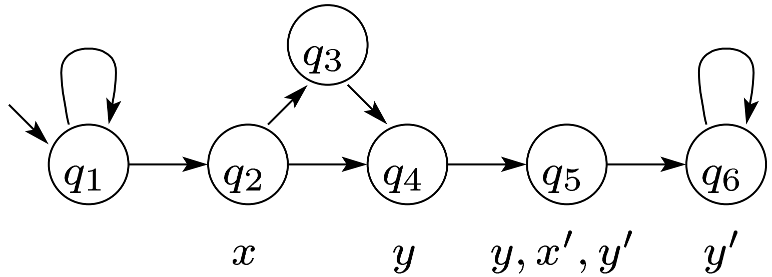

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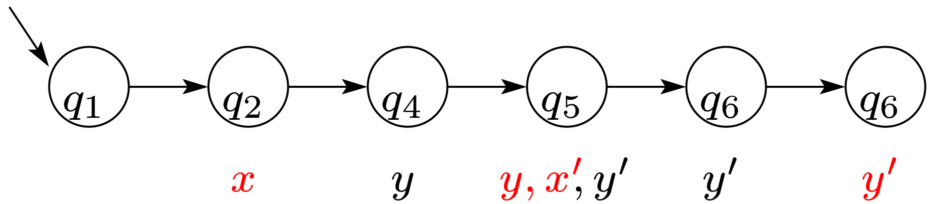

Tuples associated to this accepting run:

$$(1, 3, 3, 5),$$

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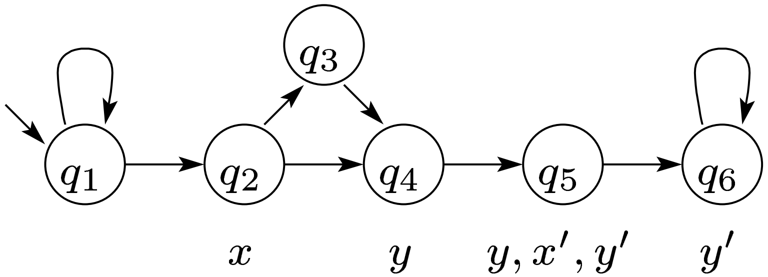

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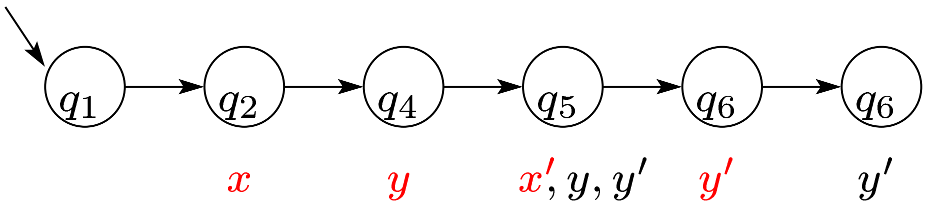

Tuples associated to this accepting run:

$$(1,3,3,5), (1,2,3,4) \ldots$$

Properties of $2n$-automata

- Emptiness is decidable (NP-complete).

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  - Algorithm with time exponential in n but linear in the number of states.

n-automata and relational composition

Relational composition:

$$\{(x = 1, y = 2, x' = 4, y' = 3)\} \circ$$
$$\{x = 4, y = 3, x' = 6, y' = 4)\} =$$
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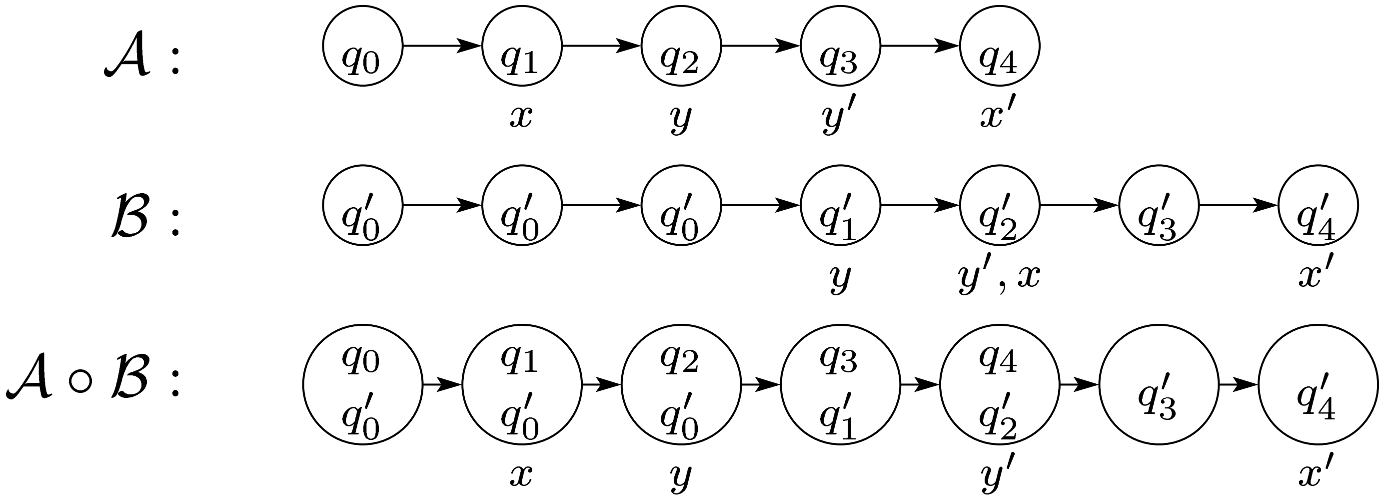

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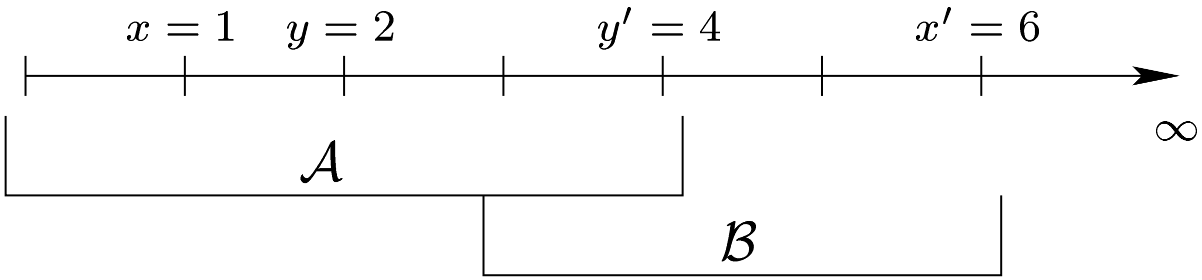
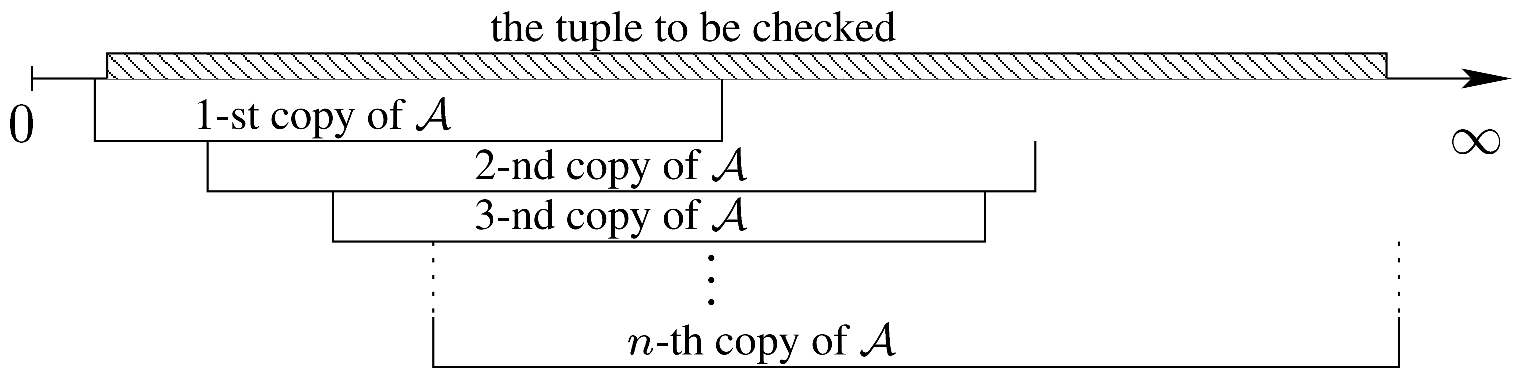


The star closure problem

In general, $2n$-automata are not closed under star.



“Non-elastic” relations

Relation $R \subseteq \mathbb{R}^{2n}$ which contains only tuples $(x_1, \ldots, x_n, x'_1, \ldots, x'_n)$ for which:

$$\text{if } x'_i \neq x_i \text{ and } x'_j \neq x_j \text{ then } x'_i - x_j \geq 0$$

If, along a chain of transitions, two clocks x_i and x_j are both reset, then their new reset points are both greater than their former reset points.

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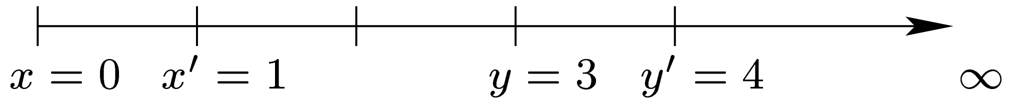
“Elastic” tuple:



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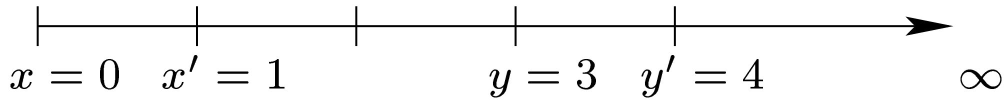

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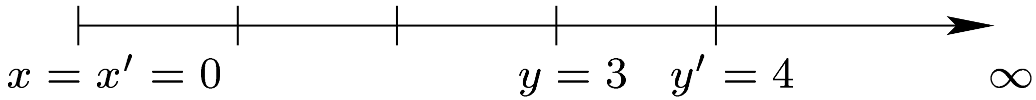

“Non-elastic” relations (2)

Theorem. Given a $2n$-automaton $\mathcal{A}$, if $L(\mathcal{A})^k$ contains only of non-elastic tuples for all $k \in \mathbb{N}$ then $L(\mathcal{A})^*$ is recognizable by a $2n$-automaton.

Complexity = $O(2^{card(Q_\mathcal{A}) \cdot 4^n})$

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Theorem. The union of all reachability relations for all timed automata is a non-elastic relation.

Representing clock relations – dense time case

Each boolean combination over “difference constraints” is a disjunction of “basic blocks”:

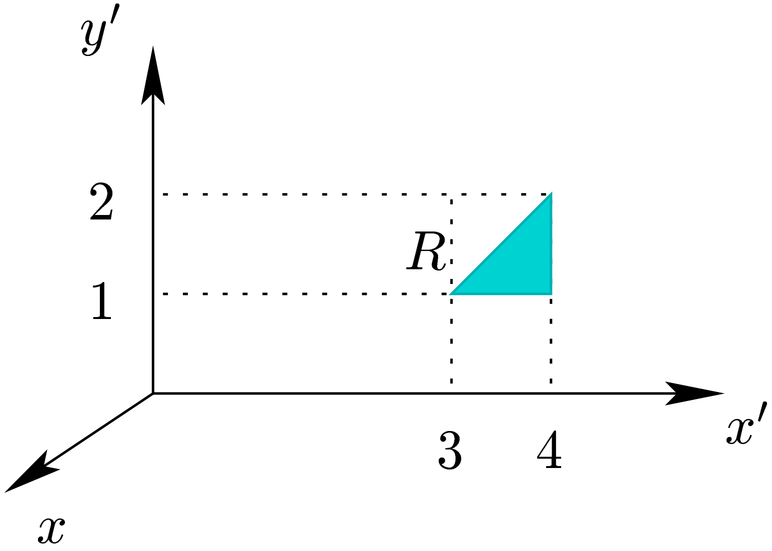
Region: $\bigwedge_{1 \le i < j \le n} x_i - x_j \in I_{ij}$ → point interval / open unit interval

How to use $2n$-automata for regions?

Regions

$$R : x = y \wedge x' - y' \ \in]2,3[\wedge y' - x \ \in]1,2[\wedge x' - x \ \in]3,4[$$

If we set $y = 0$ then we can represent the respective “projection” as:



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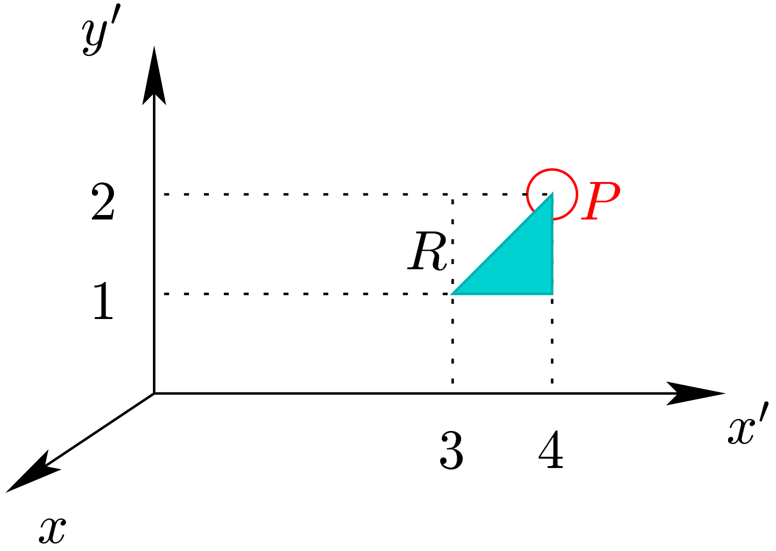

Idea: instead of working with R, work with one of its three vertices.

Vertex P: tuple of integers $(x = 0, y = 0, x' = 4, y' = 2)$.

But P is also a vertex in other 15 regions!

Representing regions

Relative positioning of P w.r.t. R:

$$P : (0, 0, 4, 2) \mapsto \begin{array}{cc} & \begin{array}{cccc} x & y & x' & y' \end{array} \\ \begin{array}{c} x \\ y \\ x' \\ y' \end{array} & \begin{pmatrix} 0 & 0 & -4 & -2 \\ 0 & 0 & -4 & -2 \\ 4 & 4 & 0 & 2 \\ 2 & 2 & -2 & 0 \end{pmatrix} \end{array}$$

Representing regions

Relative positioning of P w.r.t. R:

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$$\begin{aligned} R : \quad & x = y \wedge x'-y' \in \,]2,3[\, \wedge \\ & y'-x \in \,]1,2[\, \wedge x'-x \in \,]3,4[\end{aligned} \qquad \begin{pmatrix} \{0\} & \{0\} &]-4,-3[&]-2,-1[\\ \{0\} & \{0\} &]-4,-3[&]-2,-1[\\ ]3,4[&]3,4[& \{0\} &]2,3[\\ ]1,2[&]1,2[&]-3,-2[& \{0\} \end{pmatrix}$$

Representing regions

$$P : \quad \begin{pmatrix} 0 & 0 & -4 & -2 \\ 0 & 0 & -4 & -2 \\ 4 & 4 & 0 & 2 \\ 2 & 2 & -2 & 0 \end{pmatrix} \quad M = \begin{pmatrix} `=` & `=` & `>` & `>` \\ `=` & `=` & `>` & `>` \\ `<` & `<` & `=` & `>` \\ `<` & `<` & `<` & `=` \end{pmatrix}$$

$$R : \begin{pmatrix} \{0\} & \{0\} &]-4,-3[&]-2,-1[\\ \{0\} & \{0\} &]-4,-3[&]-2,-1[\\ ]3,4[&]3,4[& \{0\} &]2,3[\\ ]1,2[&]1,2[&]-3,-2[& \{0\} \end{pmatrix}$$

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Endow $2n$-automata with such matrix labels.

$2n$-automata and timed automata

The reachability relation R_A defined by a timed automaton A can be computed using $2n$-automata.

The result is a $2n$-automaton whose accepting runs represent the regions in R_A.

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- Timed automata and clock relations
- Representing clock relations with n-automata
- Discretizations of timed languages
- Stack automata for discretizations of timed languages

Timed words

- Sequences of actions and durations:

$$2\,a\,1.5\,1.5\,b\,1\,a$$

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- Sequences of actions and durations:

$$2\,a\,1.5\,1.5\,b\,1\,a = 2\,a\,3\,b\,1\,a$$

- The monoid of timed words : $TW(\Sigma) = \Sigma^* \oplus \mathbb{R}_{\geq 0}$.

Timed words

- Sequences of actions and durations:

$$2\,a\,1.5\,1.5\,b\,1\,a = 2\,a\,3\,b\,1\,a$$

- The monoid of timed words : $TW(\Sigma) = \Sigma^* \oplus \mathbb{R}_{\geq 0}$.
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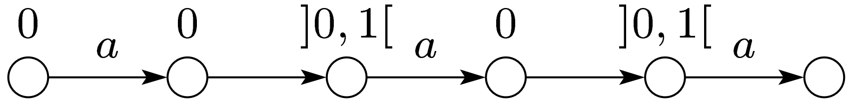
- Actions serve for communication/synchronization.
- Synchronization $=$ intersection of timed languages.

The “region discretization”

$$L_1 = \{at_1at_2a \mid t_1, t_2 \ \in\ ]0,1[\} = \left\| \langle a\underline{\mathbf{t}}\rangle_{]0,1[} \langle a\underline{\mathbf{t}}a\rangle_{]0,1[} \right\|$$

$\mathcal{A}_1$:



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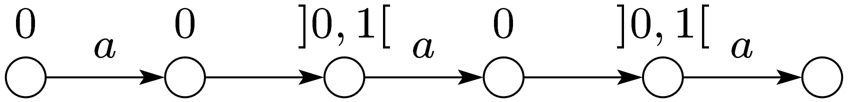
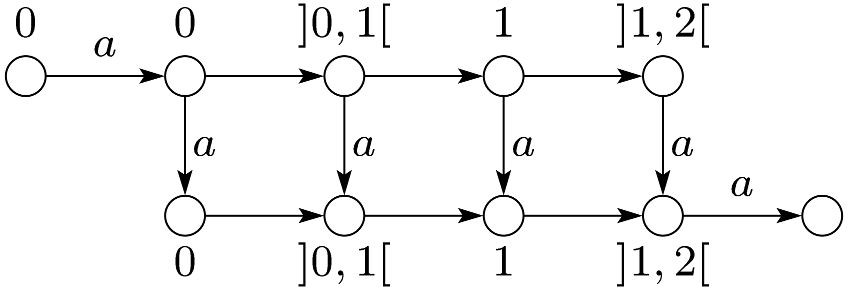


$$L_2 = \{at_1at_2a \mid t_1 + t_2 \in]1,2[\} = \left\| \langle a\underline{\mathbf{t}}a\underline{\mathbf{t}}a\rangle_{]1,2[} \right\|$$

$\mathcal{A}_2$:



The region discretization and intersection

$$L_1 \cap L_2 = \left\| \left[\langle a\underline{\mathbf{t}} \rangle_{]0,1[} \langle a\underline{\mathbf{t}}a \rangle_{]0,1[} \; \right]_{]1,2[} \right\|$$

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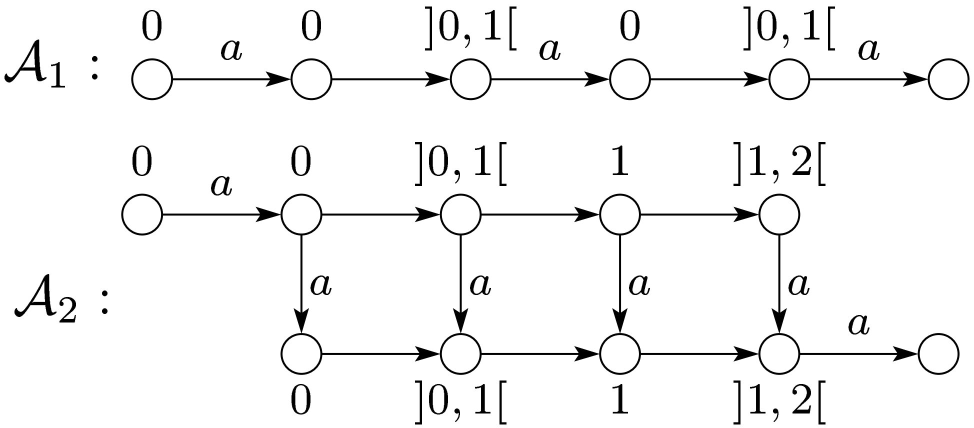

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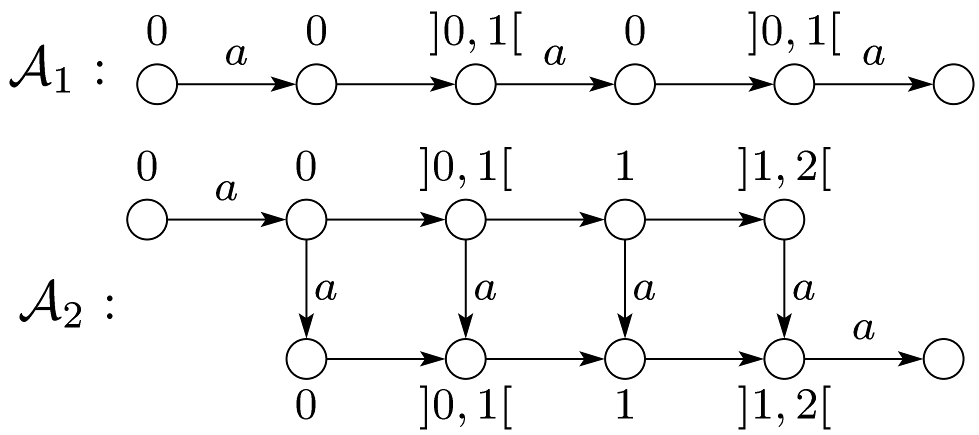

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Obviously not !

A discretization with nonarchimedian delays

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- Each symbol in Σ acts like a “time hook”.
- Words over $\Sigma^* \sqcup\!\sqcup\, 1^*$ are timed words over a discrete time domain.
- All is needed then is, for each word, a “pattern” for the fractional parts between all pairs of time hooks.
- Practically, what we need is the shuffle $\Sigma^* \sqcup\!\sqcup\, 1^* \sqcup\!\sqcup\, \mathcal{G}_3$, where $\mathcal{G}_3$ is the three-element group.

Properties of nonarchimedian delay discretizations

- Nonarchimedian delay discretizations induce a partition on TW(Σ): for all $w_1, w_2 \in \Sigma^* \sqcup\!\sqcup 1^* \sqcup\!\sqcup (\delta^+ + \delta^-)^*$,

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$$\text{if } \|w_1\| \cap \|w_2\| \neq \emptyset \text{ then } \|w_1\| = \|w_2\|.$$

- Each class can be represented by several (nonarchimedian) delay discretizations:

$$\|a\delta^+\delta^+a1\delta^-a\| = \|a1\delta^-a\delta^+\delta^+a\| = \|a1\delta^-a1\delta^-a\|$$

Nonarchimedian delay discretizations for timed languages

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- **Basic blocks:** shuffles of a regular language $L \subseteq \Sigma^*$, a regular language *Ticks* $\subseteq 1^*$ and a (?) language $D \subseteq \left(\delta^+ + \delta^-\right)^*$.
- The number of positive/negative delays in between two “hook” points may be in general unbounded:

We cannot bind the number of δ^+ in the discretization of

$$\left[a\left(\langle b\underline{\mathbf{t}}\rangle_{]0,1[}\right)^* a\left(\langle b\underline{\mathbf{t}}\rangle_{]0,1[}\right)^*\right]_{]1,2[}$$

Languages of nonarhimedian delay discretizations

$$L_1 \cap L_2 = \Big\|\Big\{a1w_1a1w_2a \mid w_1, w_2 \in (\delta^+ + \delta^-)^*, \#_{\delta^+}(w_1) < \#_{\delta^-}(w_1)$$
$$\#_{\delta^+}(w_2) < \#_{\delta^-}(w_2),\ \#_{\delta^+}(w_1w_2) < \#_{\delta^-}(w_1w_2)\Big\}\Big\|$$

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$$L = \{ aw_1aw_2aw_3a \mid w_1, w_2, w_3 \in (\delta^+ + \delta^-)^*,$$
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$$L \cap\ a(\delta^+)^*a(\delta^+)^*a(\delta^+)^*a =$$
$$= \left\| \{ a(\delta^+)^m a(\delta^+)^n a(\delta^+)^p a \mid m < n, n > p \} \right.$$

Context sensitive language !

Summary of the presentation

- Timed automata and clock relations
- Representing clock relations with n-automata
- Discretizations of timed languages
- Stack automata for discretizations of timed languages

Stack automata

A (one-way) **stack automaton** = Turing machine with two tapes – the input tape and the working tape – and n sets of distinguished states Q_i $(1 \leq i \leq n)$.

- The input tape is parsed in one pass.
- The working tape is unbounded in both directions.
- A given word is accepted iff, when its parse is finished, the machine arrives in a final state.

(Not exactly like Ginsburg's stack automata...)

Example

$$L = \left\{aw_1aw_2aw_3a \mid w_1, w_2, w_3 \in (1 + \delta^+ + \delta^-)^*, \right.$$
$$\left. \#_{\delta^+}(w_1w_2) < \#_{\delta^-}(w_1w_2), \#_{\delta^+}(w_2w_3) > \#_{\delta^-}(w_2w_3)\right\}$$

- Symbols on the working tape: $\perp, \delta^+, \delta^-$ plus four markers, 1, 2, 3 and 4.
- When meeting the i-th a we put an i on the working tape ($i = 1..4$).
- 3 must be to the right of 1 and 4 to the left of 2.

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Conjecture: The class of stack automata that are used in the nonarhimedian discretization of timed automata have a decidable emptiness problem.

Intersection

Theorem. The stack automata that represent timed automata are closed under intersection.

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Lemma. Given two stack automata, suppose that in each jointly reachable configuration both automata move their head in the same direction and/or write the same number of symbols and/or erase the same number of symbols.

Then the intersection of their languages can be accepted by a stack automaton too.

Conclusions

New discretization techniques for timed systems.

- Compositionality – concatenation, intersection.
- Classes of automata.
- Connections to trace theory ?