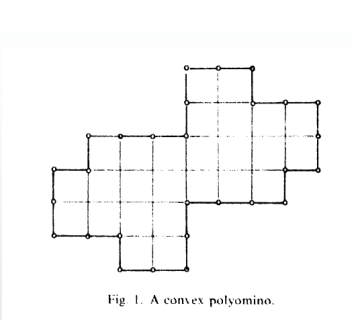
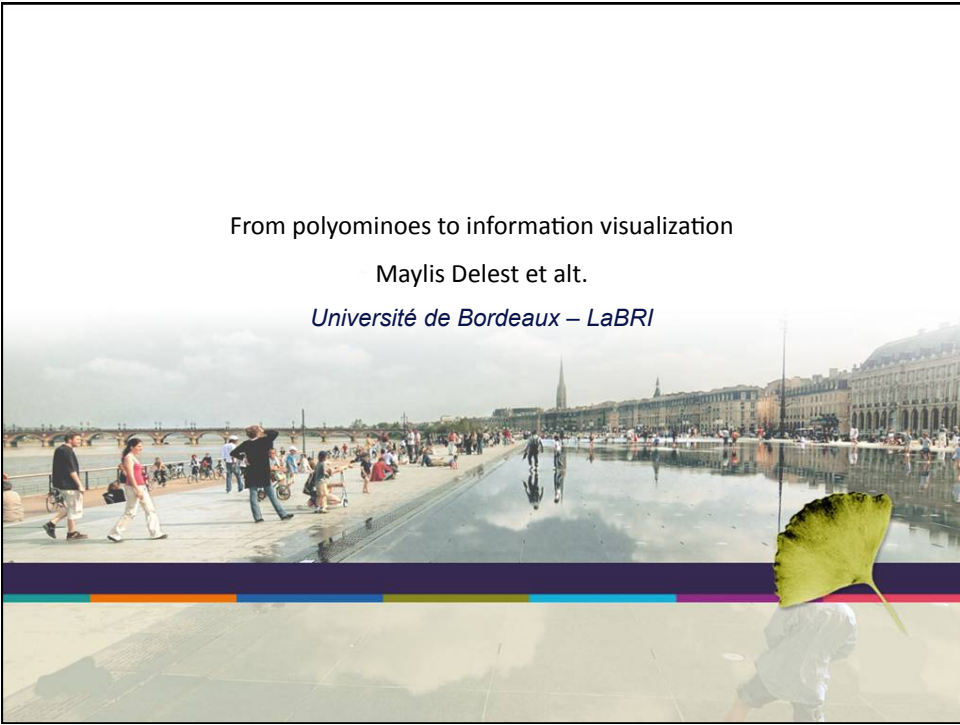
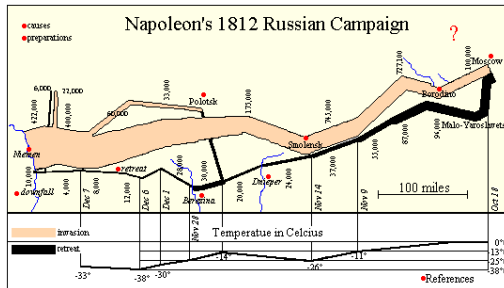


From polyominoes to information visualization

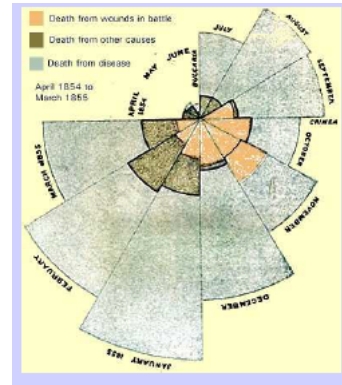
Maylis Delest et al.

Université de Bordeaux – LaBRI

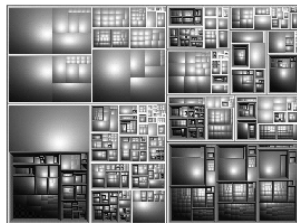




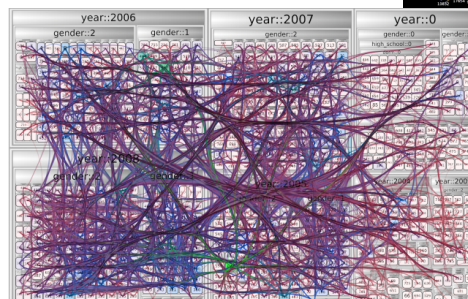
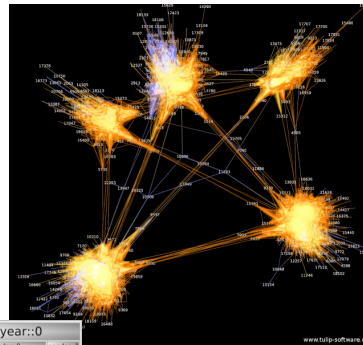
J. Minard (1850)



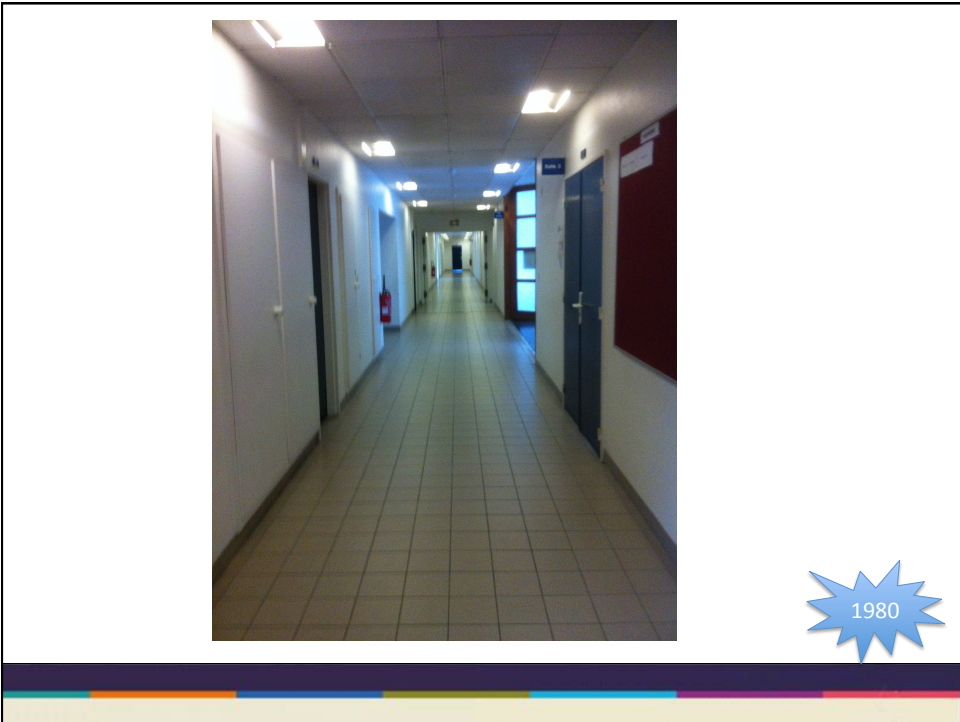
F. Nightingale (1850)



Cushion scarified (Van Wijk, 01)



Auber&Alt



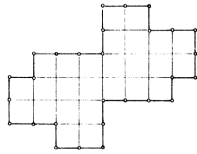


Fig. 1. A convex polyomino.

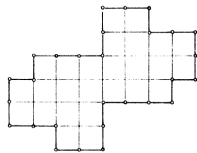
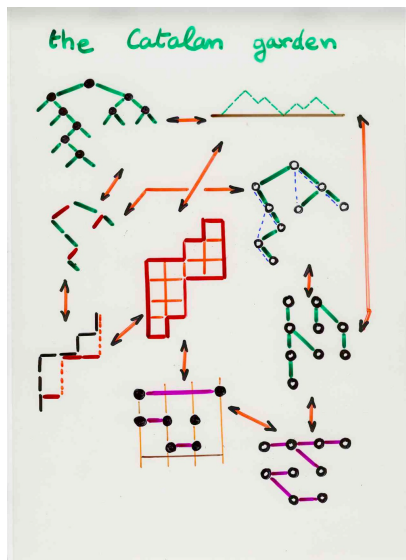


Fig. 1. A convex polyomino.





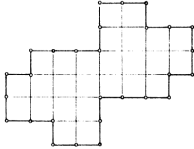
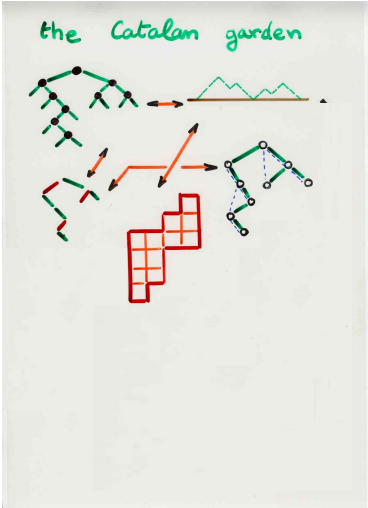



Fig. 1. A convex polyomino.

Macsyma



the Catalan garden

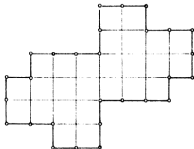
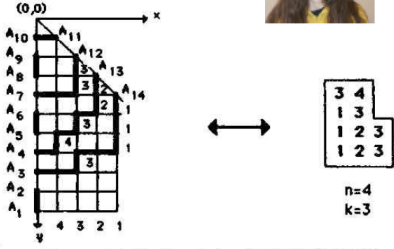



Fig. 1. A convex polyomino.



$(0,0)$

$x$

$y$

$A_{10}$

$A_9$

$A_8$

$A_7$

$A_6$

$A_5$

$A_4$

$A_3$

$A_2$

$A_1$

$A_{11}$

$A_{12}$

$A_{13}$

$A_{14}$

4 3 2 1

3 4

1 3

1 2 3

1 2 3

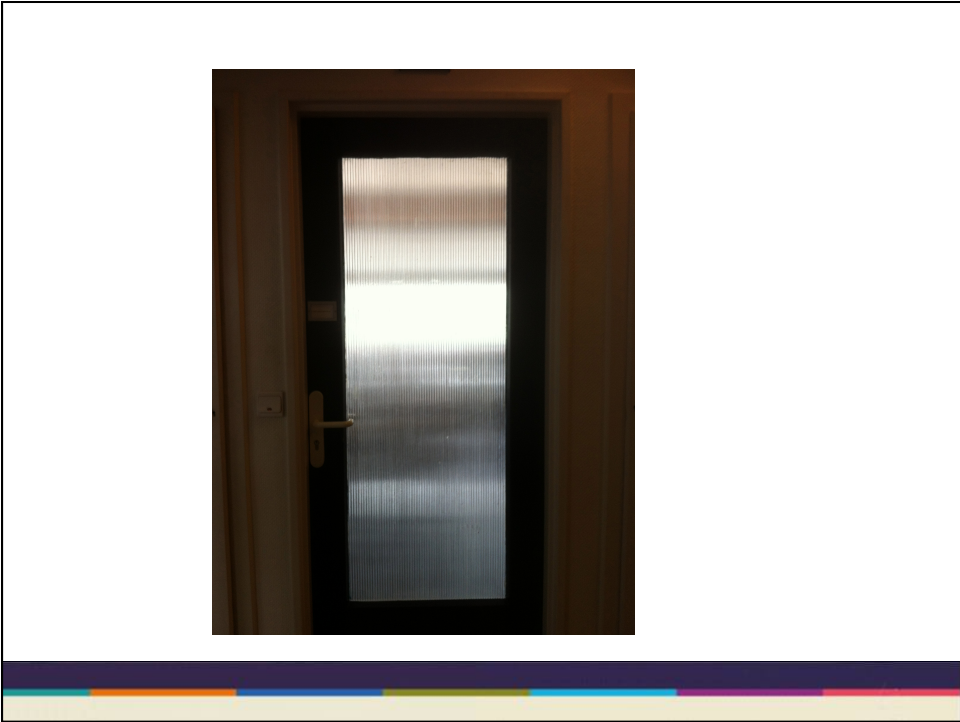
$n=4$

$k=3$

Figure 7. Bijection between Young tableaux and configurations of paths interpreting the coefficient  $Pf_{n,k}$ .

1981





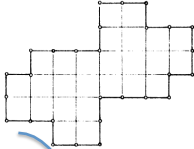
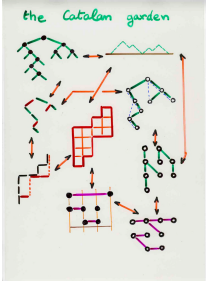


Fig. 1. A convex polyomino.



the Catalan garden

Macsyma

**Theorem 1.1.** The number  $p_{2n}$  of convex polyominoes having a perimeter  $2n$  is

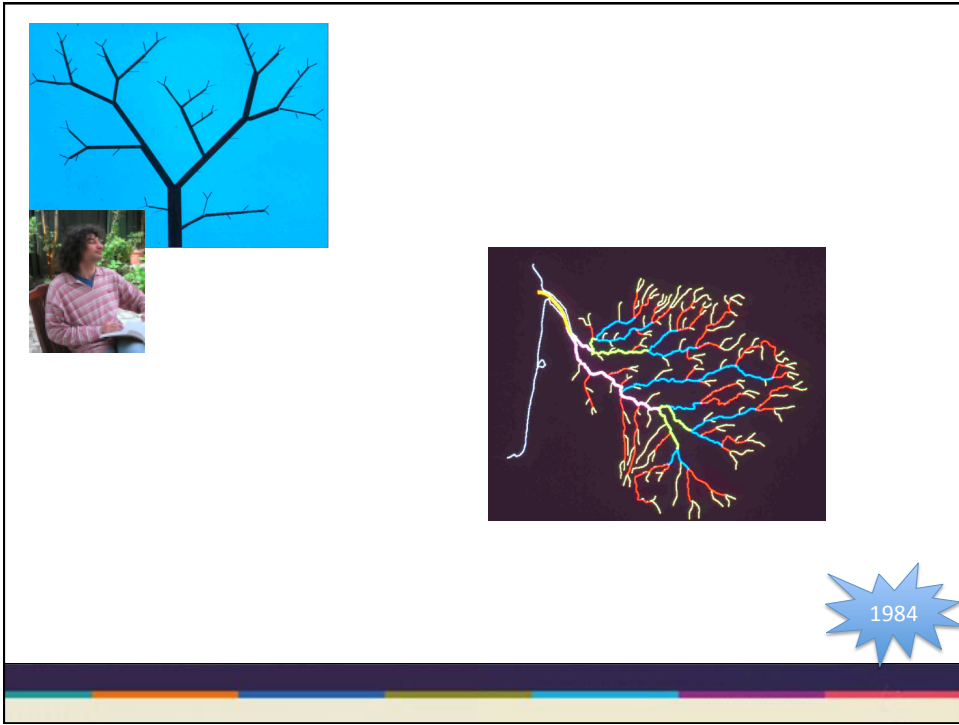
$$p_4 = 1, \quad p_6 = 2$$

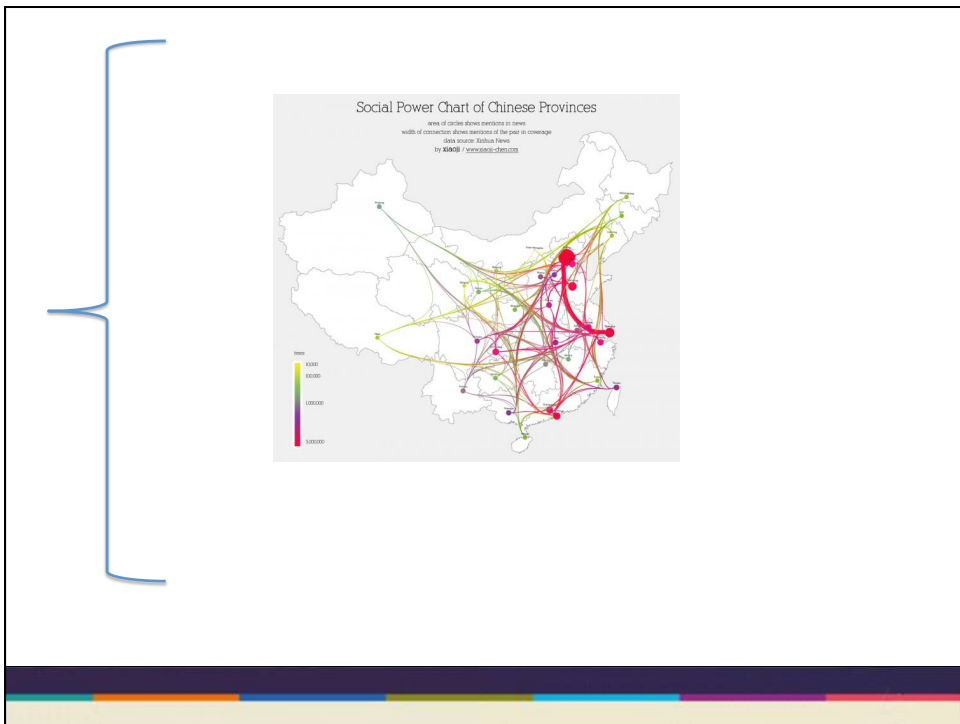
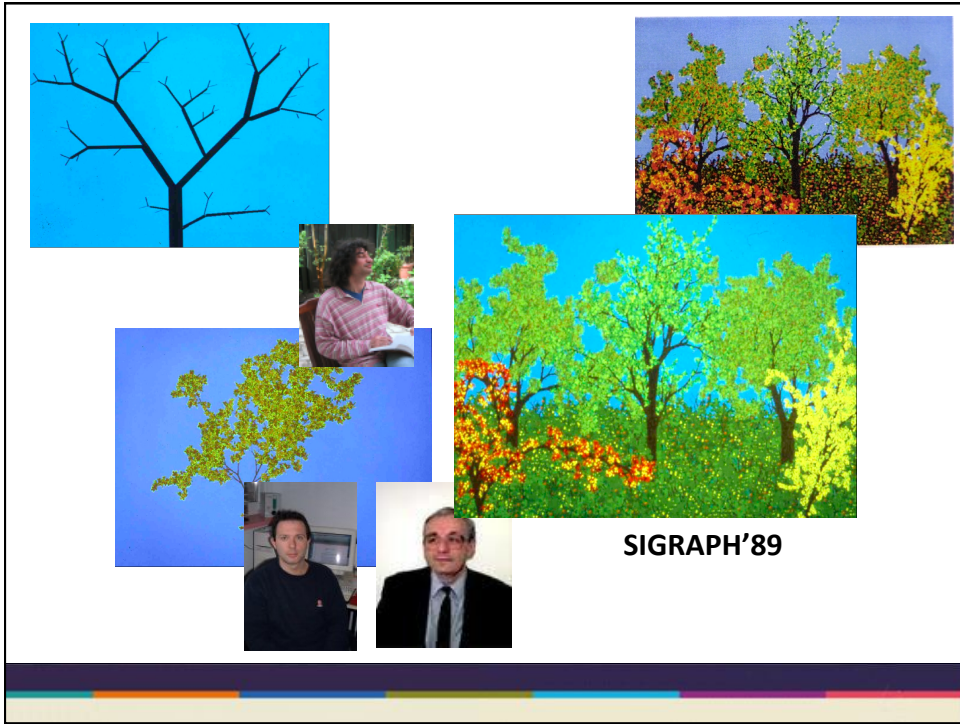
and for  $n \geq 0$ ,

$$p_{2n+8} = (2n+1)4^n - 4(2n+1)\binom{2n}{2}.$$





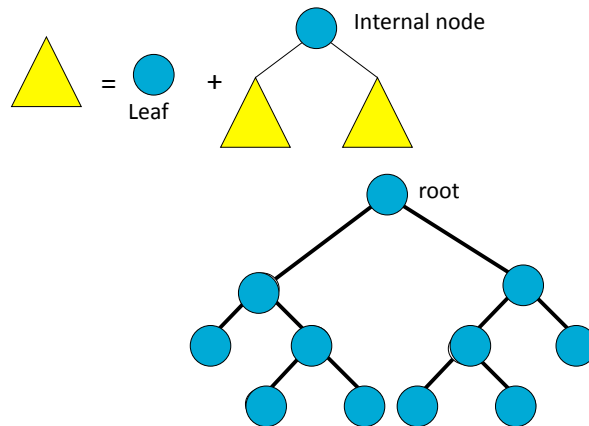





## What are Strahler numbers ?

Ershov 1958

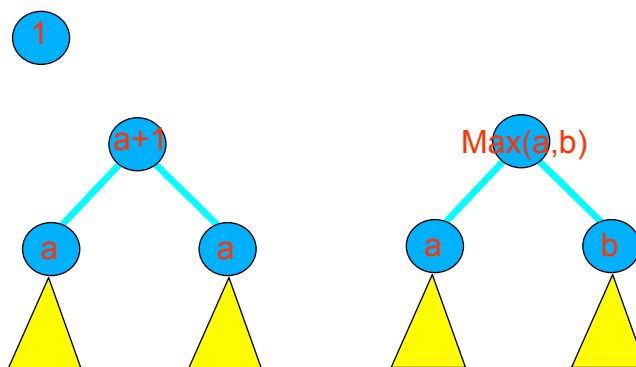
- Defined on **binary** trees



## What are Strahler numbers ?

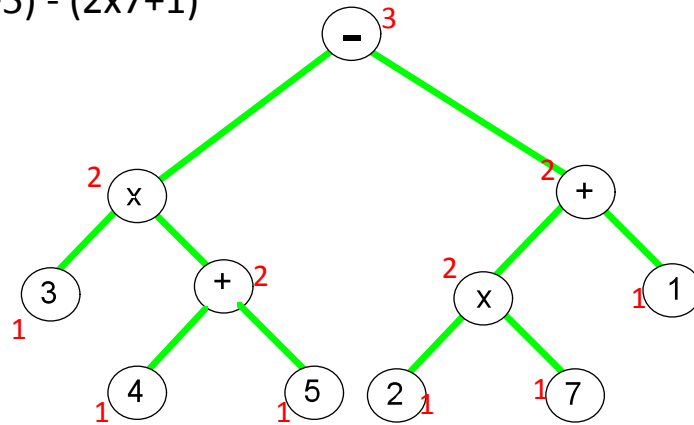
Ershov 1958

- Defined on **binary** trees



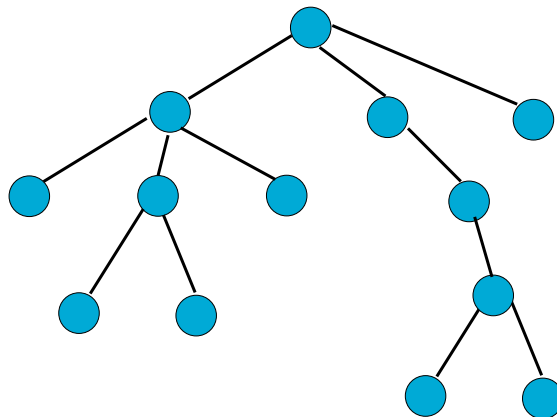
## What are Strahler numbers ?

- $3 \times (4+5) - (2 \times 7 + 1)$



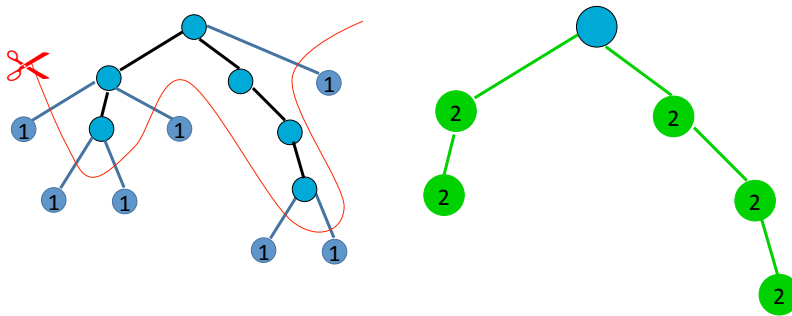
## Strahler – pruning

Vauchaussade Viennot (1985), Zeilberger (1990)  
Defined on rooted planar trees



## Strahler – pruning

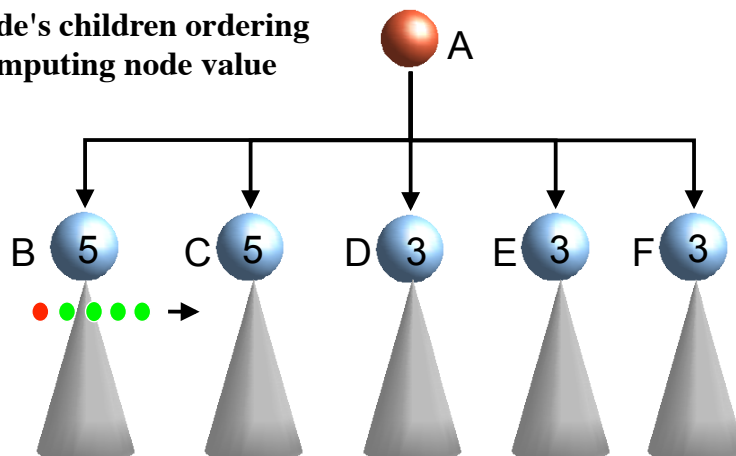
Vauchaussade Viennot (1985), Zeilberger (1990)  
 Définied on rooted planar maps



## Strahler – an other way

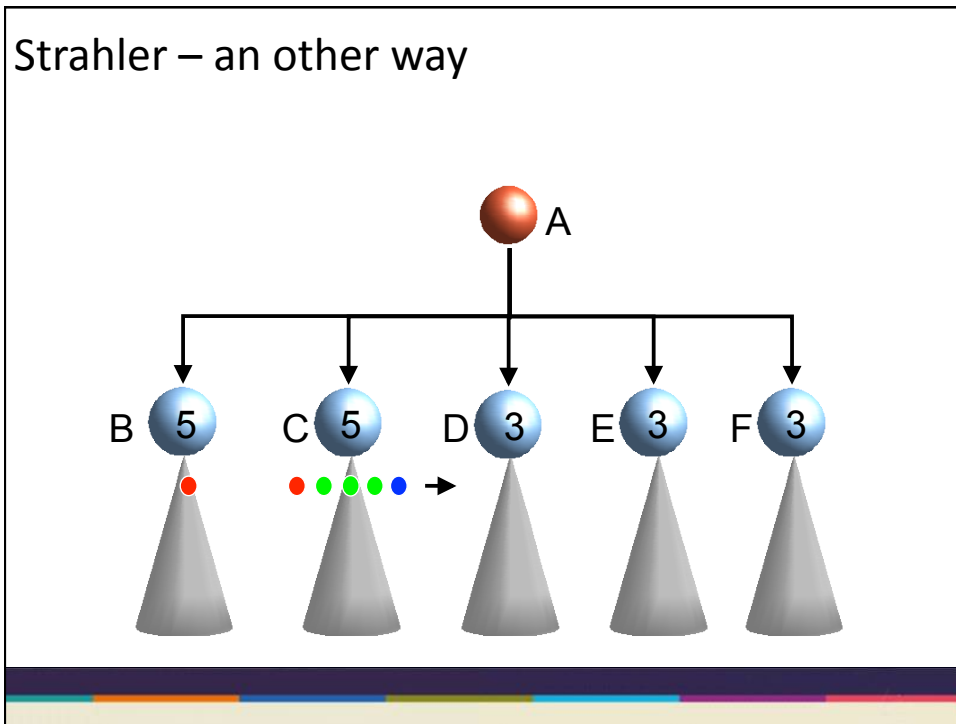
Auber&alt 2003

- Node's children ordering
- Computing node value

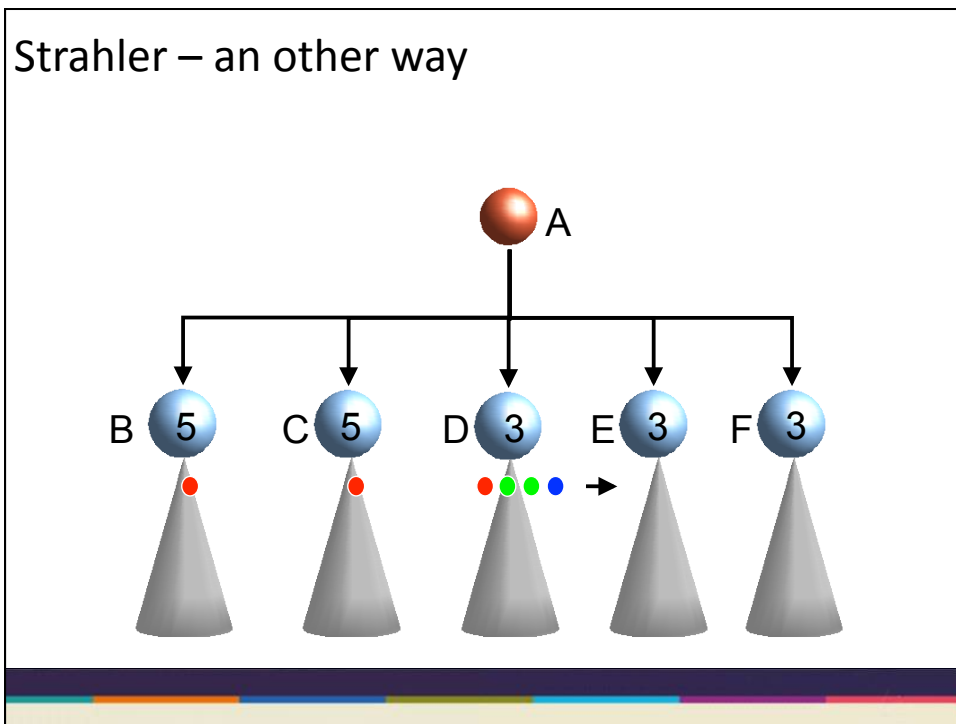


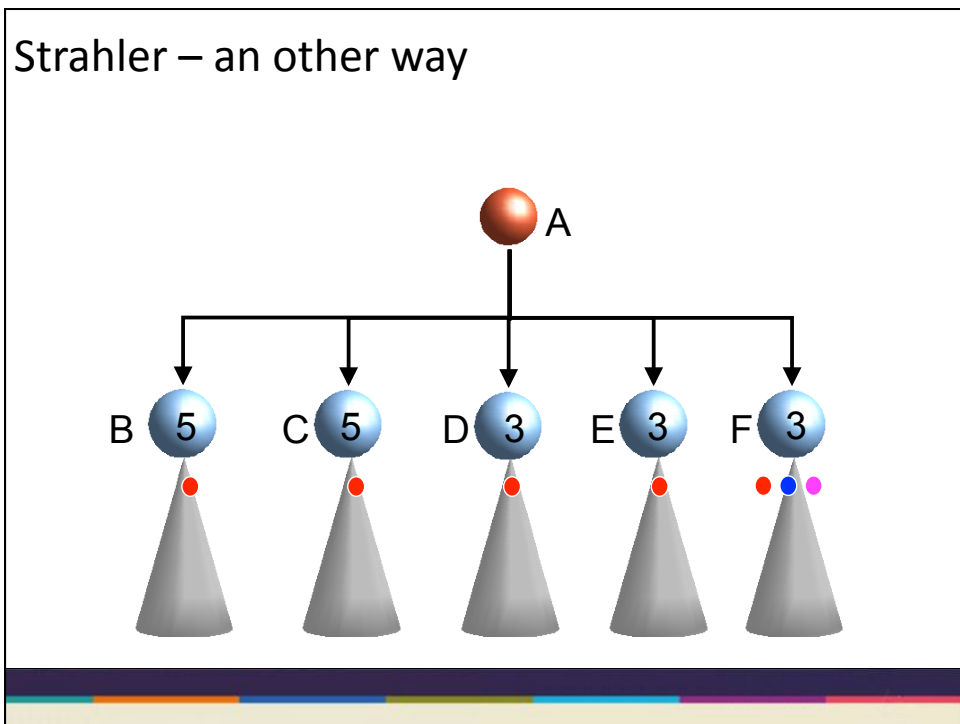
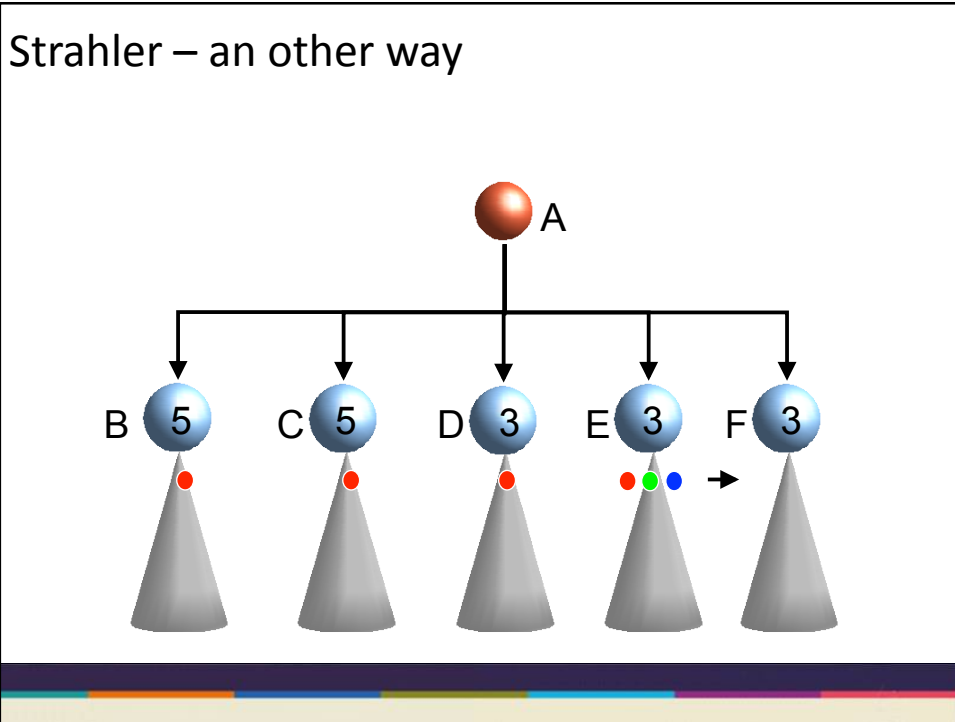


## Strahler – an other way

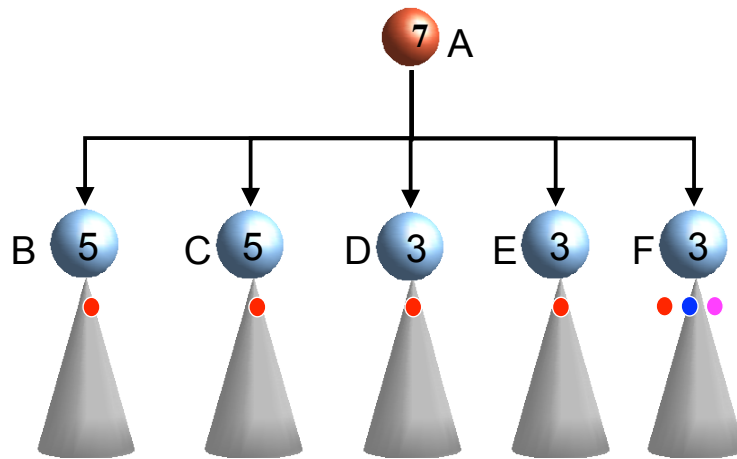


## Strahler – an other way





## Strahler – an other way



### New Strahler numbers for rooted plane trees

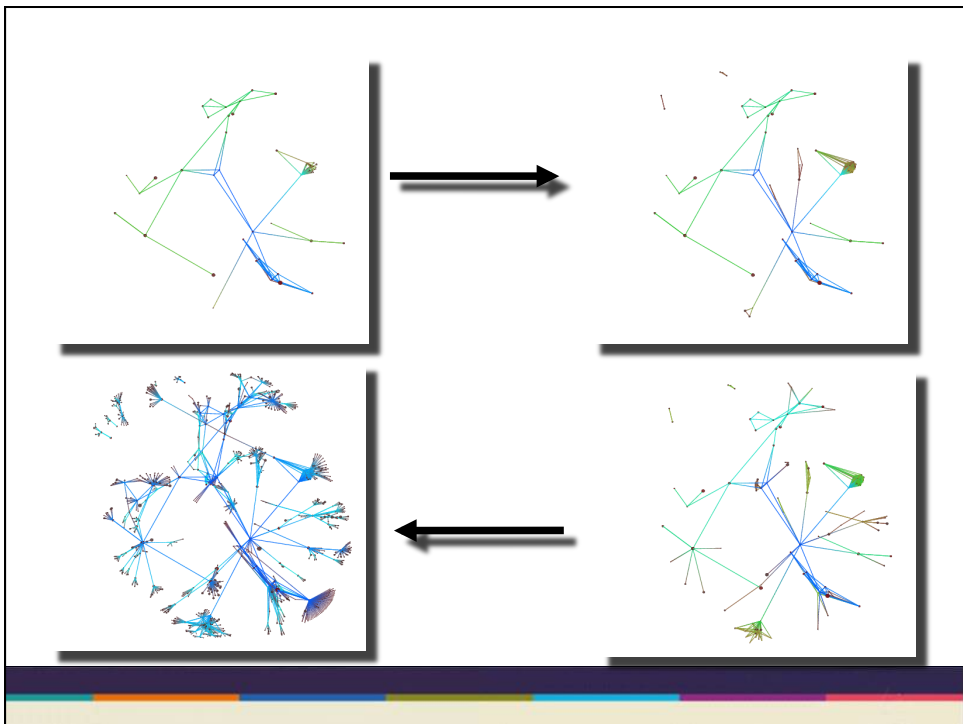
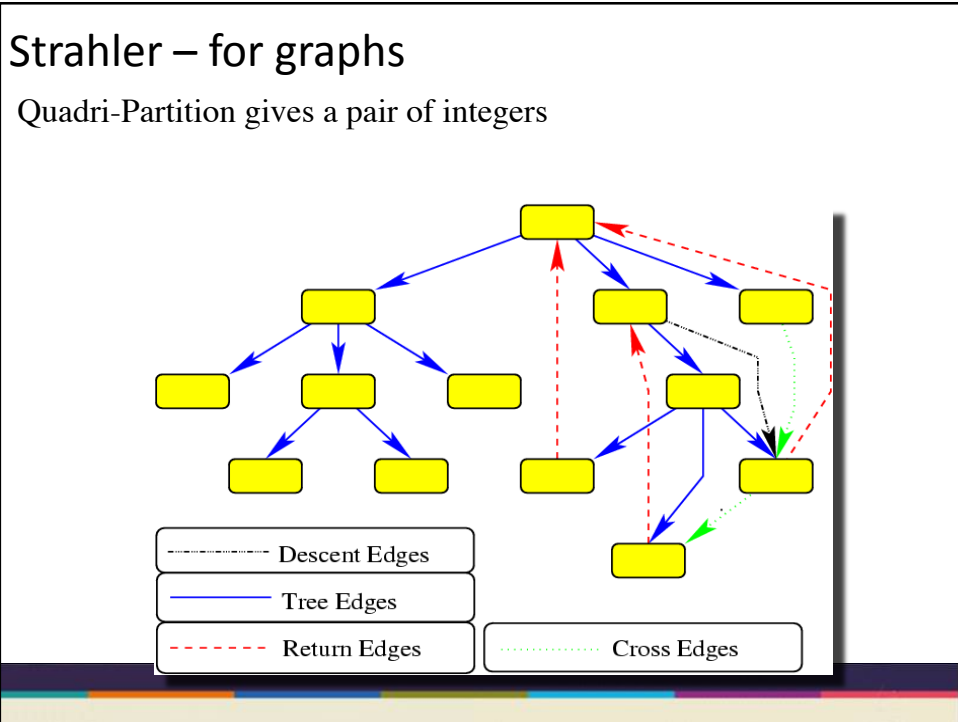
D. Auber, J.P. Domenger, M. Delest, P. Duchon, J.M. Fédou

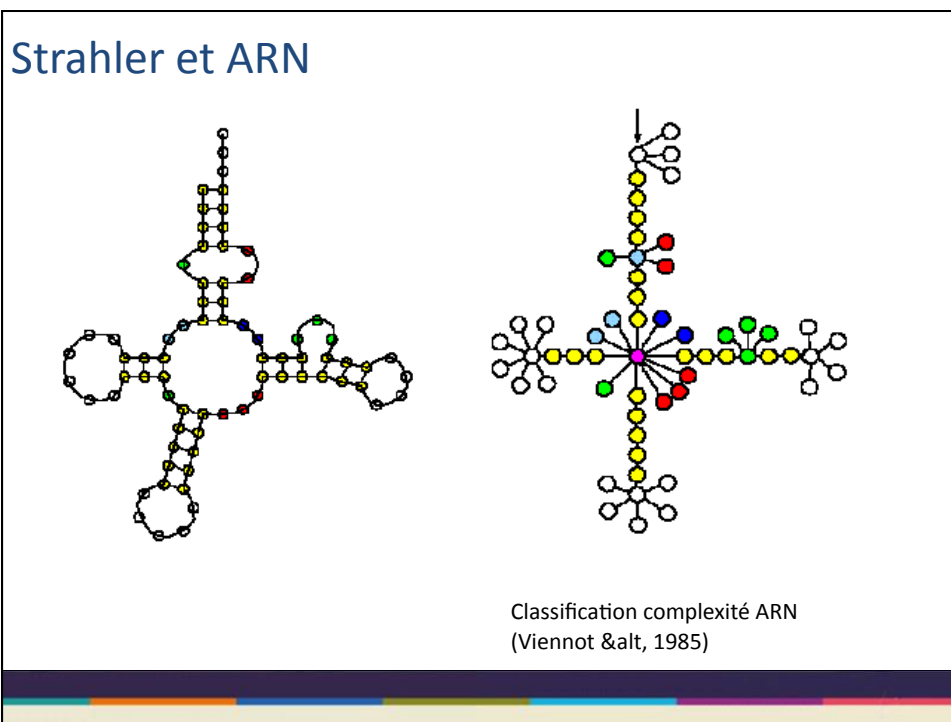
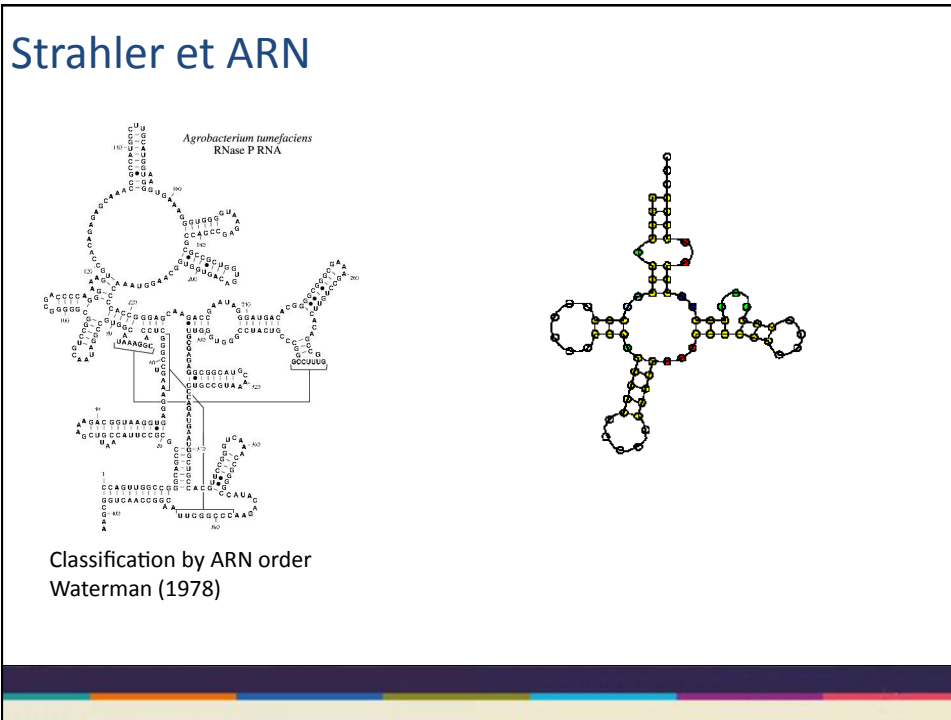
Proceedings of Third Colloquium on Mathematics and Computer Science  
Algorithms, Trees, Combinatorics and Probabilities 2004

**Theorem 5.5** *The branching ratio of trees of any family of finite degree  $D$  is asymptotically 4.*



1991

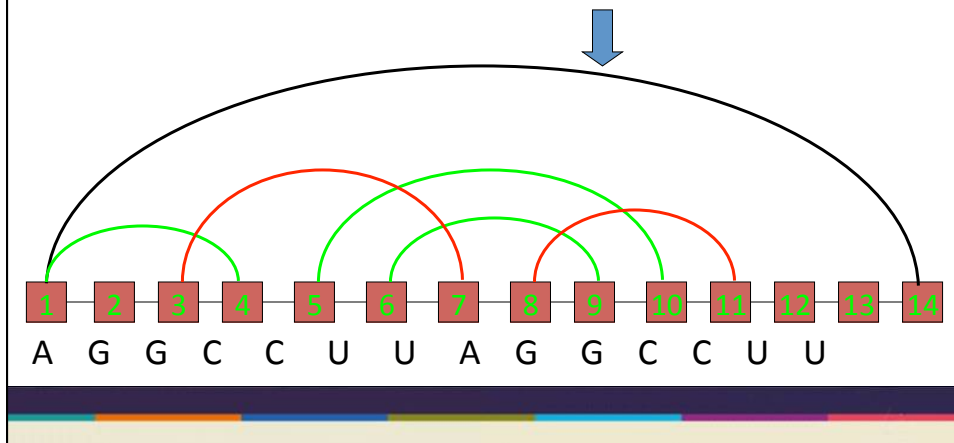




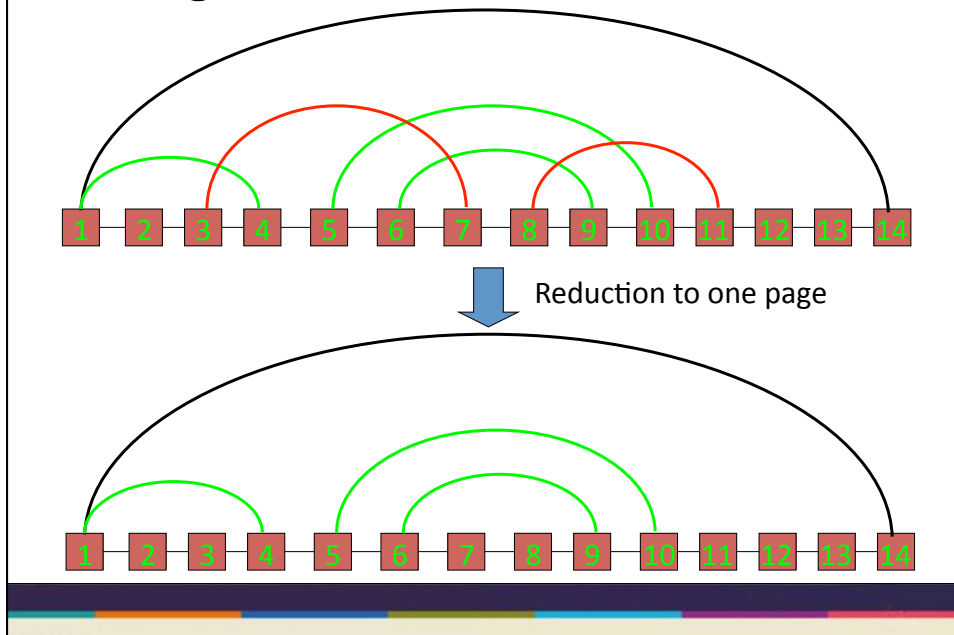


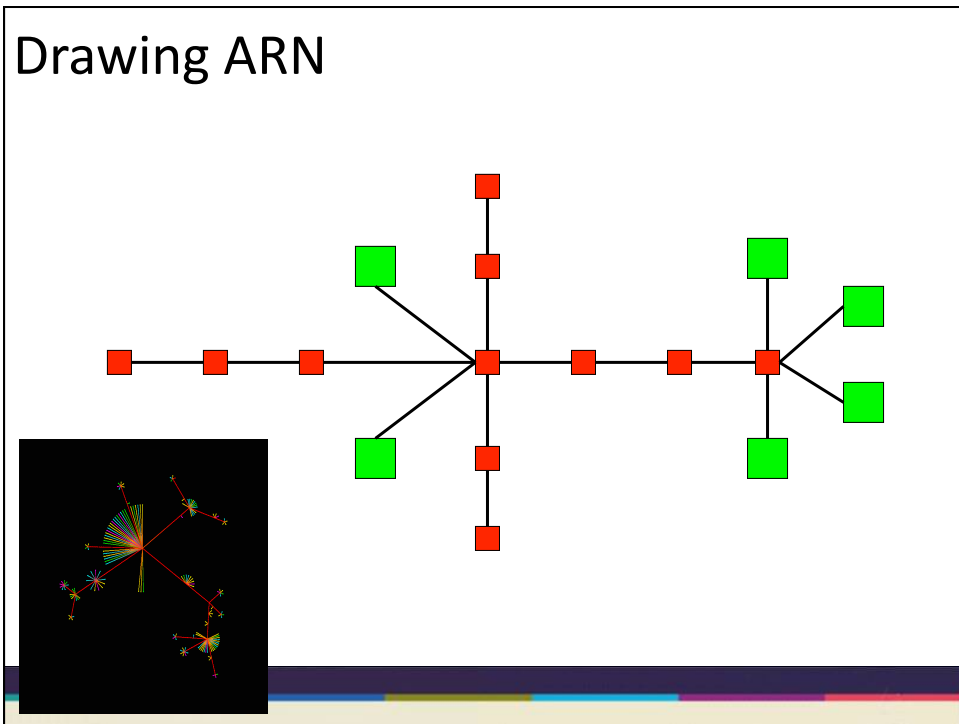
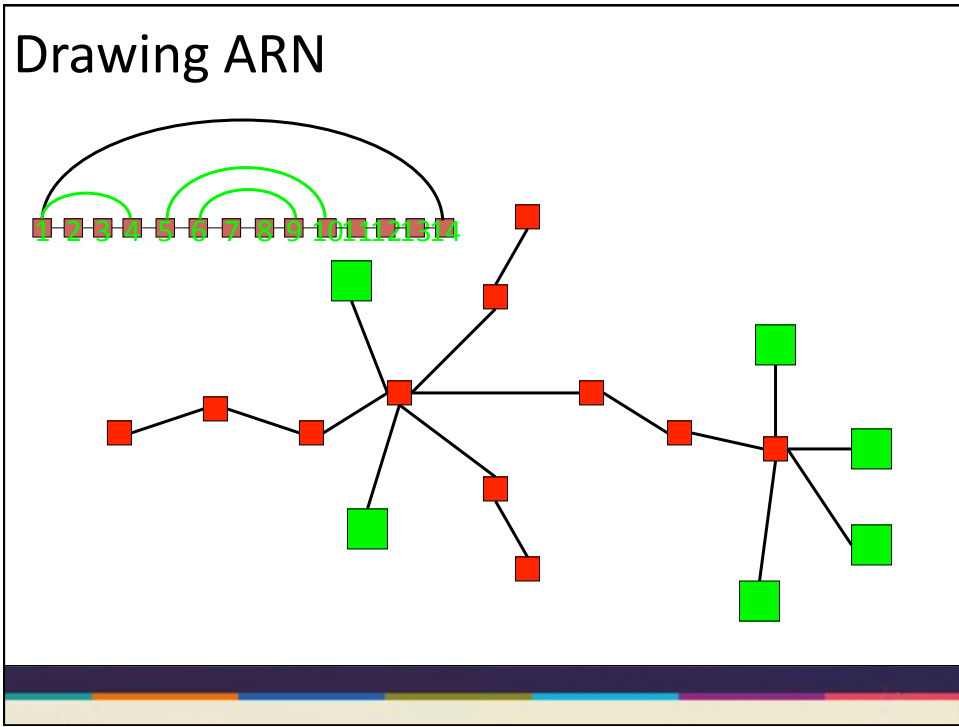
## Drawing ARN

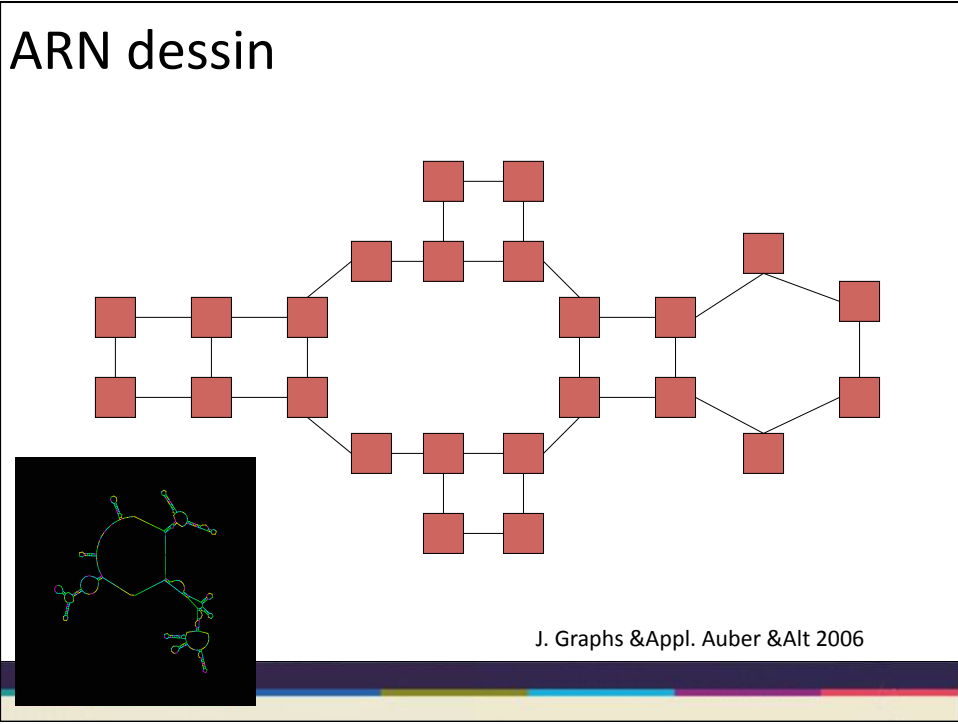
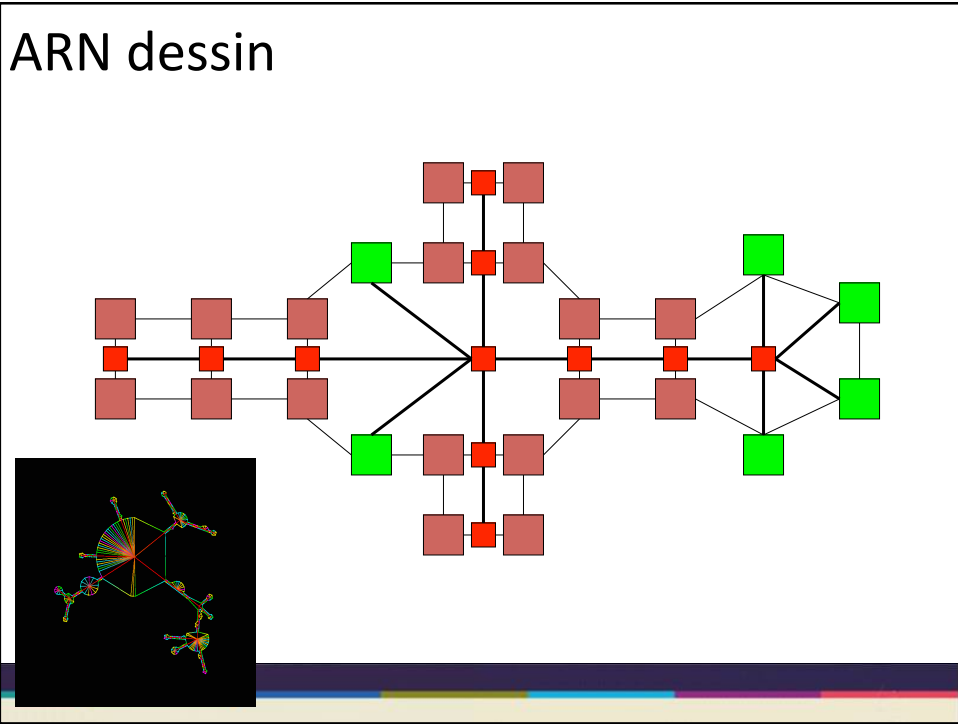
- Sequence : AGGCCUUAGGCCUU
- liens: 1, 14, 1, 4, 3, 7, 5, 10, 6, 9, 8, 11



## Drawing ARN







## ARN Visualization?

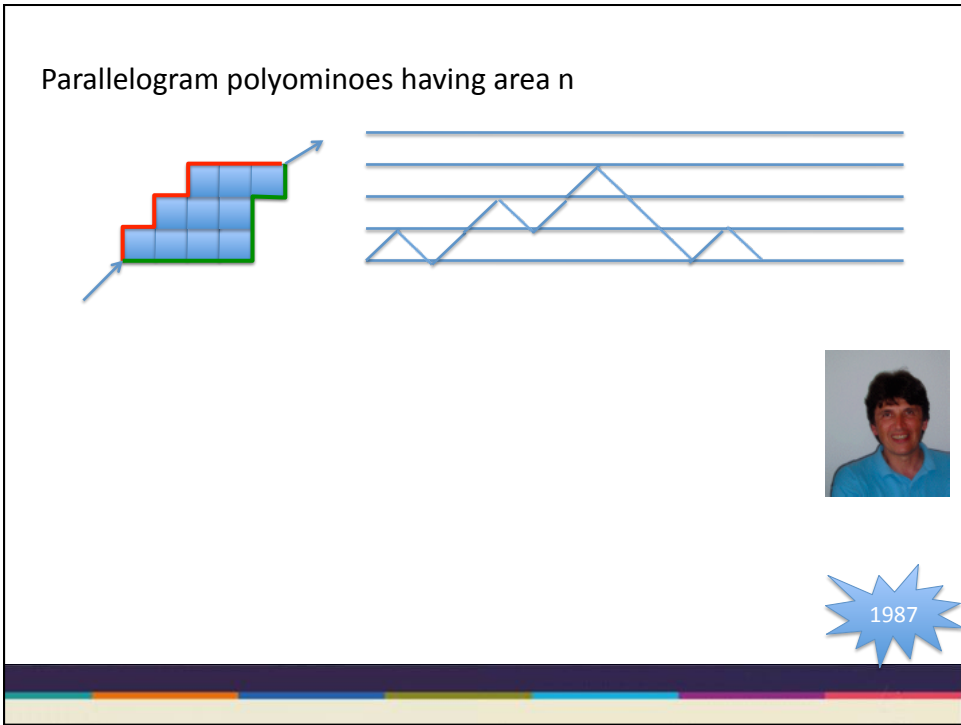
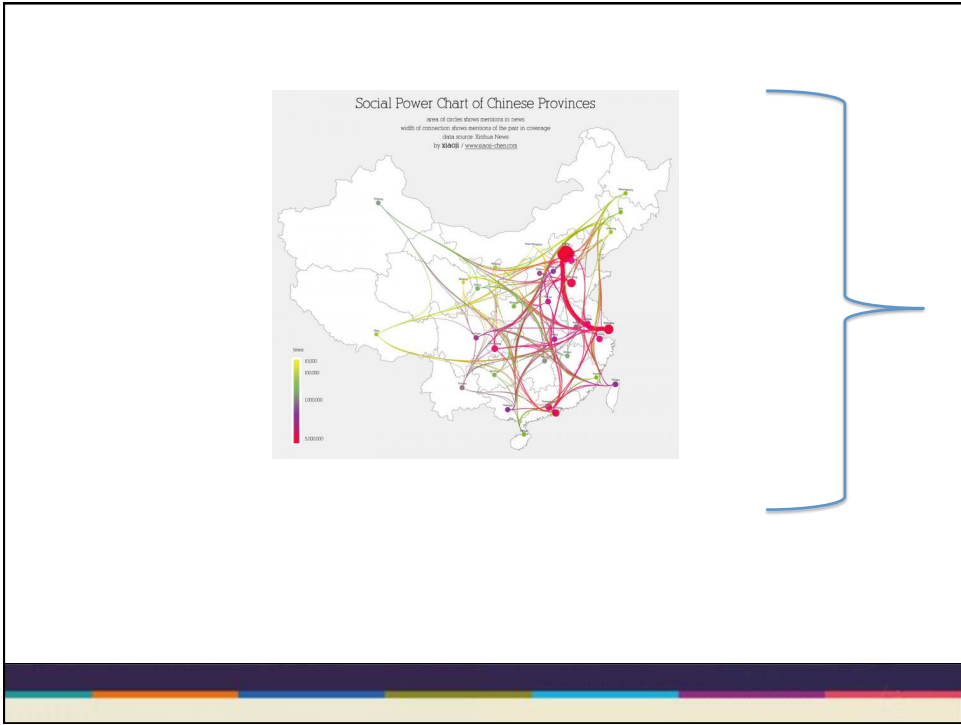
A U R S T

```
GGUUAAGUUAGAAGGGCGCACGGUGGUAUGCCUUGGCACUAGGAGCCGAUGAAGGACGGGACGAACACCGAUUAGCUUCGGGGAGCUUUAAGCAAGCUUUGAUCCGGAGAUUUCGGAUUGGGAAACCCACCACUCUAAU  
UCGGAUAGUUAUCCUGACGUJAUUACAUAGCGUUAAGGAAGGCAGACCCAGGGAACUGAAACAUCUAAGUACCUGGAGGAGAGAAGAAAAUUCGAUUUCUGAGUAGCGCGGAGCAACGGAAAGAGCCAAACCAAGAA
```

## ARN Visualization?

A U R S T

```
GGUUAAGUUAGAAGGGCGCACGGUGGUAUGCCUUGGCACUAGGAGCCGAUGAAGGACGGGACGAACACCGAUUAGCUUCGGGGAGCUUUAAGCAAGCUUUGAUCCGGAGAUUUCGGAUUGGGAAACCCACCACUCUAAU  
UCGGAUAGUUAUCCUGACGUJAUUACAUAGCGUUAAGGAAGGCAGACCCAGGGAACUGAAACAUCUAAGUACCUGGAGGAGAGAAGAAAAUUCGAUUUCUGAGUAGCGCGGAGCAACGGAAAGAGCCAAACCAAGAA
```





Parallelogram polyominoes having area n

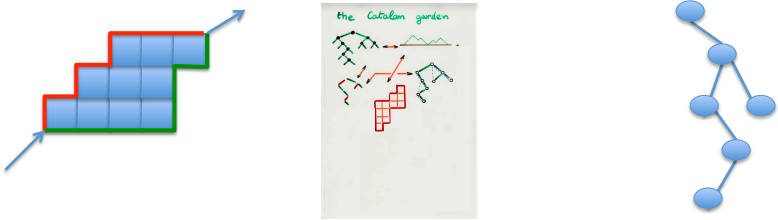
The diagram illustrates the correspondence between parallelogram polyominoes and Dyck paths. On the left, a polyomino is shown with a red top boundary and a green bottom boundary. On the right, a Dyck path is shown between two horizontal lines. In the center, a photograph shows a hand-drawn diagram titled "the Catalan garden" with various paths and polyominoes.

Parallelogram polyominoes having area n

The diagram illustrates the correspondence between parallelogram polyominoes and Dyck paths. On the left, a polyomino is shown with a red top boundary and a green bottom boundary. In the center, a photograph shows a hand-drawn diagram titled "the Catalan garden" with various paths and polyominoes. On the right, a Dyck path is shown between two horizontal lines.

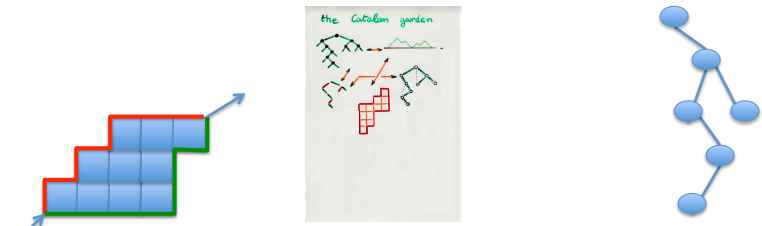
$$D = xD\bar{x}D + xt\bar{x}D + xD\bar{x} + xt\bar{x}$$

1988




$$D = xD\bar{x}D + x\bar{t}D + xD\bar{x} + x\bar{t}$$

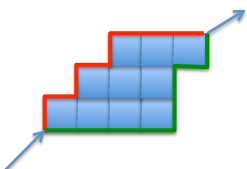
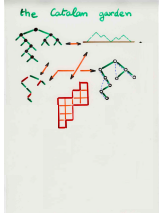

perimeter	Catalan
area	
Area and columes	



perimeter	Catalan
area	q-analog of Catalan
Area and columes	q-analog of Bessel functions

Maylis! Jean-Marc! It's great ! Fantastic !



<b>perimeter</b>	<b>Catalan</b>
area	q-analog of Catalan
Area and columes	q-analog of Bessel functions

Bijection with paths in binary trees (Fedou 89)

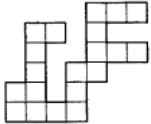
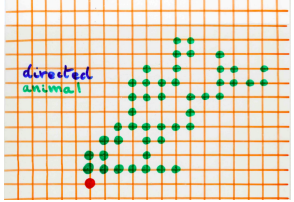

**Proposition 2.10** — Let  $T_{d,n}(x, y)$  be the area generating function for animals on the  $n$ -decorated triangular lattice:


$$T_{d,n}(x, y) = \sum_A x^{|T(A)|} y^{|A| - |T(A)|}$$

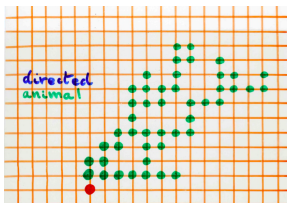
where  $|T(A)|$  is the number of cells of  $A$  on the underlying triangular lattice. We have

$$T_{d,n}(x, y) = T_1\left(x \frac{1-y^{n+1}}{1-y}, y^n \frac{1-y}{1-y^{n+1}}, 1, 1, 1\right)$$

where  $T_1(t, u, v, w, x)$  is defined by (8).

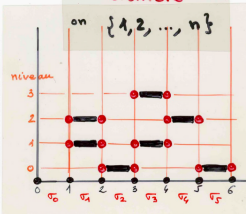




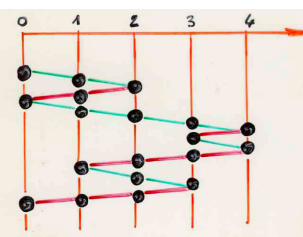


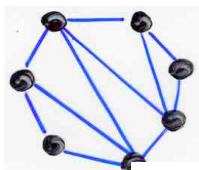


directed animal

heap of dimers  
on  $\{1, 2, \dots, n\}$

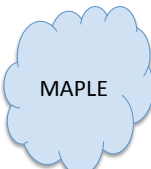




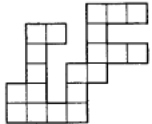


**Théorème 4.6** Les ensembles de permutations à motifs exclus  $S_n(1234, 1243, 1423, 4123)$ ,  $S_n(1324, 1342, 1432, 4132)$ ,  $S_n(2134, 2143, 2413, 4213)$ ,  $S_n(2314, 2413, 3142, 3241)$ ,  $S_n(1234, 1324, 2134, 2314)$ ,  $S_n(1234, 2134, 2314, 3124)$ ,  $S_n(1324, 2134, 2314, 3124)$ ,  $S_n(1324, 2134, 3124, 3214)$ ,  $S_n(1324, 2314, 3124, 3214)$ ,  $S_n(1342, 2341, 3142, 3241)$  et  $S_n(1324, 1342, 2314, 2341)$ , sont en bijection avec les mots de  $GD_{\leq 3}$  de longueur  $2n - 2$ , et sont donc énumérés par le  $(n - 1)^{\text{ème}}$  coefficient binomial central

$$\binom{2n - 2}{n - 1}$$



MAPLE



**Proposition 2.10** Let  $T_{n,r}(x, y)$  be the area generating function for excursions on the  $n$ -diamonded triangular lattice:

$$T_{n,r}(x, y) = \sum_{\pi \in \mathcal{E}(n,r)} x^{|\pi|} y^{area(\pi)}$$

where  $|\pi|$  is the number of cells of  $\pi$  on the underlying triangular lattice. We have

$$T_{n,r}(x, y) = \frac{1 - x^{n+1} y^r}{1 - x y^r} \cdot \frac{1 - x}{1 - x^{n+1} y^r}$$

where  $T_{n,r}(n, n, x, y)$  is defined by (9).













Combinatoire



BioInfo



1991







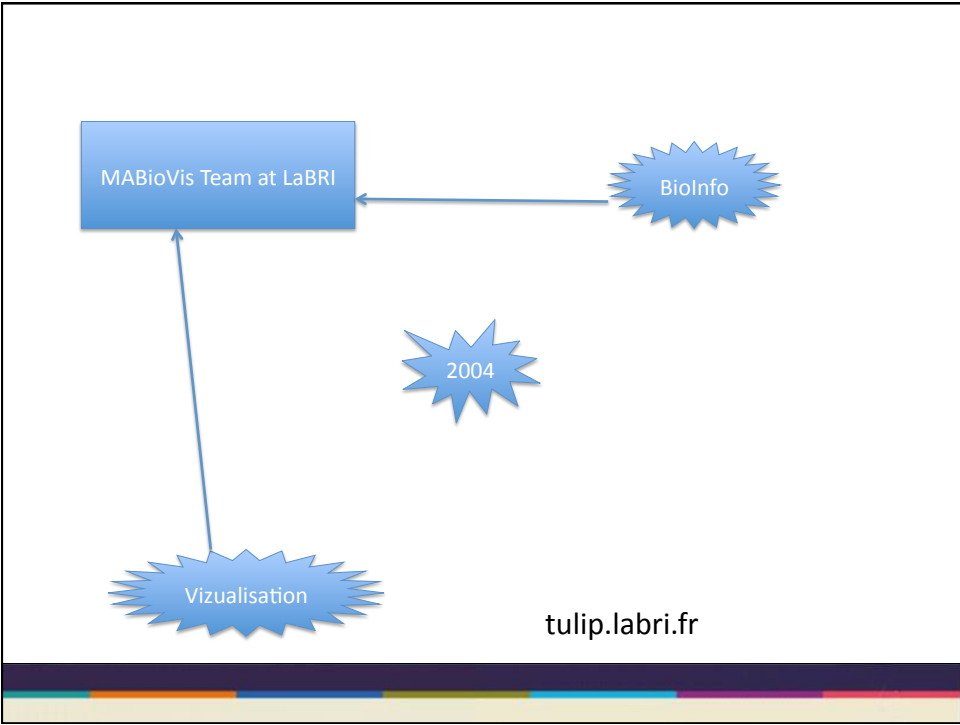


Vizualisation



Random generation



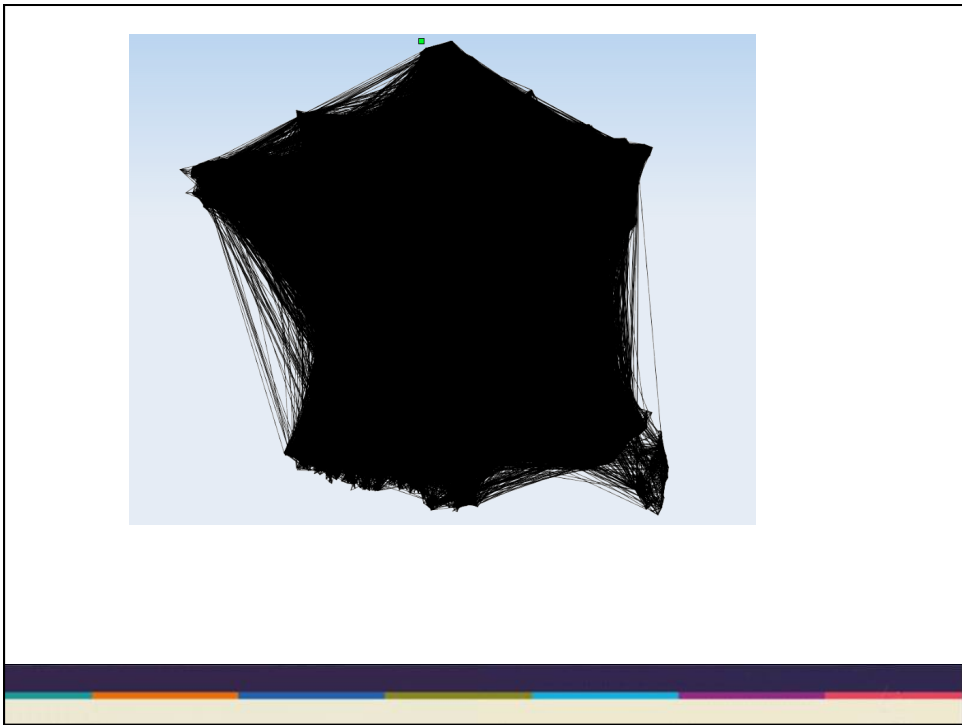
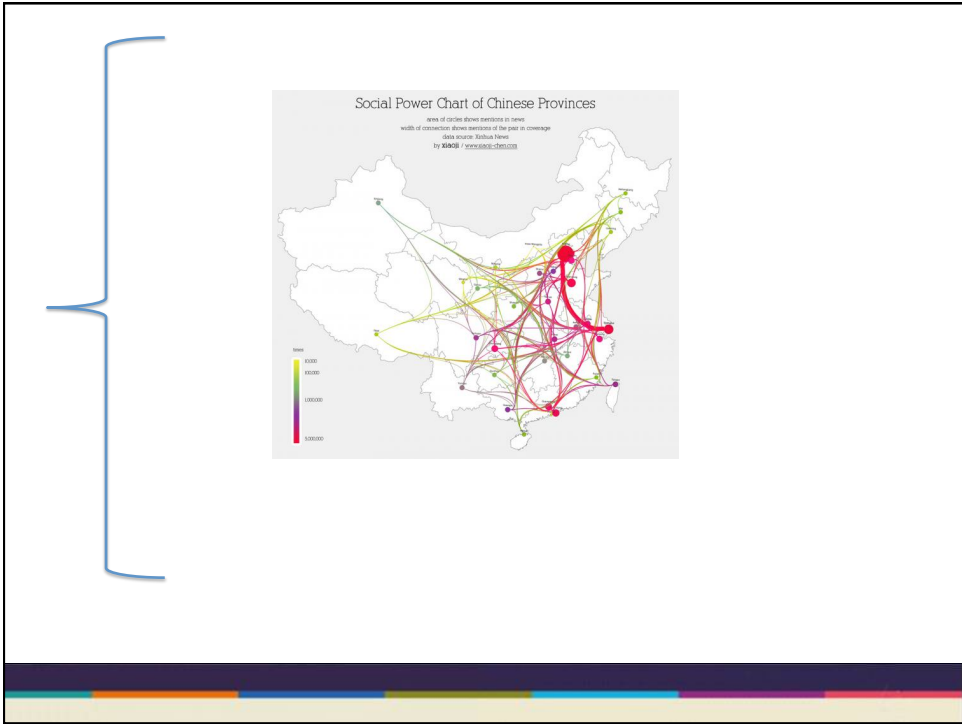


tulip.labri.fr

Vizualisation

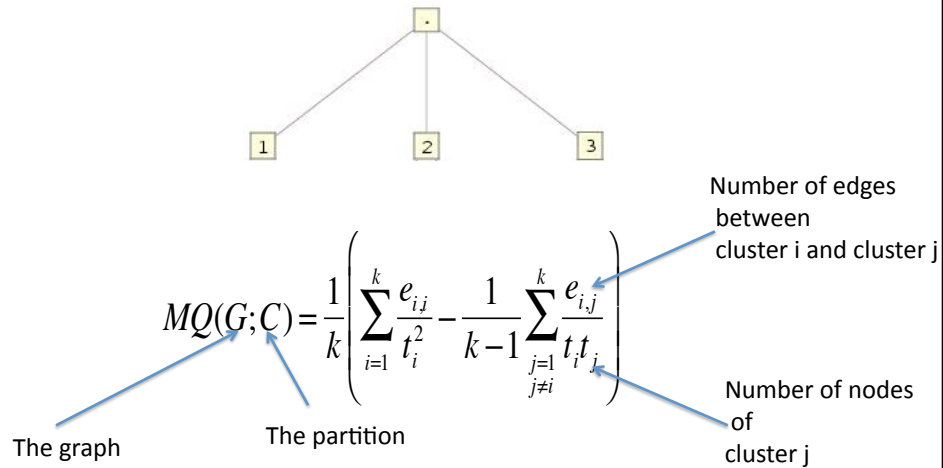
2003

The screenshot shows a complex software interface with multiple panels. On the left, there is a table with columns for 'Name', 'Type', 'Value', and 'Unit'. The main area contains several plots: a large network graph, a bar chart, a line graph, and a 3D molecular model. Below the screenshot is a portrait of a person with curly hair and glasses. The year '2003' is indicated in a blue starburst at the bottom right. The website 'tulip.labri.fr' is written below the screenshot. The word 'Vizualisation' is written in a blue starburst at the bottom left.

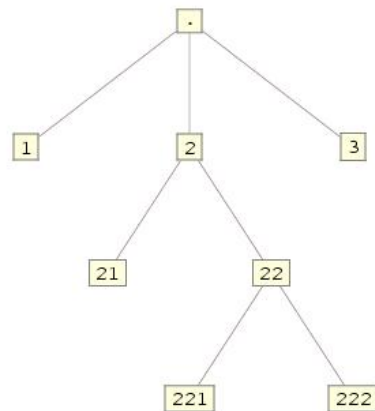


Many algorithms split data based on a quality measure

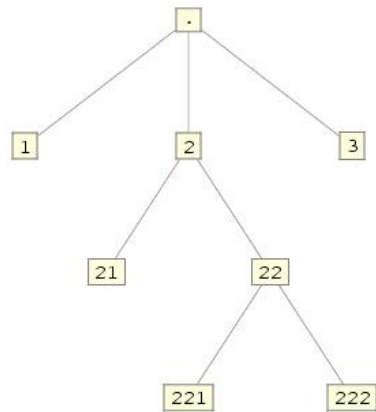
Manconridis (98)



Usually one algorithm is run recursively on the graph  
-> fragmentation tree



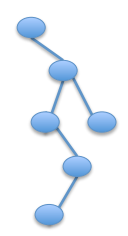
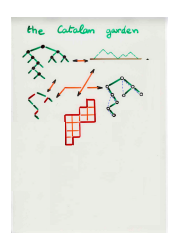
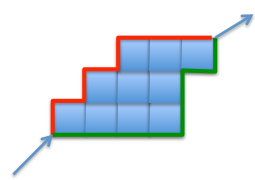
Usually one algorithm is run recursively on the graph  
 -> fragmentation tree



Is it possible to measure the quality of the whole tree ?



2008



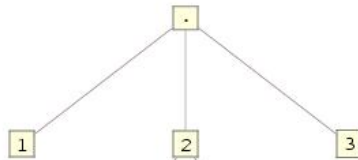
perimeter	Catalan
area	q-analog of Catalan
Area and columns	q-analog of Bessel functions

Bijection with paths in binary trees (Fedou 89)



First : modify MQ

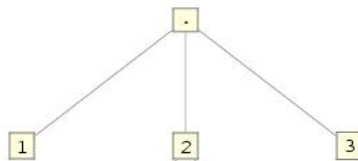
$$MQ(G;C) = \frac{1}{k} \left( \sum_{i=1}^k \frac{e_{ii}}{t_i^2} - \frac{1}{k-1} \sum_{\substack{j=1 \\ j \neq i}}^k \frac{e_{i,j}}{t_i t_j} \right)$$



$$MQ(G;C) = \frac{1}{\sum_{i=1}^k t_i} \sum_{i=1}^k \left( \frac{t_i e_{ii}}{t_i^2} \right) - \frac{1}{\sum_{i=1}^k \sum_{\substack{j=1 \\ j \neq i}}^k t_i t_j} \sum_{\substack{j=1 \\ j \neq i}}^k \frac{t_i t_j e_{i,j}}{t_i t_j}$$

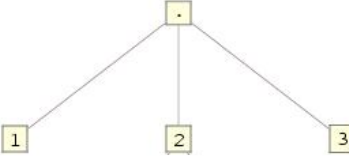
Second : simplify notations

$$MQ(G;C) = \frac{1}{\sum_{i=1}^k t_i} \sum_{i=1}^k \left( \frac{t_i e_{ii}}{t_i^2} \right) - \frac{1}{\sum_{i=1}^k \sum_{\substack{j=1 \\ j \neq i}}^k t_i t_j} \sum_{\substack{j=1 \\ j \neq i}}^k \frac{t_i t_j e_{i,j}}{t_i t_j}$$



$$MQ(G;C) = \frac{1}{\sum_{i=1}^k x_i} \left( \sum_{i=1}^k \frac{x_i e_{ii}}{\delta_i} - \frac{1}{\sum_{i=1}^k \sum_{\substack{j=1 \\ j \neq i}}^k x_i} \sum_{\substack{j=1 \\ j \neq i}}^k \frac{x_i e_{i,j}}{\delta_{i,j}} \right)$$

Second : simplify notations



$$MQ(G; C) = \frac{1}{\sum_{i=1}^k x_i} \left( \sum_{i=1}^k x_i \frac{e_{i,i}}{\delta_i} - \frac{1}{\sum_{\substack{j=1 \\ i \neq j}}^k x_i} \sum_{j=1}^k x_i \frac{e_{i,j}}{\delta_{i,j}} \right)$$

$\alpha_i$  (under  $\delta_i$ )       $\beta_i$  (under  $\sum_{i \neq j}$ )

$$MQ(G, T; q) = \begin{cases} 0 & \text{if } k=0 \\ \frac{1}{X} \sum_{i=1}^k x_i (\alpha_i - \beta_i) (1 + qMQ(G_i, T_i; q)) & \text{otherwise} \end{cases}$$

- This formula makes sense on small tractable examples
- The same idea can be applied to others quality measures
- It can be interpreted as sum of paths in the graph



