

Partitions of graphs with bounded maximum average degree

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A graph G is called *improperly* (d_1, \dots, d_k) -colorable, or just (d_1, \dots, d_k) -colorable, if the vertex set of G can be partitioned into subsets V_1, \dots, V_k such that the graph $G[V_i]$ induced by the vertices of V_i has maximum degree at most d_i for all $1 \leq i \leq k$. This notion generalizes those of proper k -coloring (when $d_1 = \dots = d_k = 0$) and d -improper k -coloring (when $d_1 = \dots = d_k = d \geq 1$).

Proper and d -improper colorings have been widely studied. As shown by Appel and Haken [1, 2], every planar graph is 4-colorable, i.e. $(0, 0, 0, 0)$ -colorable. Eaton and Hull [3] and independently Škrekovski [5] proved that every planar graph is 2-improperly 3-colorable (in fact, 2-improper choosable), i.e. $(2, 2, 2)$ -colorable.

This latter result was extended by Havet and Sereni to not necessarily planar sparse graphs as follows:

Theorem. [4] *For every $k \geq 0$, every graph G with $\text{mad}(G) < \frac{4k+4}{k+2}$ is k -improperly 2-colorable (in fact k -improperly 2-choosable), i.e. (k, k) -colorable.*

We recall that $\text{mad}(G) = \max \left\{ \frac{2|E(H)|}{|V(H)|}, H \subseteq G \right\}$ is the maximum average degree of a graph G .

In this talk we will present obtained results concerning the (k, j) -colorability for some values of k and j ($k \neq j$) for graphs with a given maximum average degree.

References

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