

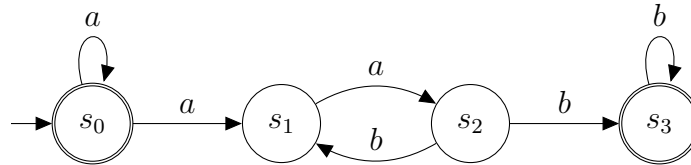
Université Bordeaux 1. Master Sciences & Technologies,
 Informatique. Examen “Games and synthesis”.

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Session 1. January 7, 2015, 14h-17h.

Documents: lecture notes and exercise class material.

Exercise 1 A non-deterministic Büchi ω -automaton is called *weak* if every strongly connected component of the automaton contains only accepting states or only non-accepting states (a strongly connected component is a maximal, non-empty set C of vertices such that for every two vertices u, v in C , there is a path from u to v). The figure below represents such an automaton \mathcal{A} with $F = \{s_0, s_3\}$:



1. Describe the language accepted by \mathcal{A} by a ω -regular expression.
2. Give a *deterministic* automaton that accepts $L(\mathcal{A})$ (you can choose the acceptance condition).
3. Show how to complement a weak Büchi automaton into a Büchi automaton (*Hint*: use the subset construction.)

Exercise 2 A Büchi automaton accepts an infinite, Σ -labelled tree t , if there is a run on t such that on every path from the root some final state is seen infinitely often.

Propose a polynomial time algorithm for checking emptiness of Büchi tree automata. Justify your answer.

Exercise 3 A request-response game is a 2-player game played on a graph arena $(V = V_0 \cup V_1, E)$, where the winning condition consists of pairs $(Q_i, R_i)_{i=1}^k$ of requests $Q_i \subseteq V$ and responses $R_i \subseteq V$. A play $\pi = v_0, v_1, \dots$ is winning if for all $1 \leq i \leq k$, whenever a state in Q_i is visited, a state in R_i is visited now or later:

$$\bigwedge_{i=1}^k \mathbf{G}(Q_i \rightarrow \mathbf{FR}_i)$$

1. Consider the arena

$$V_0 = \{1, 4, 5\}, V_1 = \{2, 3\}, E = \{(1, 2), (1, 3), (2, 4), (2, 5), (3, 4), (3, 5), (4, 2), (5, 3)\}$$

The winning condition is $(Q_1, R_1), (Q_2, R_2)$ with $Q_1 = \{1\}$, $R_1 = \{4\}$, $Q_2 = \{2\}$, $R_2 = \{4, 5\}$.

Determine the winning regions of the two players.

2. We will now reduce a request-response game to a Büchi game.
 - (a) Let $A, B \subseteq \Sigma$ be two alphabets, not necessarily disjoint. Give a *deterministic* Büchi automaton accepting the language $\mathbf{G}(A \rightarrow \mathbf{F}B)$.
 - (b) Let $A_i, B_i \subseteq \Sigma$ be subalphabets, not necessarily disjoint. Give a *deterministic* Büchi automaton accepting the language $\bigwedge_{i=1}^k \mathbf{G}(A_i \rightarrow \mathbf{F}B_i)$.
 - (c) Describe the complete reduction of the a request-response game to a Büchi game.