

TD1 “Logique et langages”

**Exercise 1** The *shuffle* product of two words  $u, v \in \Sigma^*$  is defined as a set of words  $u \parallel v$  as follows:

$$u \parallel v = \{u_0v_0u_1v_1 \cdots u_nv_n \mid u_i, v_j \in \Sigma^+ \text{ for } i > 0, j \geq 0, u = u_0u_1 \cdots u_n, v = v_0v_1 \cdots v_n\}$$

For languages  $L, K \subseteq \Sigma^*$  we define  $L \parallel K = \bigcup_{u \in L, v \in K} u \parallel v$ .

Show that if  $L, K$  are regular, then  $L \parallel K$  is regular, too.

**Exercise 2** Give formulas (MSO or FO) for the following regular languages. Try to give an FO formula, if one exists - otherwise argue that no FO formula is possible.

1. The set of all words over  $\{a, b\}$  such that there is an  $a$  at an even position.
2. The binary encodings of the (positive) solution of  $y = 5x$ .
3.  $(aa + b)^*(aba)^*a^\omega$ .

**Exercise 3** What kind of properties on words can be specified by first-order logic that uses only the labeling predicates  $P_a$  ( $a \in \Sigma$ ) and the successor as atomic predicates?

**Exercise 4** Write *first-order formulas* expressing the graph properties listed below.

- The graph is complete.
- The graph contains a clique of size 4.
- Every vertex is connected to a vertex of out-degree 3.

**Exercise 5** Write formulas of *existential second-order logic* ( $\exists$  SO) expressing the graph properties listed below.

- The graph is bipartite.
- The graph is strongly connected.
- The graph is colorable with 3 colors.
- The graph contains a clique of size  $|U|$ . Here  $U$  is a subset of nodes of  $G$ , and it is described by an (additional) unary predicate  $U(x)$ .
- The graph contains a hamiltonian cycle.