

Exercise 1 “Reachability” means below the predicate $R(x, y)$, with $R(u, v)$ iff there is a path from vertex u to vertex v .

1. Show that on undirected graphs, reachability is definable by an \exists MSO formula (i.e., of the form $\exists X_1 \dots \exists X_k \varphi(X_1, \dots, X_k)$, where φ is an FO formula).
2. Give an example for a directed graph, where the above approach fails.
3. What can we say for directed graphs of bounded degree?
4. Same question as 1. for the undirected graph properties “not a tree” and “not planar”.

Exercise 2 Let $K_{m,n}$ be the undirected graph with vertex set partitioned into U_1, U_2 , where $|U_1| = m$, $|U_2| = n$ and $E = \{\{u, v\} \mid u \in U_1, v \in U_2\}$.

- For which values of m, n does $K_{m,n}$ contain a hamiltonian cycle?
- MSO is monadic second-order logic where one can quantify over (sets of) vertices (the atomic predicates are $x = y$, $x \in X$ and $E(x, y)$). Show that *Hamiltonicity* is not a property definable in MSO.
- MSO_2 is monadic second-order logic where one can quantify over (sets of) vertices or edges (the atomic predicates are $x = y$, $x \in X$ and (x, e, y) saying that $e = (x, y)$). Show that *Hamiltonicity* is definable in MSO_2 .