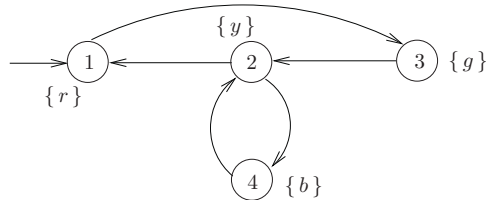


### 6.11 Exercises

EXERCISE 6.1. Consider the following transition system over  $AP = \{b, g, r, y\}$ :

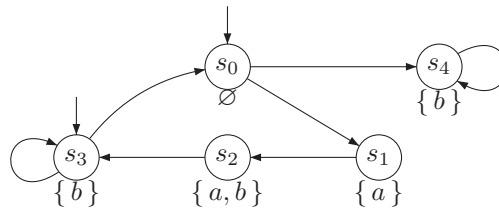


The following atomic propositions are used:  $r$  (red),  $y$  (yellow),  $g$  (green), and  $b$  (black). The model is intended to describe a traffic light that is able to blink yellow. You are requested to indicate for each of the following CTL formulae the set of states for which these formulae hold:

- |                                       |   |
|---------------------------------------|---|
| (a) $\forall \Diamond y$              | (g) $\exists \Box \neg g$                             |
| (b) $\forall \Box y$                  | (h) $\forall (b \text{ U } \neg b)$                   |
| (c) $\forall \Box \forall \Diamond y$ | (i) $\exists (b \text{ U } \neg b)$                   |
| (d) $\forall \Diamond g$              | (j) $\forall (\neg b \text{ U } \exists \Diamond b)$  |
| (e) $\exists \Diamond g$              | (k) $\forall (g \text{ U } \forall (y \text{ U } r))$ |
| (f) $\exists \Box g$                  | (l) $\forall (\neg b \text{ U } b)$                   |

EXERCISE 6.2. Consider the following CTL formulae and the transition system  $TS$  outlined on the right:

- $\Phi_1 = \forall (a \text{ U } b) \vee \exists \bigcirc (\forall \Box b)$
- $\Phi_2 = \forall \Box \forall (a \text{ U } b)$
- $\Phi_3 = (a \wedge b) \rightarrow \exists \Box \exists \bigcirc \forall (b \text{ W } a)$
- $\Phi_4 = (\forall \Box \exists \Diamond \Phi_3)$



Determine the satisfaction sets  $Sat(\Phi_i)$  and decide whether  $TS \models \Phi_i$  ( $1 \leq i \leq 4$ ).

EXERCISE 6.3. Which of the following assertions are correct? Provide a proof or a counterexample.

- (a) If  $s \models \exists \Box a$ , then  $s \models \forall \Box a$ .
- (b) If  $s \models \forall \Box a$ , then  $s \models \exists \Box a$ .
- (c) If  $s \models \forall \Diamond a \vee \forall \Diamond b$ , then  $s \models \forall \Diamond (a \vee b)$ .
- (d) If  $s \models \forall \Diamond (a \vee b)$ , then  $s \models \forall \Diamond a \vee \forall \Diamond b$ .

(e) If  $s \models \forall(a \cup b)$ , then  $s \models \neg(\exists(\neg b \cup (\neg a \wedge \neg b)) \vee \exists \square \neg b)$ .

EXERCISE 6.4. Let  $\Phi$  and  $\Psi$  be arbitrary CTL formulae. Which of the following equivalences for CTL formulae are correct?

- (a)  $\forall \bigcirc \forall \diamond \Phi \equiv \forall \diamond \forall \bigcirc \Phi$
- (b)  $\exists \bigcirc \exists \diamond \Phi \equiv \exists \diamond \exists \bigcirc \Phi$
- (c)  $\forall \bigcirc \forall \square \Phi \equiv \forall \square \forall \bigcirc \Phi$
- (d)  $\exists \bigcirc \exists \square \Phi \equiv \exists \square \exists \bigcirc \Phi$
- (e)  $\exists \diamond \exists \square \Phi \equiv \exists \square \exists \diamond \Phi$
- (f)  $\forall \square (\Phi \Rightarrow (\neg \Psi \wedge \exists \bigcirc \Phi)) \equiv (\Phi \Rightarrow \neg \forall \diamond \Psi)$
- (g)  $\forall \square (\Phi \Rightarrow \Psi) \equiv (\exists \bigcirc \Phi \Rightarrow \exists \bigcirc \Psi)$
- (h)  $\neg \forall (\Phi \cup \Psi) \equiv \exists (\Phi \cup \neg \Psi)$
- (i)  $\exists ((\Phi \wedge \Psi) \cup (\neg \Phi \wedge \neg \Psi)) \equiv \exists (\Phi \cup (\neg \Phi \wedge \Psi))$
- (j)  $\forall (\Phi \text{ W } \Psi) \equiv \neg \exists (\neg \Phi \text{ W } \neg \Psi)$
- (k)  $\exists (\Phi \cup \Psi) \equiv \exists (\Phi \cup \Psi) \wedge \exists \diamond \Psi$
- (l)  $\exists (\Psi \text{ W } \neg \Psi) \vee \forall (\Psi \cup \text{false}) \equiv \exists \bigcirc \Phi \vee \forall \bigcirc \neg \Phi$
- (m)  $\forall \square \Phi \wedge (\neg \Phi \vee \exists \bigcirc \exists \diamond \neg \Phi) \equiv \exists X \neg \Phi \wedge \forall \bigcirc \Phi$
- (n)  $\forall \square \forall \diamond \Phi \equiv \Phi \wedge (\forall \bigcirc \forall \square \forall \diamond \Phi) \vee \forall \bigcirc (\forall \diamond \Phi \wedge \forall \square \forall \diamond \Phi)$
- (o)  $\forall \square \Phi \equiv \Phi \vee \forall \bigcirc \forall \square \Phi$

EXERCISE 6.5. Consider an elevator system that services  $N > 0$  floors numbered 0 through  $N-1$ . There is an elevator door at each floor with a call button and an indicator light that signals whether or not the elevator has been called. In the elevator cabin there are  $N$  send buttons (one per floor) and  $N$  indicator lights that inform to which floor(s) is going to be sent. For simplicity consider  $N = 4$ . Present a set of atomic propositions—try to minimize the number of propositions—that are needed to describe the following properties of the elevator system as CTL formulae and give the corresponding CTL formulae:

- (a) The doors are “safe”, i.e., a floor door is never open if the cabin is not present at the given floor.
- (b) The indicator lights correctly reflect the current requests. That is, each time a button is pressed, there is a corresponding request that needs to be memorized until fulfillment (if ever).
- (c) The elevator only services the requested floors and does not move when there is no request.

- (d) All requests are eventually satisfied.

EXERCISE 6.6. Consider the single pulser circuit, a hardware circuit that is part of a set of benchmark circuits for hardware verification. The single pulser has the following informal specification: “For every pulse at the input *inp* there appears exactly one pulse of length 1 at output *outp*, independent of the length of the input pulse”. Thus, the single pulser circuit is required to generate an output pulse between two rising edges of the input signal. The following questions require the formulation of the circuit in terms of CTL. Suppose we have the proposition *rise\_edge* at our disposal which is true if the input was low (0) at time instant  $n-1$  and high (1) at time instant  $n$  (for natural  $n > 0$ ). It is assumed that input sequences of the circuit are well behaved, i.e., more than the rising edge appears in the input sequence.

Questions: specify the following requirements of the circuit in CTL:

- (a) A rising edge at the inputs leads to an output pulse.
- (b) There is at most one output pulse for each rising edge.
- (c) There is at most one rising edge for each output pulse.

(This exercise is taken from [246].)

EXERCISE 6.7. Transform the following CTL formulae into ENF and PNF. Show all intermediate steps.

$$\Phi_1 = \forall ( (\neg a) W (b \rightarrow \forall \bigcirc c) )$$

$$\Phi_2 = \forall \bigcirc ( \exists ((\neg a) U (b \wedge \neg c)) \vee \exists \square \forall \bigcirc a )$$

EXERCISE 6.8. Provide two finite transition systems  $TS_1$  and  $TS_2$  (without terminal states, and over the same set of atomic propositions) and a CTL formula  $\Phi$  such that  $Traces(TS_1) = Traces(TS_2)$  and  $TS_1 \models \Phi$ , but  $TS_2 \not\models \Phi$ .

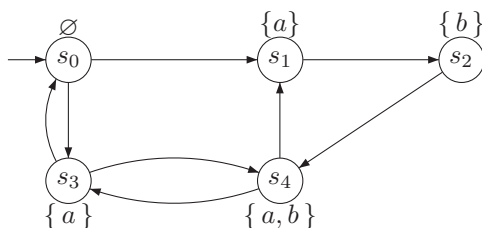
EXERCISE 6.9. Consider the CTL formula

$$\Phi = \forall \square (a \rightarrow \forall \diamond (b \wedge \neg a))$$

and the following CTL fairness assumption:

$$fair = \forall \diamond \forall \bigcirc (a \wedge \neg b) \rightarrow \forall \diamond \forall \bigcirc (b \wedge \neg a) \wedge \diamond \square \exists \diamond b \rightarrow \square \diamond b.$$

Prove that  $TS \models_{fair} \Phi$  where transition system  $TS$  is depicted below.



EXERCISE 6.10. Let  $\wp = (z_1, z_2, z_3, z_4, z_5, z_6)$ . Depict the  $\wp$ -ROBDD for the majority function

$$\text{MAJ}([z_1 = b_1, z_2 = b_2, \dots, z_6 = b_6]) = \begin{cases} 1 & \text{if } b_1 + b_2 + \dots + b_6 \geq 4 \\ 0 & \text{otherwise.} \end{cases}$$

EXERCISE 6.11. Consider the function

$$F(x_0, \dots, x_{n-1}, a_0, \dots, a_{k-1}) = x_m$$

where  $n = 2^k$ , and  $m = \sum_{j=0}^{k-1} a_j 2^j$ . Let  $k=3$ . Questions:

- Depict the  $\wp$ -ROBDD for  $\wp = (a_0, \dots, a_{k-1}, x_0, \dots, x_{n-1})$ .
- Depict the  $\wp$ -ROBDD for  $\wp = (a_0, x_0, \dots, a_{k-1}, x_{k-1}, x_k, \dots, x_{n-1})$ .

EXERCISE 6.12. Let  $\mathfrak{B}$  and  $\mathfrak{C}$  be two  $\wp$ -ROBDDs. Design an algorithm that checks whether  $f_{\mathfrak{B}} = f_{\mathfrak{C}}$  and runs in time linear in the sizes of  $\mathfrak{B}$  and  $\mathfrak{C}$ .

(Hint: It is assumed that  $\mathfrak{B}$  and  $\mathfrak{C}$  are given as separate graphs (and not by nodes of a shared OBDD).)

EXERCISE 6.13. Let  $TS$  be a finite transition system (over  $AP$ ) without terminal states, and  $\Phi$  and  $\Psi$  be CTL state formulae (over  $AP$ ). Prove or disprove

$$TS \models \exists(\Phi \cup \Psi) \quad \text{if and only if} \quad TS' \models \exists \diamond \Psi$$

where  $TS'$  is obtained from  $TS$  by eliminating all outgoing transitions from states  $s$  such that  $s \models \Psi \vee \neg \Phi$ .

EXERCISE 6.14. Check for each of the following formula pairs  $(\Phi_i, \varphi_i)$  whether the CTL formula  $\Phi_i$  is equivalent to the LTL formula  $\varphi_i$ . Prove the equivalence or provide a counterexample that illustrates why  $\Phi_i \neq \varphi_i$ .