# Complexity of Data Tree Patterns over XML Documents\*

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**Abstract.** We consider Boolean combinations of data tree patterns as a specification and query language for XML documents. Data tree patterns are tree patterns plus variable (in)equalities which express joins between attribute values. Data tree patterns are a simple and natural formalism for expressing properties of XML documents. We consider first the model checking problem (query evaluation), we show that it is DP-complete<sup>1</sup> in general and already NP-complete when we consider a single pattern. We then consider the satisfiability problem in the presence of a DTD. We show that it is in general undecidable and we identify several decidable fragments.

# 1 Introduction

The relational model and its popular query language SQL are widely used in database systems. However, it does not fit well in the ever changing Internet environment, since its structure is fixed by an initially specified schema which is difficult to modify. When exchanging and manipulating large amounts of data from different sources, a less structured and more flexible data model is preferable. This was the initial motivation for the Extensible Markup Language (XML) model which is now the standard for data exchange.

An XML document is structured as an unranked, labelled tree. The main difference with the relational model is that in XML, data is also extracted because of its position in the tree and not only because of its value. Consequently, all the tools manipulating XML data, like XML query languages and XML schema, combine navigational features with classical data extraction ones. XPath<sup>2</sup> is a typical example. It has a navigational core, known as Core-XPath and studied in [16], which is essentially a modal language that walks around in the tree. XPath also allows restricted tests on data attributes. It is the building block of most XML query languages (XQuery, XSLT...). Similarly, in order to specify integrity constraints in XML Schema, XML languages have navigational features for description of walks in the tree and selection of nodes. The nodes are for instance chosen according to a key or a foreign key [15].

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<sup>&</sup>lt;sup>1</sup> A problem DP is the intersection of a NP problem and a co-NP problem.

<sup>&</sup>lt;sup>2</sup> In all the paper, XPath refers to XPath1.0

In this paper, we study an alternative formalism as a building block for querying and specifying XML data. It is based on Boolean combinations of data tree patterns. A data tree pattern is essentially a tree with child or descendant edges, labelled nodes and (in)equality constraints on data values. Intuitively, a document satisfies a data tree pattern if there exists an injective mapping from the tree pattern into the tree that respects edges, node labels and data value constraints. Using patterns, one can express properties on trees in a natural, visual and intuitive way. These properties can express queries, as well as some integrity constraints.

At first glance, the injectivity requirement does not seem important; however, it has some consequences in terms of expressive power. As we do not consider horizontal order between siblings, without injectivity data tree patterns are invariant by bisimulation. Data tree patterns with injective semantics are strictly more expressive than with non-injective semantics. For example, it is not possible to express desirable properties such as a node has two *a*-labelled children without injectivity. Another consequence of injectivity appears when considering conjunctions of data tree patterns. With non-injective semantics, the conjunction of two patterns would be equivalent to a new pattern obtained by merging the two patterns at the root. With injectivity this no longer works and we have to consider conjunctions of tree patterns. This difference appears when we study the complexity of the satisfiability problem: for one pattern the problem is PTIME while it is untractable for a conjunction of patterns.

XPath and data tree patterns are incomparable in terms of expressiveness. Without data value, XPath queries are closed under bisimulation while data tree patterns are not. On the other hand, XPath allows negation of subformulas while we only allow negation of a full data tree pattern. For example XPath can check whether a node has *a*-labelled children but no *b*-labelled child. This is not possible with Boolean combinations of tree pattern. In terms of data comparison, Xpath allows very limited joins because XPath queries cannot compare more than two elements at a time, while a single pattern can compare simultaneously an arbitrary number of elements.

In this paper, to continue this comparison, we study the complexity of two questions related to data tree patterns: the model checking problem (query evaluation) and the satisfiability problem in the presence of schema.

The evaluation of XPath queries has been extensively studied (see [6] for a detailed survey). The evaluation problem is PTIME for general XPath queries. In our case, this problem is more difficult: the combined complexity of the model checking problem for Boolean combinations of data tree patterns is untractable. We prove that it is DP-complete in general and already NP-complete when considering only one tree pattern.

The satisfiability problem for XPath is undecidable in general [5]. However for many fragments the problem is decidable with a complexity ranging from NP to NEXPTIME. Similarly, for Boolean combinations of data tree patterns the satisfiability problem is undecidable in general. We identify several decidable fragments by restraining the expressivity of tree patterns or by bounding the depth of the documents. The corresponding complexities range from NP to 2ExPTIME.

**Related Work:** Tree patterns have already been investigated in a database context, often without data values [22, 3, 20]. The focus is usually optimisation techniques for efficient navigation [1, 12, 7]. In this work, we focus on the difficulty raised by data values and we are not interested in optimisation but in the worst case complexity for the model checking and satisfiability problems.

Several papers considered the non injective semantics of tree pattern with data constraints. First, [19] considered the satisfiability problem for one positive pattern while we consider Boolean combinations of tree pattern. Then, the authors of [2] consider the type checking problem which is more powerful that unsatisfiability but incomparable to the satisfiability problem.

Data tree patterns are used in [4] to specify data exchange settings. They study two problems: the first one is consistency of data exchange settings, the second one is query answering under data exchange settings. Given a conjunction of data tree patterns and a DTD, we can construct a data exchange setting such that the consistency of this setting is equivalent to the satisfiability of the conjunction of patterns in the presence of the DTD. However the data tree patterns they consider are less expressive than ours, in that they can not express inequality constraints on data values nor Boolean combinations of data tree patterns. The other problem considered in this paper is query answering. This problem seems related to our model checking problem. However it does not seem possible to use their result or their proof techniques.

Fragment of XPath: In [14], the authors consider an XPath fragment (simple XPath) allowing only vertical navigation but augmented with data comparisons. Negation is disallowed, both in the navigation part and in the comparison part. A simple XPath expression can be viewed as a pattern with non-injective semantics and only data equality. They study the inclusion problem of such expressions wrt special schemes (SXIC) containing integrity constraints like inclusion dependency. We cannot simulate inclusion dependency even with Boolean combinations of data tree patterns. Hence, their framework is incomparable to ours.

Conjunctive queries on trees: Conjunctive queries on trees can be expressed by tree patterns. They were considered in [17,8] without data values. Very recently [9], an extension by schema constraints is proposed and in very few cases they allow data comparison. Notice that, without sibling predicate, those conjunctive queries are strictly less expressive than our framework because they do not allow negation and do not have an injective semantic. It is shown that the query satisfiability problem is NP-complete, whereas the query validity problem is 2EXPTIME-complete. Moreover, the validity of a disjunction of conjunctive queries is shown to be undecidable. This last result corresponds to our undecidability result but the proof is different.

Logics over infinite alphabets: Another related approach is to consider logic for trees over an infinite alphabet. In [10, 11], the authors study an extension of First Order Logic with two variables. In [13, 18], the focus is on temporal logic and  $\mu$ -calculus. These works are very elegant, but the corresponding complexities are non primitive recursive. Our work can be seen as a continuation of this work aiming for lower complexities.

Structure: Section 2 contains the necessary definitions. In Section 3, we consider the model-checking problem. In Section 4, we consider the satisfiability problem in general and the restricted cases. Section 5 contains a summary of our results and a discussion. Omitted proofs can be found in the appendix available at http://www.liafa.jussieu.fr/~cdavid/publi/mfcs08.pdf.

# 2 Preliminary

In this paper, we consider XML documents that are modeled as unordered, unranked *data trees*, as considered e.g. in [10].

**Definition 1** A data tree over a finite alphabet  $\Sigma$  is an unranked, unordered, labelled tree with data values. Every node v has a *label*  $v.l \in \Sigma$  and a *data value*  $v.d \in D$ , where D is an infinite domain.

We only consider equality tests between data values. The data part of a tree can thus be seen as an equivalence relation  $\sim$  on its nodes. In the following, we write  $u \sim v$  for two nodes u, v, if u.d = v.d and we use the term *class* without more precision to denote an equivalence class for the relation  $\sim$ .

The **data erasure** of a data tree t over  $\Sigma$  is the tree obtained from t by ignoring the data value v.d of each node v of t.



Fig. 1. Examples

Data tree patterns are a natural way to express properties of data trees, or to query such trees. They describe a set of nodes through their relative positions in the tree, and (in)equalities between their data values.

#### **Definition 2** A data tree pattern $P = (p, C_{\sim}, C_{\approx})$ consists of:

- an unordered, unranked tree p, with nodes labelled either by  $\Sigma$  or by the wildcard symbol \*, and edges labelled either by | (child edges) or by || (descendant edge), and
- two binary relations  $C_{\sim}$  and  $C_{\sim}$  on the set of nodes of p.

A data tree t satisfies a pattern  $P = (p, C_{\sim}, C_{\approx})$ , and we write  $t \models P$ , if there exists an *injective* mapping f from the nodes of p to the nodes of t that is consistent with the labelling, the relative positions of nodes, the branching structure and the data constraints. Formally, we require the following:

- for every node v from p with  $v.l \in \Sigma$ , we have v.l = f(v).l,
- for every pair of nodes (u,v) from p, if  $(u,v) \in C_{\sim}$  (resp.  $(u,v) \in C_{\nsim}$ ) then  $f(u) \sim f(v)$  (resp.  $f(u) \nsim f(v)$ ),
- for every pair of nodes (u,v) from p, if (u,v) is an edge of p labelled by | (resp. by ||), then f(v) is a child (resp. a descendant) of f(u),
- for any nodes u,v,z from p, if (u,v) and (u,z) are both edges of p labelled by  $\|$ , then f(v) and f(z) are not related by the descendant relation in t.

A mapping f as above is called a **witness** of the pattern P in the data tree t.

Notice that the semantic does not preserve the least common ancestor and asks for an injective mapping between the nodes of a pattern and those of the tree. This enables patterns to express integrity constraints. We will discuss the impact of those choices in Section 5. Data tree patterns can describe properties that XPath cannot, see *e.g.* the pattern in Fig 1 (XPath cannot talk simultaneously about the two *r*-nodes and the two *b*-nodes).

We denote by  $Ptn(\sim, |, ||)$  the set of data tree patterns and by  $BC(\sim, |, ||)$  the set of Boolean combinations of data tree patterns. We will also consider restricted patterns, that do not use child relations or do not use descendant relations (denoted respectively by  $Ptn(\sim, ||)$ ,  $Ptn(\sim, ||)$ ). From these, we derive the corresponding classes of Boolean combinations. Finally,  $BC^+$  (resp.  $BC^-$ ) denotes conjunctions of patterns (resp. negations of patterns).

In proofs, we consider the *parse tree* of a Boolean formula  $\varphi$  over patterns, denoted by  $\mathcal{T}(\varphi)$ . The leaves of this tree are labelled by (possibly negation of) patterns and inner nodes are labelled by conjunctions or disjunctions. Such trees are of linear size in the size of the formula and can be computed in PTIME.

Given a pattern formula from  $BC(\sim, |, ||)$ , the main problems we are interested in are the model-checking on a data tree (evaluation), the satisfiability problem, in the general case as well as for interesting fragments. Because the general structure of XML documents is usually constrained, we may consider DTDs as additional inputs. DTDs are essentially regular constraints on the finite structure of the tree. Since we work on unordered, unranked trees, we use as DTDs an unordered version of hedge automata. A DTD is a bottom-up automaton  $\mathcal{A}$  where the transition to a state q' with label a is given by a Boolean combination of clauses of the form  $\#q \leq k$  where q is a state and k a constant (unary encoded). A clause  $\#q \leq k$  is satisfied if there are at most k children in state q. Adding a DTD constraint does not change the complexity results for the model-checking, since checking whether the data erasure of a tree satisfies a DTD is PTIME. Therefore, we do not mention DTDs in the model-checking part. We consider the following problems:

Problem 1. Given a data tree t and a pattern formula  $\varphi$ , the **model-checking** problem asks whether t satisfies  $\varphi$ .

Problem 2. Given a pattern formula  $\varphi$  and a DTD L, the **satisfiability** problem in the presence of a DTD asks whether  $\varphi$  is satisfied by some data tree whose data erasure belongs to L.

# 3 Model Checking

Patterns provide a formalism for expressing properties. In this section, we see how efficiently we can evaluate them. Our main result is the exact complexity of the model-checking problem for pattern formulas from  $BC(\sim, |, ||)$ .

**Theorem 3** The model-checking problem for  $BC(\sim, |, ||)$  is DP-complete.

The class of complexity DP is defined as the class of problems that are the conjunction of a NP problem and a co-NP problem [21]. In particular, DP includes both NP and co-NP. A typical DP-complete problem is SAT/UNSAT: given two propositional formulas  $\varphi_1, \varphi_2$ , it asks whether  $\varphi_1$  is satisfiable, and  $\varphi_2$  is unsatisfiable.

The key to the proof of Theorem 3 is the case where only one pattern is present. This problem is already NP-complete.

**Proposition 4** The model-checking problem for a single pattern from  $Ptn(\sim, |, ||)$  is NP-complete.

**Proof.** The upper bound is obtained by an algorithm guessing a witness for the pattern in the data tree and checking in PTIME whether the witness is correct. The lower bound is more difficult. It is obtained by a reduction of 3SAT.

Given a propositional formula  $\varphi$  in 3-CNF, we build a data tree  $t_{\varphi}$  and a pattern  $P_{\varphi}$  of polynomial size, such that  $t_{\varphi} \models P_{\varphi}$  iff  $\varphi$  is satisfiable. Because we consider the model-checking problem, the data tree is fixed in the input. Thus, it must contain all possible valuations of the variables and at least all possible true valuations of each variable. Moreover, one positive data tree pattern should identify a true valuation of the formula and check its consistency. Hence, it does not seem possible to use previously published encodings of 3SAT into trees.

The pattern selects one valuation per variable and per clause. Its structure ensures that only one valuation per variable and per literal is selected. The constraints on data ensure the consistency of the selection. The data tree and the tree of the pattern depend only on the number of variables and clauses of the formula. Only the constraints on data of the pattern are specific to the formula. They encode the link between variables and clauses.

Let  $\Sigma = \{r, \overline{r}, X, Y, Z, \#, \$\}$  be the finite alphabet. Assume that  $\varphi$  has k variables and n clauses. The data tree  $t_{\varphi}$  is composed of k copies of the tree  $t_v$  and n copies of the tree  $t_c$  as depicted in Figure 2. Even if we consider unordered trees, each copy of  $t_v$  corresponds to a variable of the formula and each copy of  $t_c$  to a clause. The tree  $t_{\varphi}$  involves exactly three classes, denoted as  $0, \top, \bot$ .

Each subtree  $t_v$ , see Figure 2(b), contains the two possible values for a variable. The left (right) branch of the tree represents true (resp. false).



**Fig. 2.** The data tree  $t_{\varphi}$ 

A clause is viewed here as the disjunction of three literals, say X, Y, and Z. Each subtree  $t_c$ , see Figure 2(c), is formed by three subtrees. Each of them represents one of the three disjoint possibilities for a clause to be true: (1) X is true, or (2) X is false and Y is true, or (3) X and Y are false and Z is true.

We now turn to the definition of the tree pattern  $P_{\varphi} = (tp_{\varphi}, C_{\sim}, C_{\varkappa})$ , depicted in Figure 3. Similarly to  $t_{\varphi}$ , the tree  $tp_{\varphi}$  is formed by k copies of  $tp_v$  (each of them implicitly corresponding to a variable) and n copies of  $tp_c$  (each of them implicitly corresponding to a clause).



**Fig. 3.** The tree  $tp_{\varphi}$ 

The form of the data erasures of  $t_{\varphi}$  and  $tp_{\varphi}$  ensures that any witness of  $P_{\varphi}$  in  $t_{\varphi}$  selects exactly one value per variable and one (satisfying) valuation for each clause. Note that this is ensured by the definition of witness, since the witness mapping is injective.

It remains to define the data constraints  $C_{\sim}$  and  $C_{\sim}$  in order to guarantee that each clause is satisfied. Assume that the first literal of clause c is a positive variable x (resp. the negation of x). Then we add in  $C_{\sim}$  the r-position (resp. the  $\overline{r}$ -position) of the subtree  $tp_v$  corresponding to the variable x together with the X-position of the subtree  $tp_c$  corresponding to the clause c. The same can be done with the literals Y and Z. Figure 4 gives the example of the pattern for the formula  $\varphi$  with only the clause  $a \vee \neg b \vee c$ .



**Fig. 4.** Pattern for  $\varphi = a \lor \neg b \lor c$ 

We now prove that  $t_{\varphi} \models P_{\varphi}$  iff  $\varphi$  is satisfiable. Assume that the formula  $\varphi$  is satisfiable. From any satisfying assignment of  $\varphi$  we derive a mapping of  $P_{\varphi}$  into  $t_{\varphi}$ : the subtree  $p_v$  corresponding to the variable v is mapped on the left branch of the corresponding  $t_v$  if the value of v is true, and on the right branch otherwise. Since each clause is satisfied, one of the three cases represented by the subtree  $t_c$  happens, and we can map the  $t_{p_c}$  corresponding to the clause on the branch of the corresponding  $t_c$ . The converse is similar.

In the proofs of Proposition 4 and Theorem 3, the patterns use only the child predicate. We can do the same with similar patterns using only the descendant predicate. As a consequence, we have:

**Theorem 5** The model-checking problem for both fragments  $BC(\sim, \parallel)$ ,  $BC(\sim, \mid)$  is DP-complete.

**Corollary 6** The model-checking problem for  $BC^+(\sim, ||)$ ,  $BC^+(\sim, ||)$  and for  $BC^+(\sim, |, ||)$  is NP-complete.

Similarly, we can see that the model-checking problem of a (conjunction of) negated pattern(s) is co-NP-complete. Notice that in the proof of Theorem 3, the pattern formula of the lower bound is a conjunction of one pattern and the negation of one pattern. Thus, the model-checking problem is already DP-complete for a conjunction of one pattern and one negated pattern.

The model checking problem for conjunctive queries is also exponential (NP) in relational databases. However, the algorithms work very well in practice, when models or queries are simple. In particular, when the query is acyclic, the problem becomes polynomial. The worst cases that lead to exponential behaviors do not appear often. It would be interesting to know how the algorithms following from our proofs behave on practical cases, and whether we can find some restriction on the patterns that would lead to efficient evaluation in practice.

#### 4 Satisfiability

In this section, we study the satifiability problem in the presence of DTDs. Checking satisfiability of a query is useful for optimization of query evaluation or minimization techniques. In terms of schema design, satisfiability corresponds to checking the consistency of the specification.

We show that the satisfiability problem is undecidable in general. However the reduction needs the combination of negation, child and descendant operations. Indeed, removing any one of these features yields decidability, and we give the corresponding precise complexities.

#### 4.1 Undecidability

**Theorem 7** The satisfiability problem for  $BC(\sim, |, ||)$  in the presence of DTD is undecidable.

**Proof sketch.** We prove the undecidability by a reduction from the acceptance problem of two-counter machines (or Minsky machines). Our reduction builds a DTD and a pattern formula of size polynomial in the size of the machine whose models are exactly the encodings of the accepting runs.

The encoding of a run can be split in three parts:

- 1. The general structure of the tree, which depends only on the data erasure, and is controlled by the DTD.
- 2. The internal consistency of a configuration.
- 3. The evolution of counter values between two successive configurations.

The global structure contains a branch that is labelled by the sequence of transitions. Ensuring that a tree is of this shape is done by the DTD. It recognizes the data erasure of sequences of configurations. In particular it checks that a counter is zero when this is required by the transition. It also ensures that the sequence of transitions respects the machine's rules (succession of control states, initial and final configurations).

The data values allow us to control the evolution of the counters between two consecutive configurations. In order to do so, we need to guarantee a certain degree of structure and continuity of the values through a run. The data structure and the evolution of counters are ensured by the pattern formula.  $\Box$ 

The proof uses only conjunctions of negated patterns. Thus, the satisfiability problem is already undecidable for the  $BC^-(\sim, |, ||)$  fragment in the presence of a DTD. Alternatively, the DTD can be replaced by a pattern formula. To do so, we need a few positive patterns to constrain initial and final configurations in the coding. Thus, the satisfiability problem is undecidable for  $BC(\sim, |, ||)$  without DTDs. It is interesting to notice that the satisfiability problem of  $BC(\sim, |, ||)$  is undecidable on word models. We will discuss this in Section 5.

#### 4.2 Decidable Restrictions

We can obtain decidability by restraining either the expressive power of pattern formulas or the data trees considered. For the first part, using only one kind of edge predicate (| or ||) leads to decidability. For the second part, restricting the trees to bounded depth leads to decidability. We provide the exact complexities.

**Restricted Fragments:** The proof of undecidability uses both || and | in the pattern to count unbounded values of the counters. If we restrict expressivity of patterns to use either || or |, we can't do this anymore and the problem becomes decidable. The key to both lower bounds is that patterns can still count up to a polynomial value and thus compare positions of a tree of polynomial depth. We use this idea to encode exponential size configurations of a Turing machine into the leaves of polynomial depth subtrees.

**Theorem 8** The satisfiability problem of  $BC(\sim, |)$  in the presence of a DTD is 2ExpTIME-complete.

**Proof sketch.** The upper bound is obtained by a small model property. We can prove that a pattern formula  $\varphi$  of BC( $\sim$ , |) is satisfiable in the presence of a given DTD iff it has a model with a number of classes that is doubly exponential in the size of the formula. We can recognize the data erasure of such small models with an automaton of size doubly exponential in the size of the formula. Because emptiness of such automata is PTIME, we have the 2EXPTIME upper bound.

The lower bound is obtained by a coding of accepting runs of AEXPSPACE Turing machines. We can build a DTD and a pattern formula from  $BC(\sim, |)$  such that a data tree is a model on the pattern formula and respects the DTD iff it is the encoding of an accepting run of the machine.

**Theorem 9** The satisfiability problem of  $BC(\sim, \parallel)$  in the presence of a DTD is NEXPTIME-complete.

**Bounded Depth restrictions:** In the context of XML documents, looking at the satisfiability problem restricted to data trees of bounded depth is a crucial restriction. This restriction leads to decidability for  $BC(\sim, <, +1)$ .

Problem 3. Consider a pattern formula  $\varphi$ , an integer d and a DTD L. The problem of **bounded depth satisfiability in the presence of a DTD** asks whether  $\varphi$  is satisfiable by a data tree of depth smaller than d whose data erasure belongs to L.

**Theorem 10** If d is fixed, the bounded depth satisfiability problem in the presence of a DTD for  $BC(\sim, ||, |)$  is  $\Sigma_2$ -complete.

**Theorem 11** If d is part of the input, the bounded depth satisfiability problem in the presence of a DTD for  $BC(\sim, ||, |)$  is NEXPTIME-complete.

**Other remarks:** All the lower bound results of this section only use conjunctions of negated patterns. Thus these results hold for the BC<sup>-</sup> fragments.

**Proposition 12** The satisfiability problem of a single pattern is PTIME.

**Proposition 13** The satisfiability problem for  $BC^+(\sim, |, ||)$  is NP-complete in the presence of DTD.

# 5 Conclusion

The table below summarizes our results. bnd (resp.  $bnd_f$ ) Sat stands for Bounded depth Satisfiability when the bound is part of the input (resp. fixed). The gray parts of the table gives complexity results for data words models. Data words are the linear model corresponding to data trees. This model is studied in the verification area [11, 13]. Data patterns can also be considered for data words. The proofs are more complex and will be available in a longer version.

Fragments	Model-Checking	Satisfiability	bnd Sat	$bnd_f$ Sat
$BC(\sim,   ,  )$	DP-complete	Undecidable	NEXPTIME-complete	$\Sigma_2$ -complete
$BC(\sim,  )$	DP-complete	2 ExpTime-complete	NExpTime-complete	$\Sigma_2$ -complete
Data Word	PTIME	PSpace-complete		
$BC(\sim, \parallel)$	DP-complete	NExpTime-complete	NExpTime-complete	$\Sigma_2$ -complete
Data Word	DP-complete	$\Sigma_2$ -complete		
$\mathrm{BC}^-(\sim,\ , )$	$\operatorname{coNP-complete}$	Undecidable	NExpTime-complete	$\Sigma_2$ -complete
Data Word	$\operatorname{coNP-complete}$	undecidable		
$\mathrm{BC}^+(\sim,\ , )$	NP-complete	NP-complete	NP-complete	NP-complete
Data Word	NP-complete	NP-complete		

Discussion:

- In our framework we use the unordered version of trees. If we consider the next-sibling predicate, the situation is different. For the model checking problem all results hold with similar proofs. However, the complexity of the satisfiability problem can increase when negation is allowed. In particular the satisfiability problem for bounded depth tree becomes undecidable since we can encode data words.
- Recall that our pattern formalism does not preserve the least common ancestor. All results hold if we add the least common ancestor.
- An important issue of semi-structured databases is the containment problem. Given a DTD and two pattern formulas we want to know whether every tree satisfying the DTD and the first formula also satisfies the second one. When the set of formulas we consider is closed under negation, we can decide whether a formula  $\varphi_1$  is more constraining than  $\varphi_2$  by checking the satisfiability of  $\varphi_2 \wedge \neg \varphi_1$ . In Boolean combinations, we have closure under negation, hence the inclusion problem reduces to the satisfiability problem. For the positive fragment, the precise complexity seems harder to state and the question is left open.
- In terms of expressiveness, our pattern formalism is incomparable to XPath. In terms of tractability, evaluation of XPath queries is PTIME whereas model-checking of one data tree pattern is already NP-hard. A question is to find good notions of constraints in order to isolate interesting fragments with lower complexity. Considering the complexity of the satisfiability problem, XPath and our pattern formalism behave similarly.
- In this paper, we only consider patterns as filters in order to define properties on the data trees. Defining a query language would be a natural extension of this work. To do this, some of the variables of the patterns can be chosen as output variables.

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