

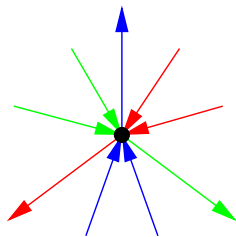
# Structure of Schnyder woods in higher genus

Benjamin Lévêque

CNRS, LIRMM, Montpellier, France

# Schnyder woods (planar triangulations)

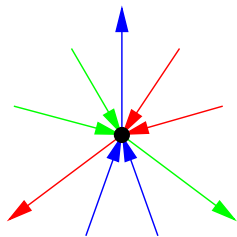
Schnyder (1989)



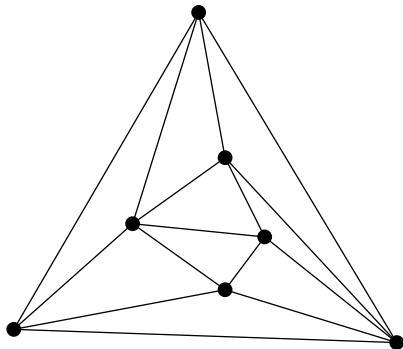
For every inner vertex

# Schnyder woods (planar triangulations)

Schnyder (1989)

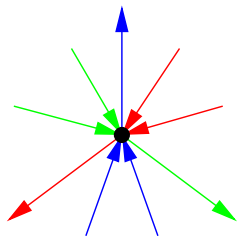


For every inner vertex

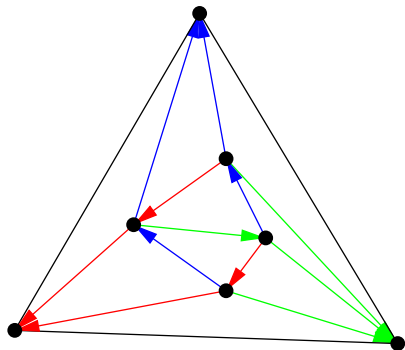


# Schnyder woods (planar triangulations)

Schnyder (1989)

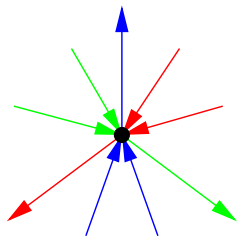


For every inner vertex

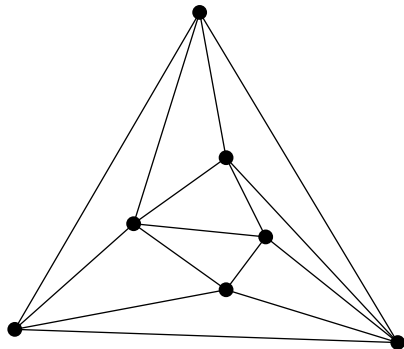


# Schnyder woods (planar triangulations)

Schnyder (1989)

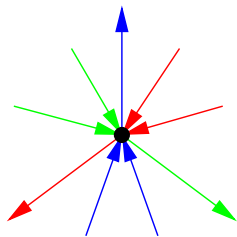


For every inner vertex

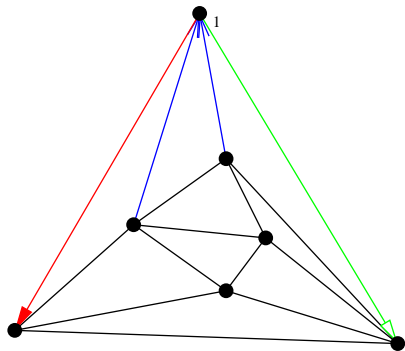


# Schnyder woods (planar triangulations)

Schnyder (1989)

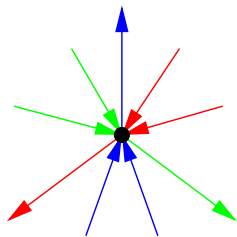


For every inner vertex

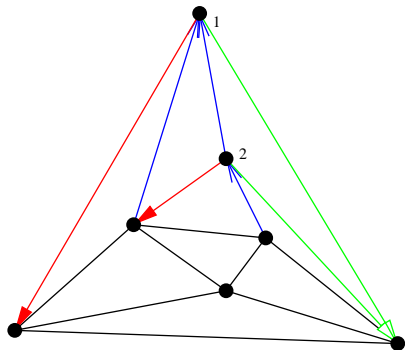


# Schnyder woods (planar triangulations)

Schnyder (1989)

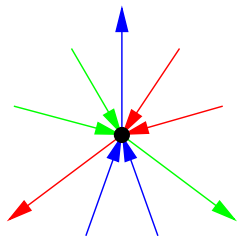


For every inner vertex

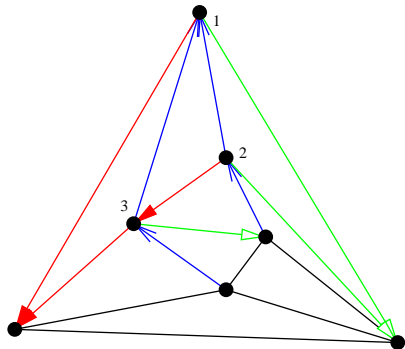


# Schnyder woods (planar triangulations)

Schnyder (1989)



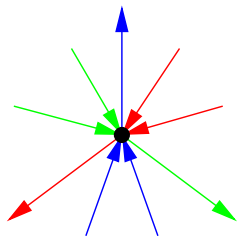
For every inner vertex



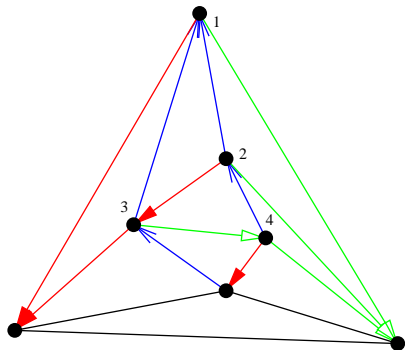


# Schnyder woods (planar triangulations)

Schnyder (1989)

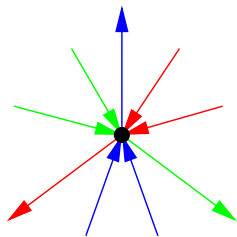


For every inner vertex

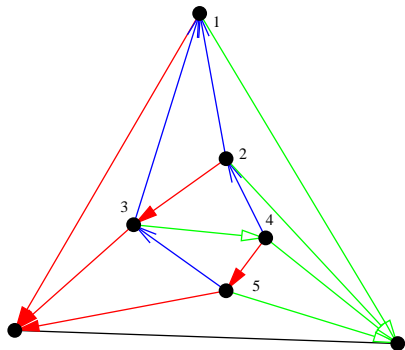


# Schnyder woods (planar triangulations)

Schnyder (1989)

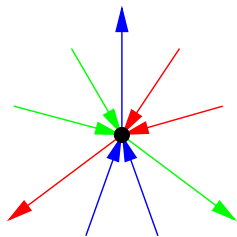


For every inner vertex

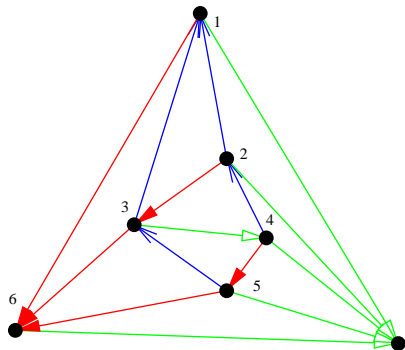


# Schnyder woods (planar triangulations)

Schnyder (1989)

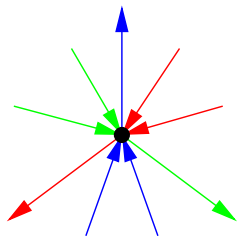


For every inner vertex

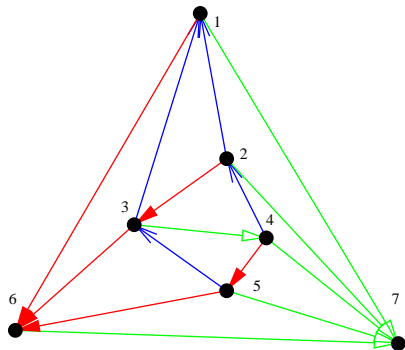


# Schnyder woods (planar triangulations)

Schnyder (1989)

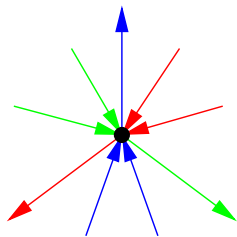


For every inner vertex

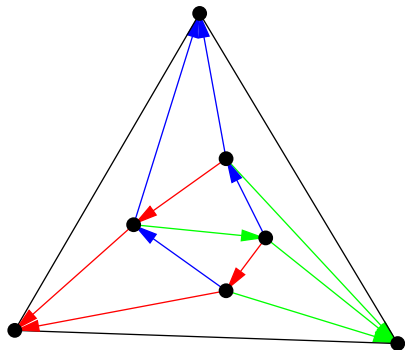


# Schnyder woods (planar triangulations)

Schnyder (1989)

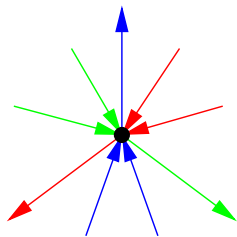


For every inner vertex

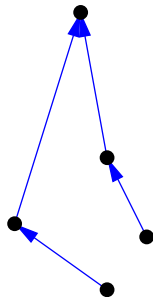


# Schnyder woods (planar triangulations)

Schnyder (1989)

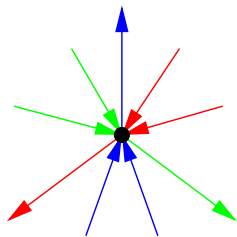


For every inner vertex

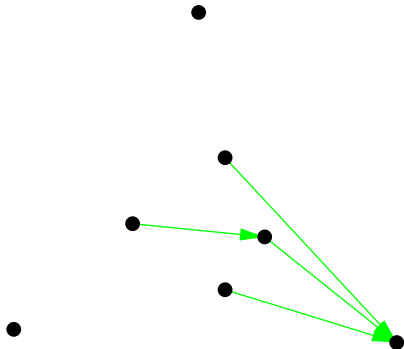


# Schnyder woods (planar triangulations)

Schnyder (1989)

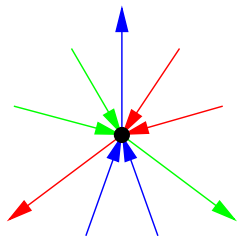


For every inner vertex

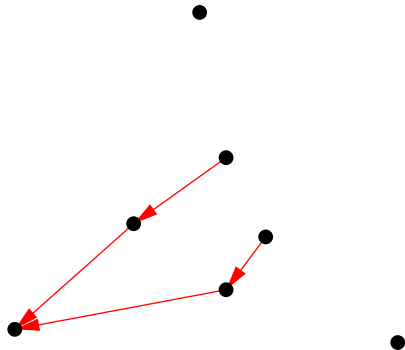


# Schnyder woods (planar triangulations)

Schnyder (1989)



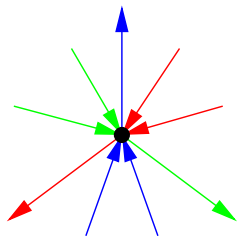
For every inner vertex



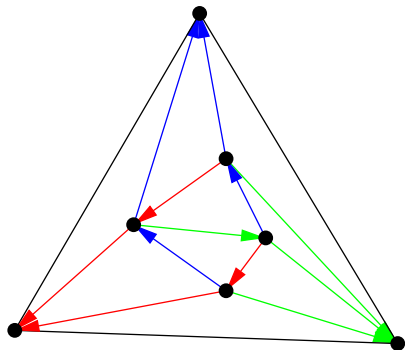


# Schnyder woods (planar triangulations)

Schnyder (1989)

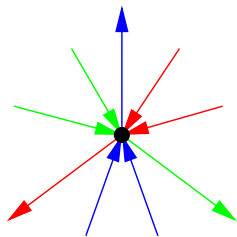


For every inner vertex

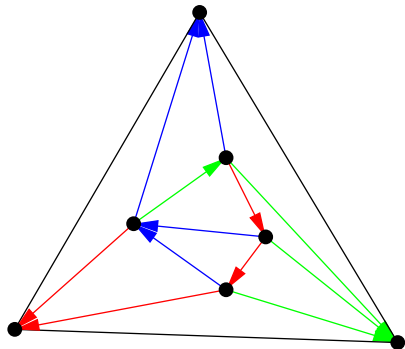


# Schnyder woods (planar triangulations)

Schnyder (1989)

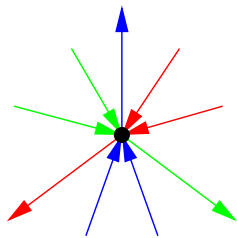


For every inner vertex

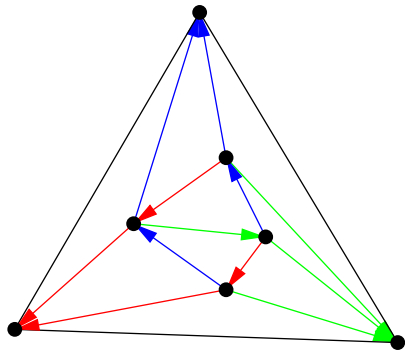


# Schnyder woods (planar triangulations)

Schnyder (1989)

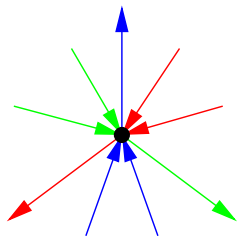


For every inner vertex

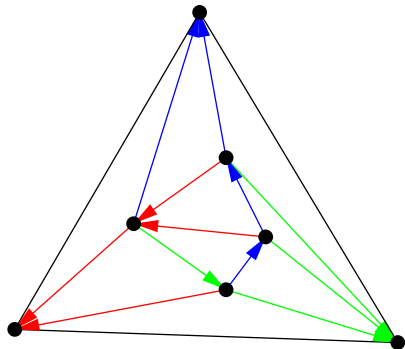


# Schnyder woods (planar triangulations)

Schnyder (1989)

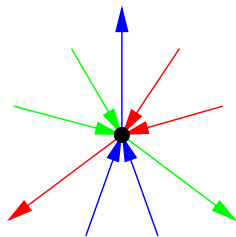


For every inner vertex

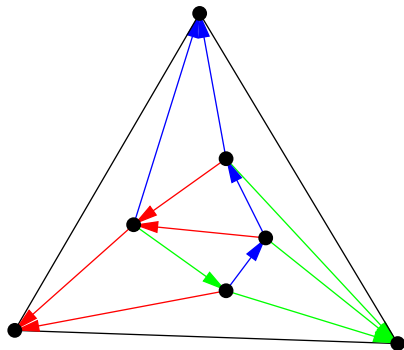


# Schnyder woods (planar triangulations)

Schnyder (1989)



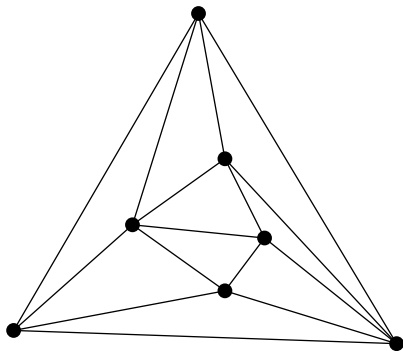
For every inner vertex



Planar graph  $\iff$  Incident poset has DM-dimension  $\leq 3$

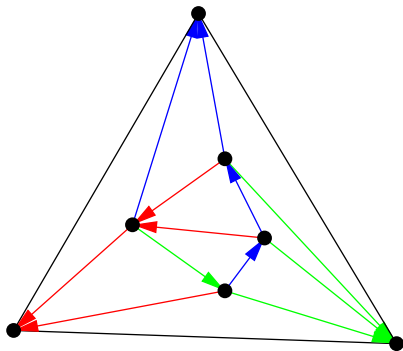
# Optimal encoding

Poulalhon and Schaeffer (2003)



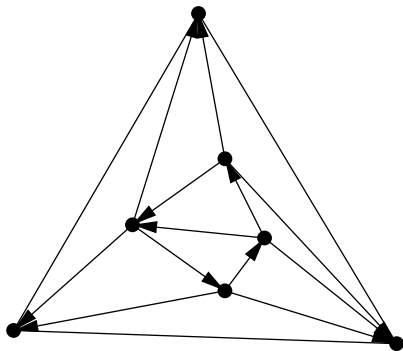
# Optimal encoding

Poulalhon and Schaeffer (2003)



# Optimal encoding

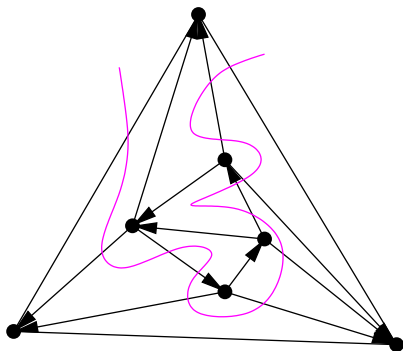
Poulalhon and Schaeffer (2003)





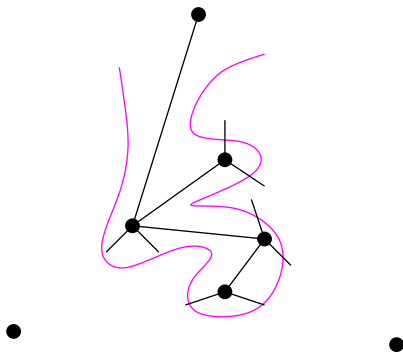
# Optimal encoding

Poulalhon and Schaeffer (2003)



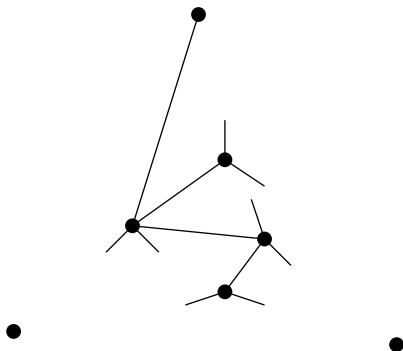
# Optimal encoding

Poulalhon and Schaeffer (2003)



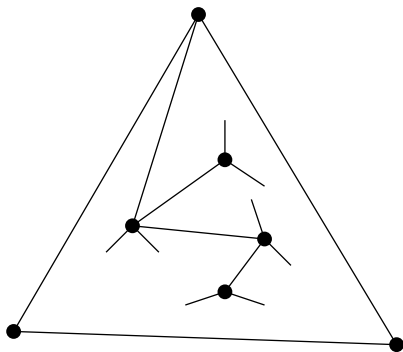
# Optimal encoding

Poulalhon and Schaeffer (2003)



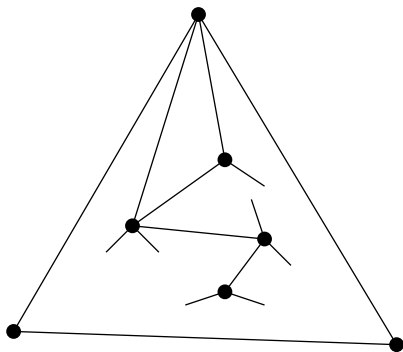
# Optimal encoding

Poulalhon and Schaeffer (2003)



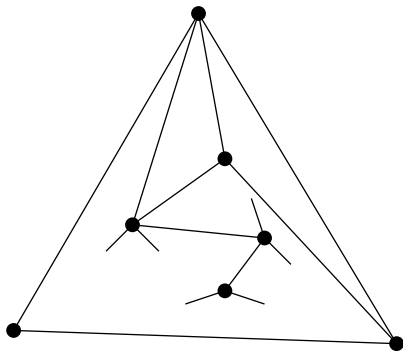
# Optimal encoding

Poulalhon and Schaeffer (2003)



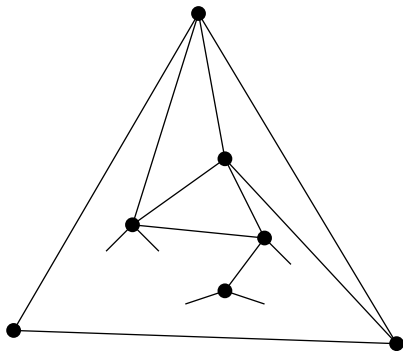
# Optimal encoding

Poulalhon and Schaeffer (2003)



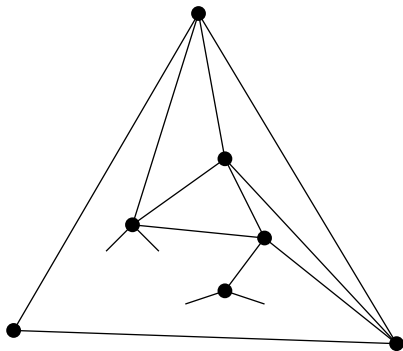
# Optimal encoding

Poulalhon and Schaeffer (2003)



# Optimal encoding

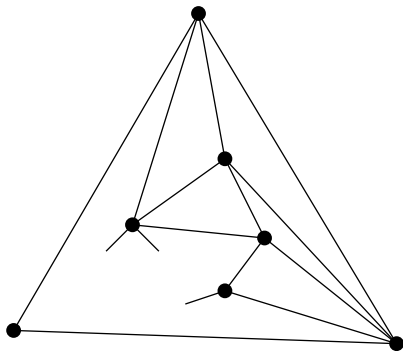
Poulalhon and Schaeffer (2003)





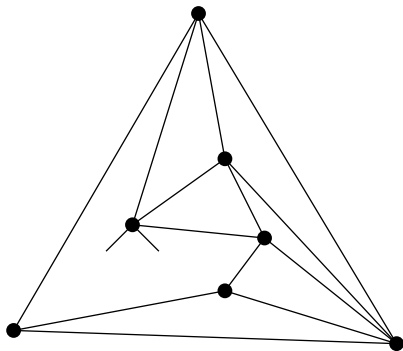
# Optimal encoding

Poulalhon and Schaeffer (2003)



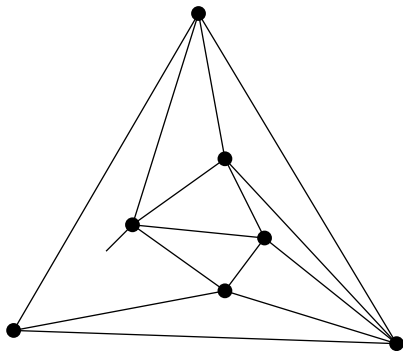
# Optimal encoding

Poulalhon and Schaeffer (2003)



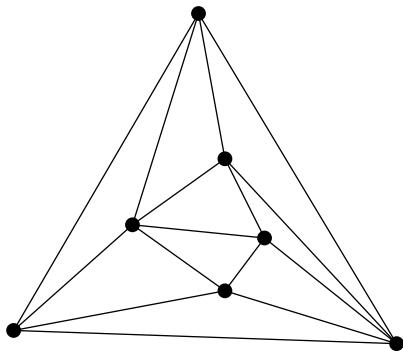
# Optimal encoding

Poulalhon and Schaeffer (2003)



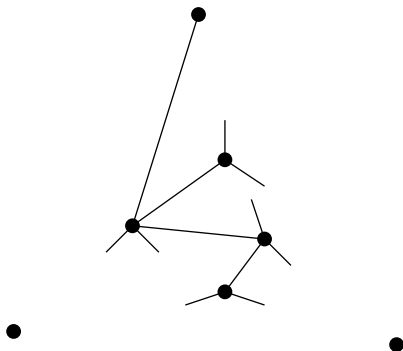
# Optimal encoding

Poulalhon and Schaeffer (2003)



# Optimal encoding

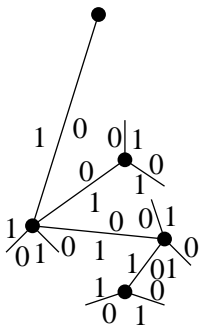
Poulalhon and Schaeffer (2003)





# Optimal encoding

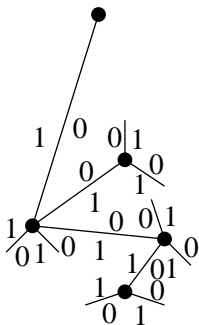
Poulalhon and Schaeffer (2003)



110101110100101001101000

# Optimal encoding

Poulalhon and Schaeffer (2003)

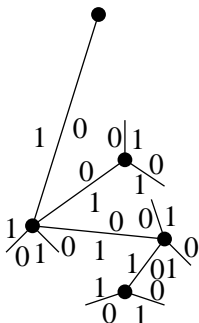


110101110100101001101000  $\rightsquigarrow$  6n bits



# Optimal encoding

Poulalhon and Schaeffer (2003)

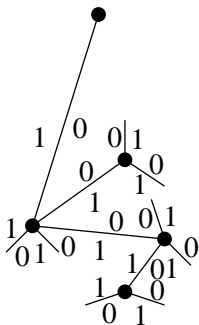


110101110100101001101000  $\rightsquigarrow$  6n bits

...1w10w10w0...

# Optimal encoding

Poulalhon and Schaeffer (2003)

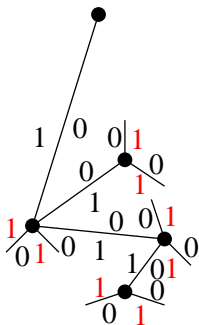


110101110100101001101000  $\rightsquigarrow$  6n bits

...1w10w10w0...

# Optimal encoding

Poulalhon and Schaeffer (2003)

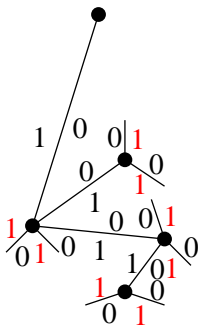


110101110100101001101000  $\rightsquigarrow$  6n bits

...1w10w10w0...

# Optimal encoding

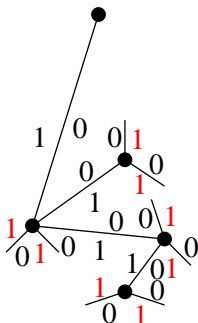
Poulalhon and Schaeffer (2003)



110101110100101001101000  
...1w10w10w0...

# Optimal encoding

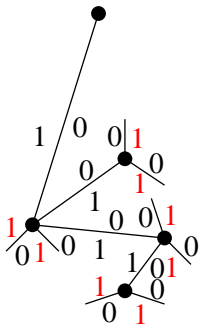
Poulalhon and Schaeffer (2003)



1001100000010000  
...1w10w10w0...

# Optimal encoding

Poulalhon and Schaeffer (2003)

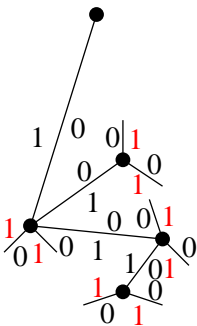


1001100000010000  $\rightsquigarrow$  4n bits

...1w10w10w0...

# Optimal encoding

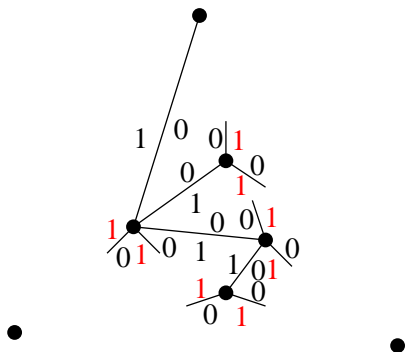
Poulalhon and Schaeffer (2003)



1001100000010000  $\rightsquigarrow$   $4n$  bits ( $n$  bits 1)  
...1w10w10w0...

# Optimal encoding

Poulalhon and Schaeffer (2003)

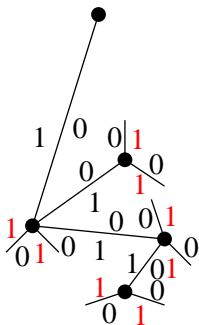


100110000010000  $\rightsquigarrow$  4n bits (n bits 1)  $\rightsquigarrow$  3,25n bits  
...1w10w10w0...



# Optimal encoding

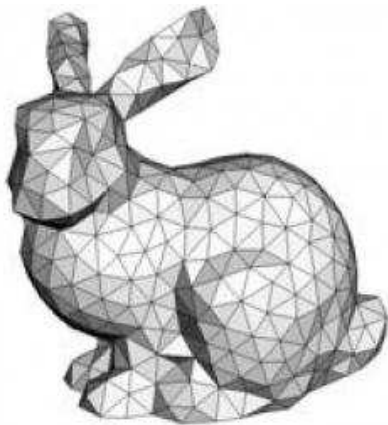
Poulalhon and Schaeffer (2003)



1001100000010000  $\rightsquigarrow$  4n bits (n bits 1)  $\rightsquigarrow$  3,25n bits  
...1w10w10w0... **OPTIMAL !**

# Optimal encoding

Poulalhon and Schaeffer (2003)



1001100000010000  $\rightsquigarrow$  4n bits (n bits 1)  $\rightsquigarrow$  3,25n bits

...1w10w10w0...

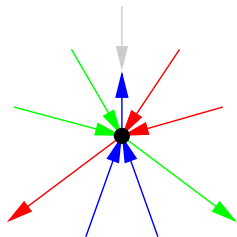
**OPTIMAL !**

# Generalization to 3-connected planar map

Felsner (2001), Miller (2002)

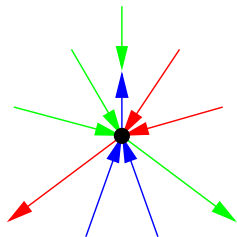
# Generalization to 3-connected planar map

Felsner (2001), Miller (2002)



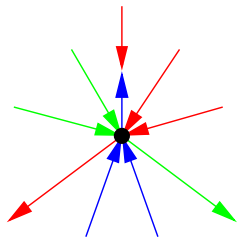
# Generalization to 3-connected planar map

Felsner (2001), Miller (2002)



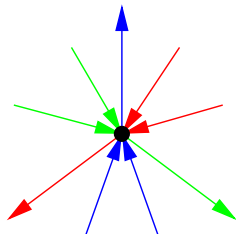
# Generalization to 3-connected planar map

Felsner (2001), Miller (2002)



# Generalization to 3-connected planar map

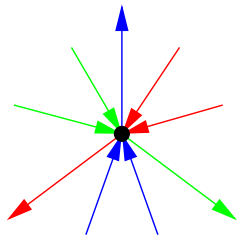
Felsner (2001), Miller (2002)



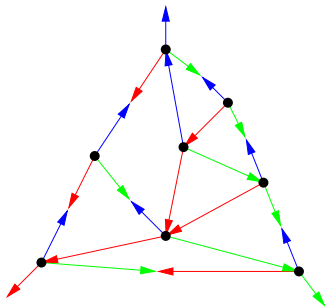
For every vertex

# Generalization to 3-connected planar map

Felsner (2001), Miller (2002)



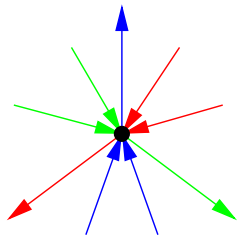
For every vertex





# Generalization to 3-connected planar map

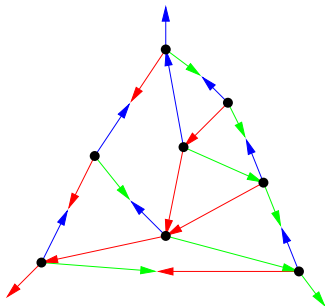
Felsner (2001), Miller (2002)



For every vertex

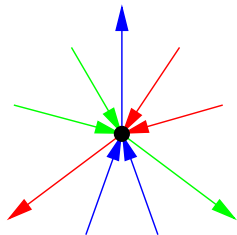
AND

No monochromatic cycles



# Generalization to (internally) 3-connected planar map

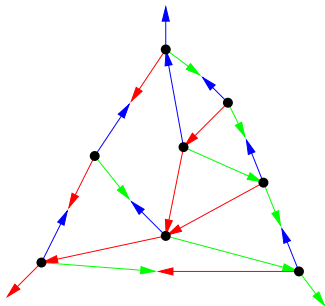
Felsner (2001), Miller (2002)



For every vertex

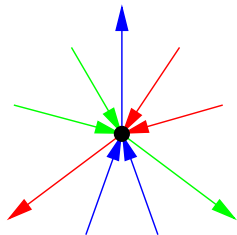
AND

No monochromatic cycles

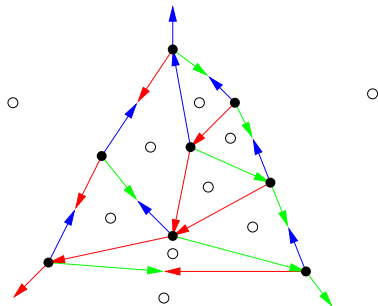


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Felsner (2001), Miller (2002)



For every vertex

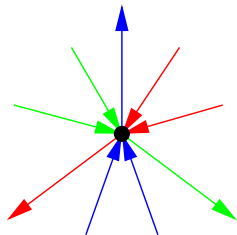


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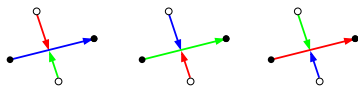
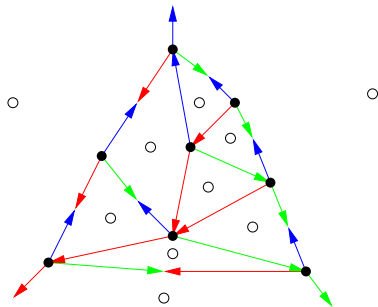
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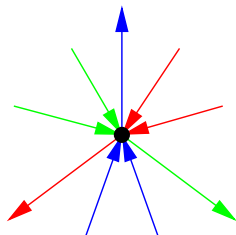
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Dual rules

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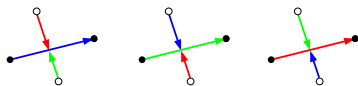
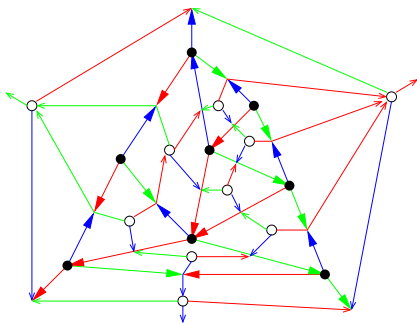
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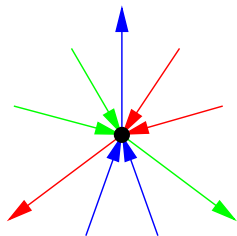
No monochromatic cycles



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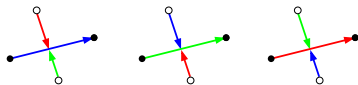
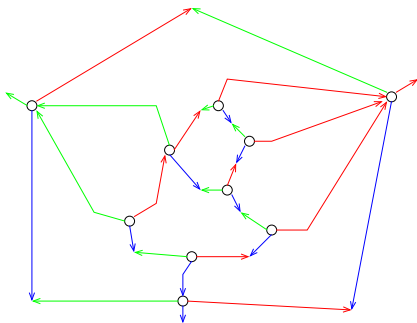
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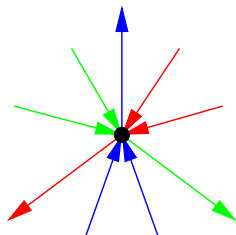
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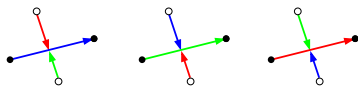
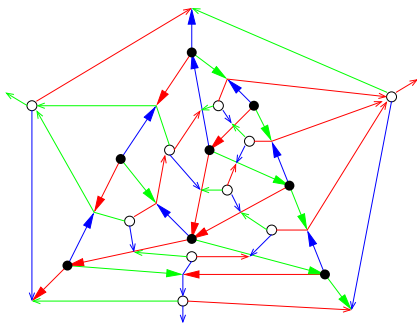
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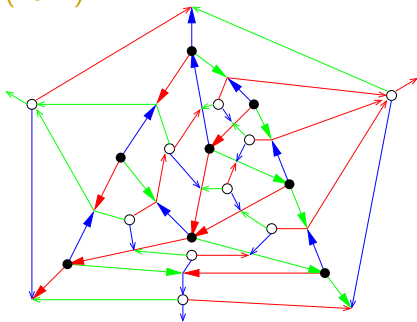
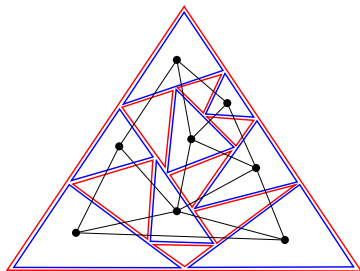
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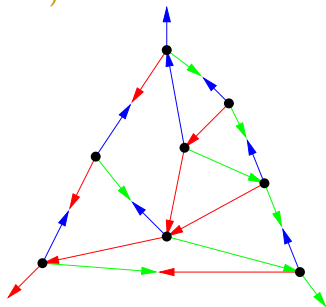
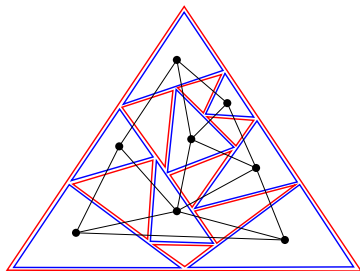
Gonçalves, Lévêque, Pinlou (2012)





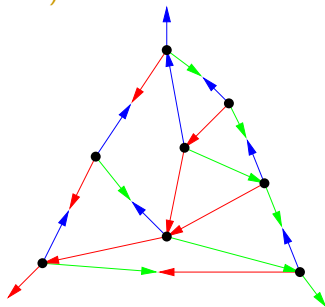
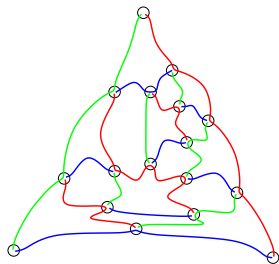
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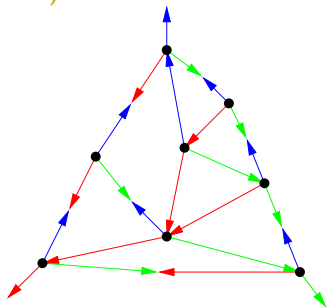
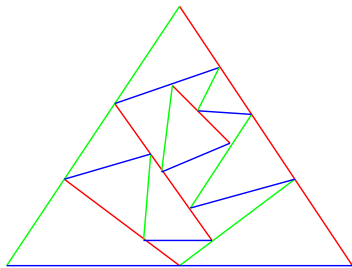
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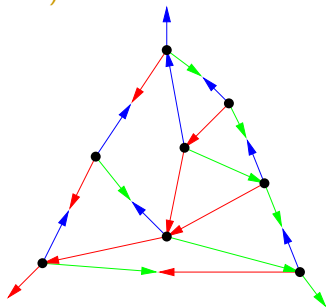
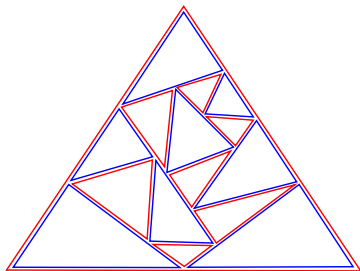
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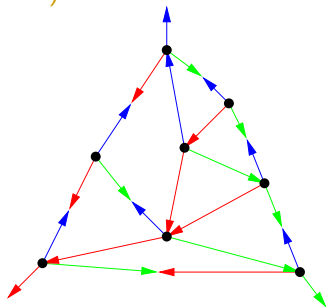
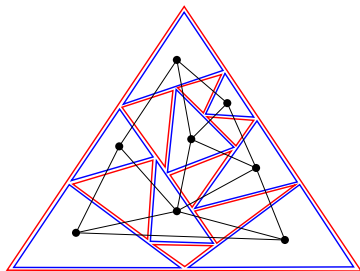
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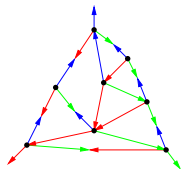
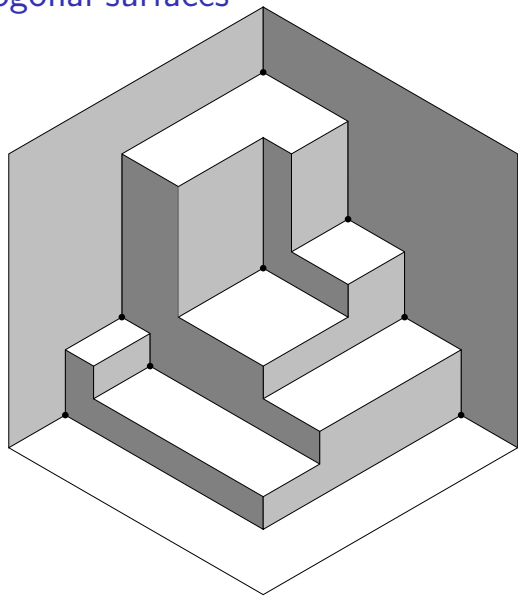


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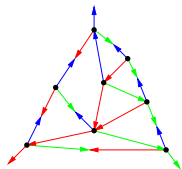
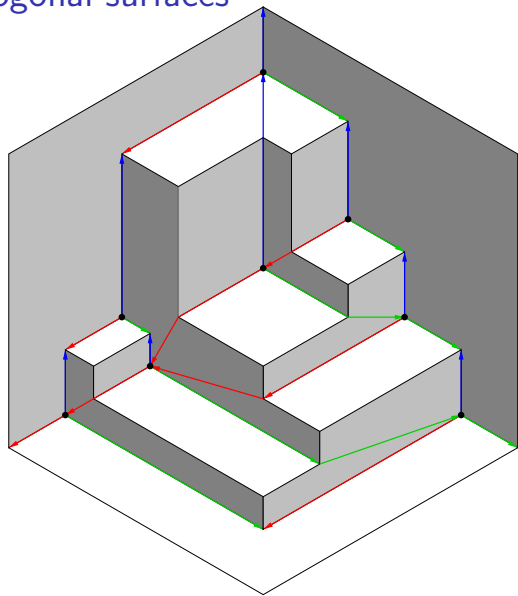
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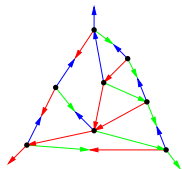
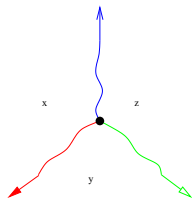
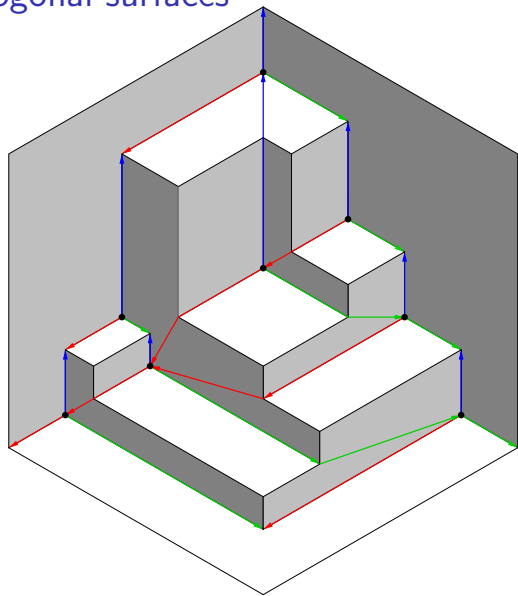
# Orthogonal surfaces



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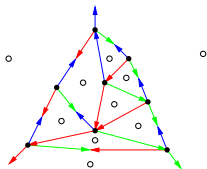
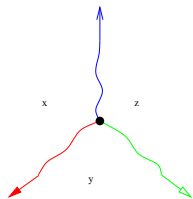
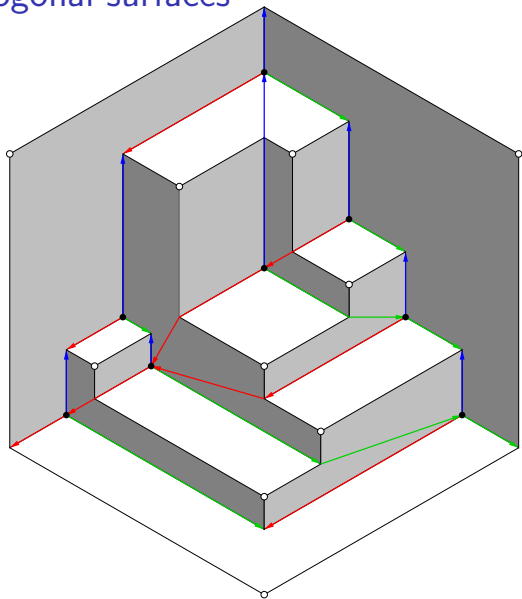


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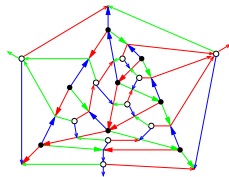
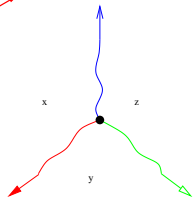
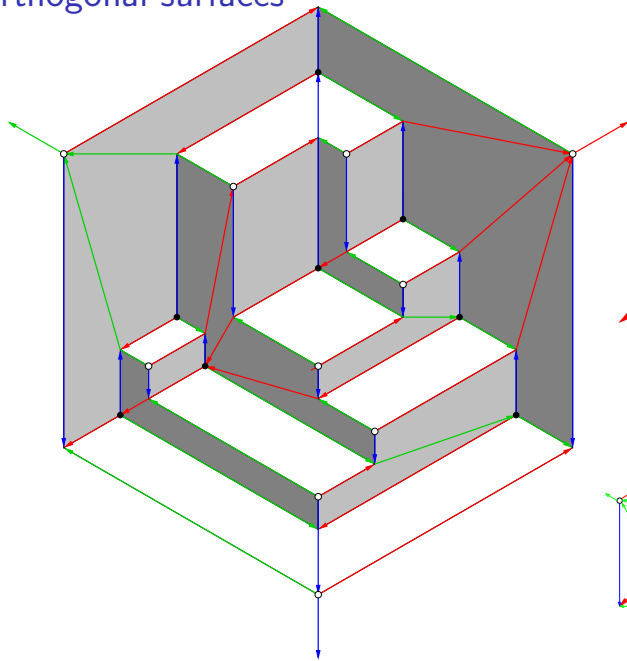




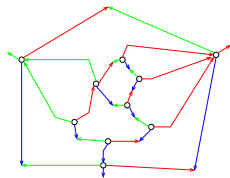
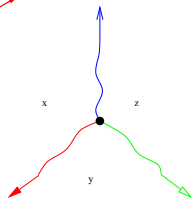
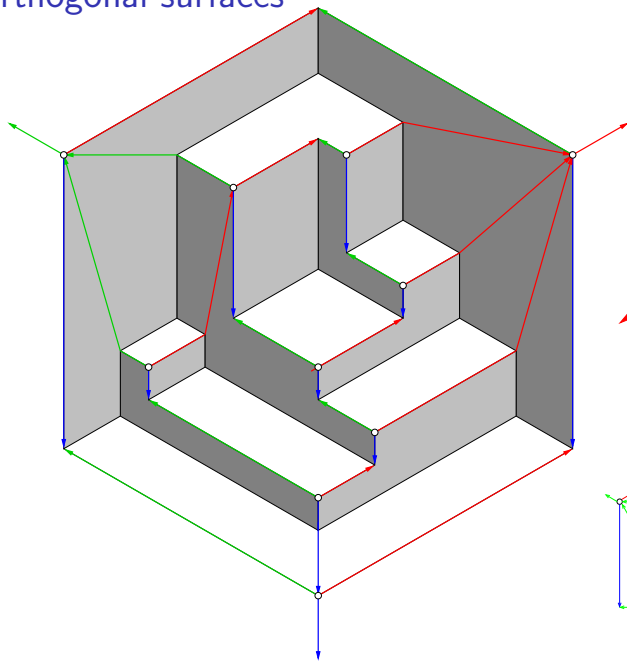
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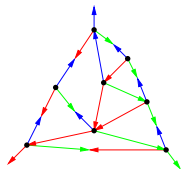
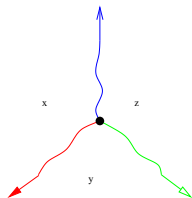
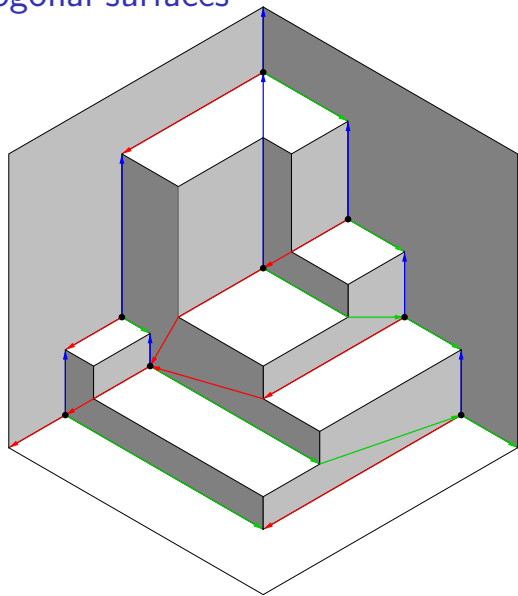
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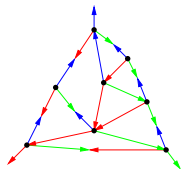
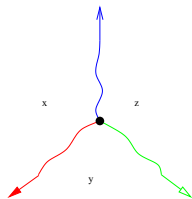
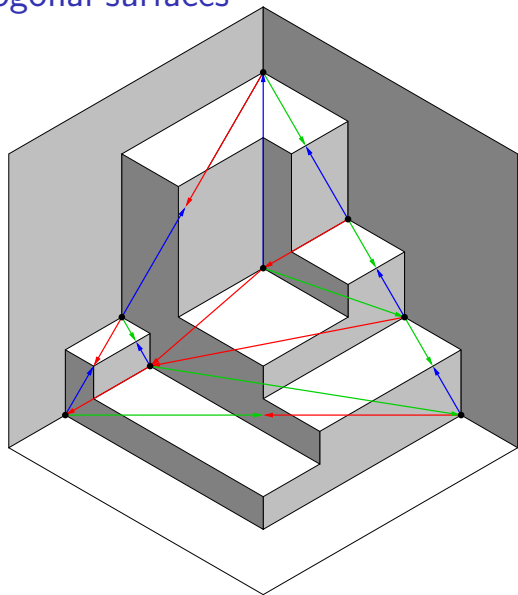
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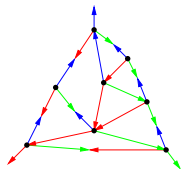
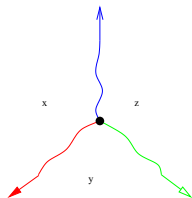
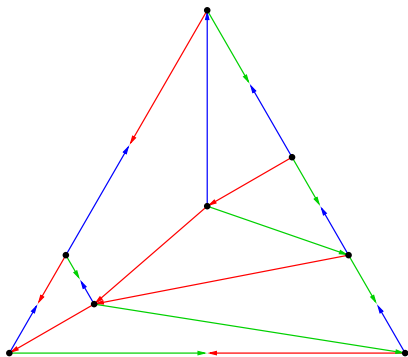
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# A generalization of Schnyder woods to higher genus

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Castelli Aleardi, Fusy, Lewiner (2009) : applications to encoding

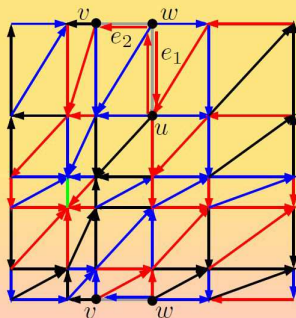
i) a small set  $\mathcal{E}^s$  of *special edges*, doubly oriented and colored

$(u, v, w)$

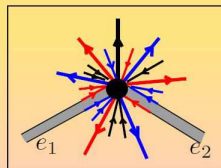
at most  $2 \cdot 2g$  multiple vertices (incident to special edges)



$\mathcal{E}^s = \{e_1, e_2\}$



ii) a new local condition for edges in a sector incident to a multiple vertex





## A different approach

## A different approach

Euler's formula in genus  $g$  :  $n - m + f = 2 - 2g$

## A different approach

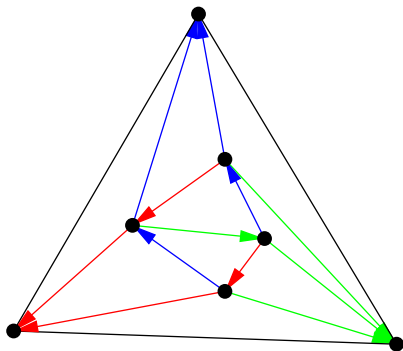
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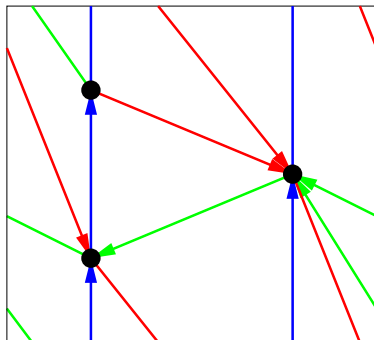
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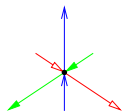
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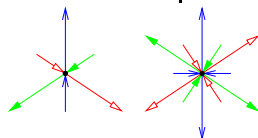
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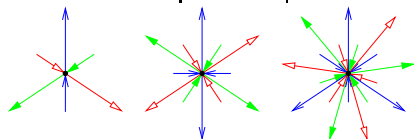
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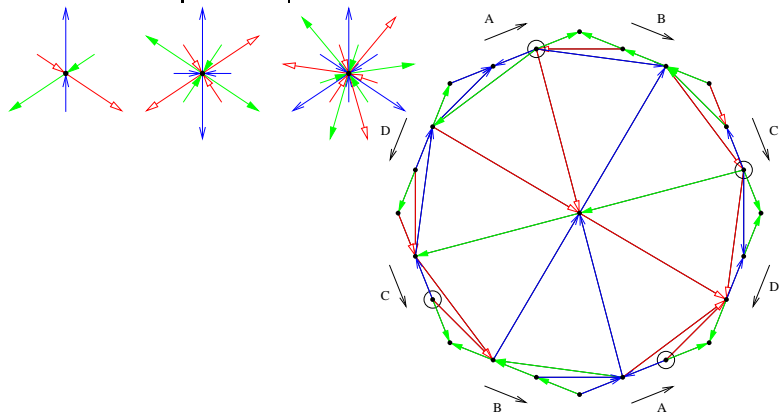
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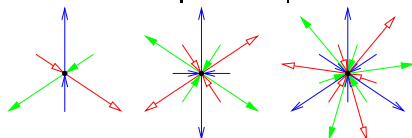
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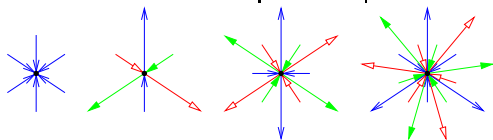
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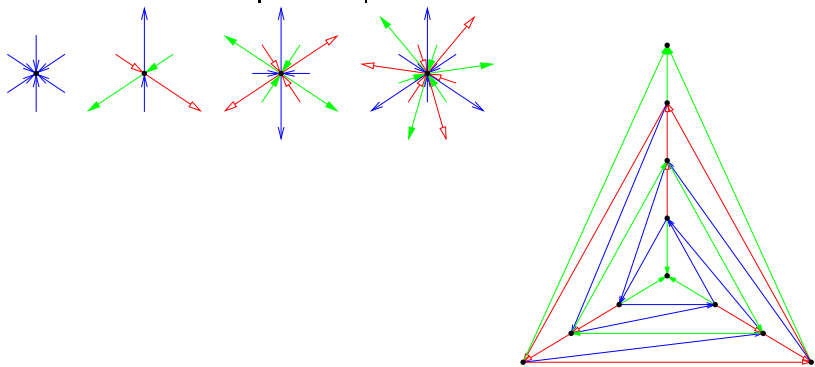
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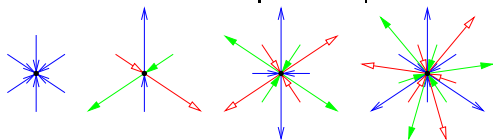
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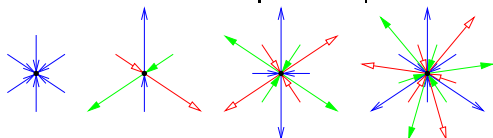




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**Theorem Barát, Thomassen (2006)**

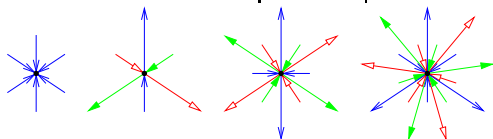
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**Theorem** Albar, Gonçalves, Knauer (2014)

Triangulation on a surface  $\implies$

orientation of the edges such that  $d^+(v) \equiv 0 \pmod{3}$ ,  $d^+(v) > 0$

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Plane : Schnyder wood  $\iff$  3-orientation

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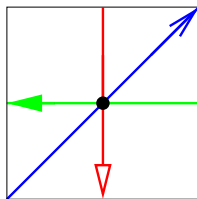
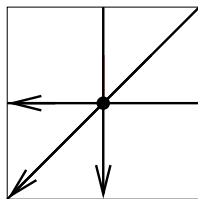
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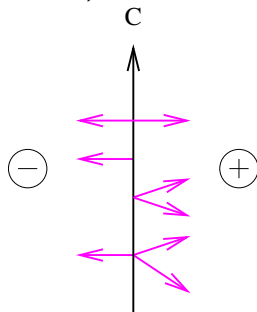
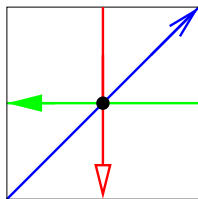
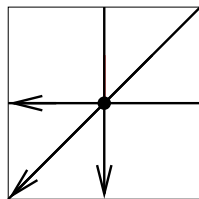


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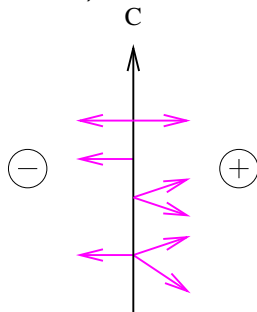
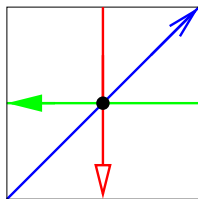
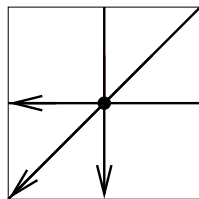


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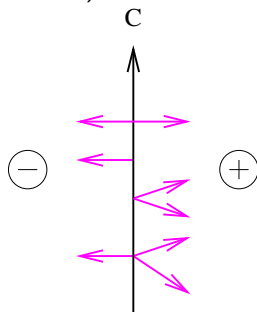
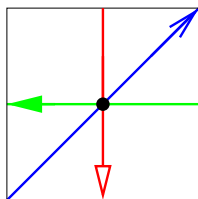
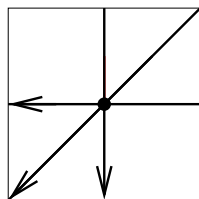
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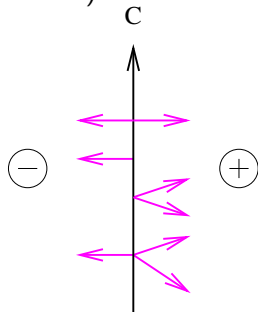
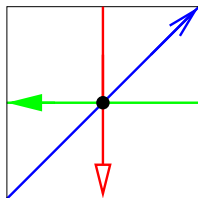
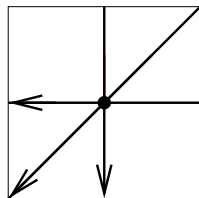
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Homology  $\rightsquigarrow$  Check  $\gamma$  only for a base

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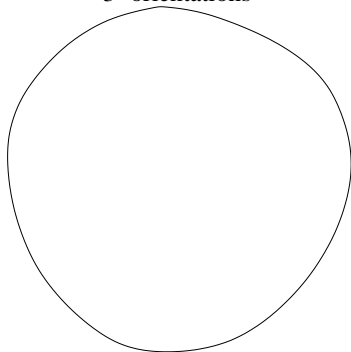
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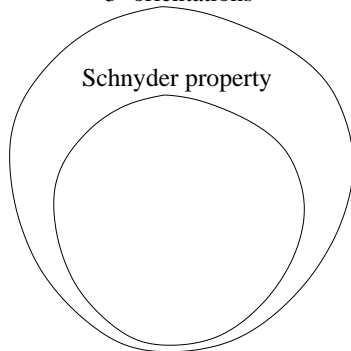
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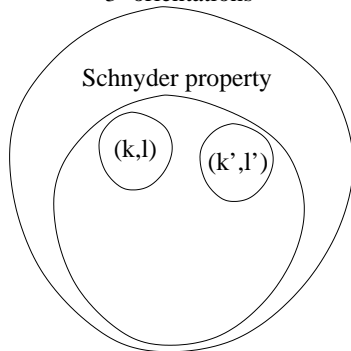
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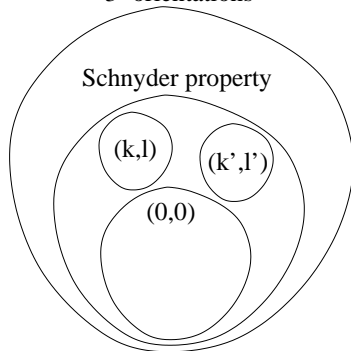
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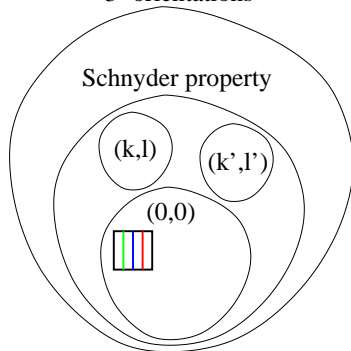




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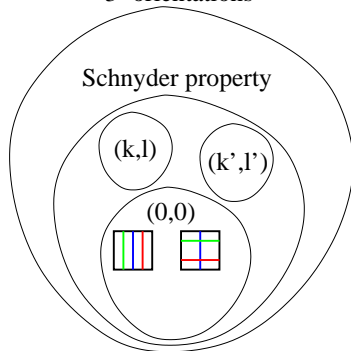
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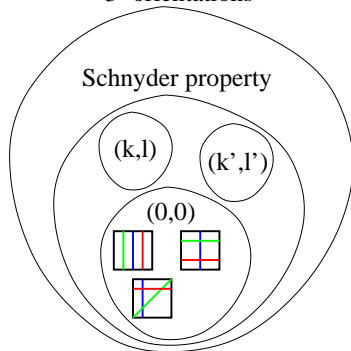
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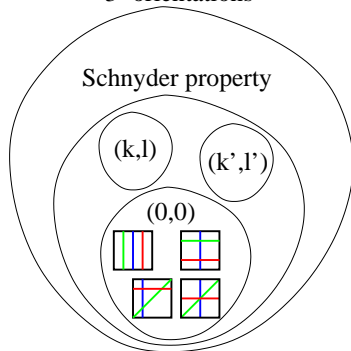
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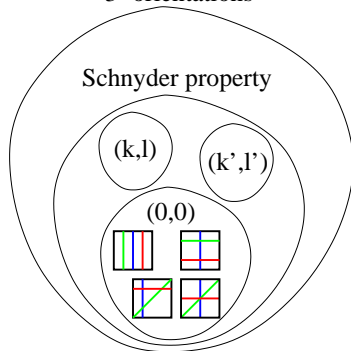
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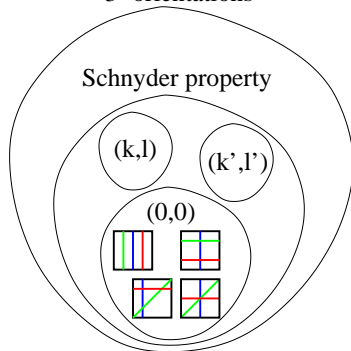


Torus : 3 proofs of existence

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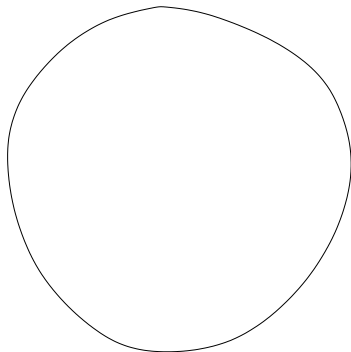


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### **Theorem Fijavz**

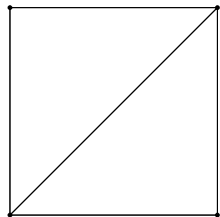
**Simple** toroidal triangulation  $\implies$  Three non contractible and non homotopic cycles that all intersect on one vertex and disjoint otherwise.

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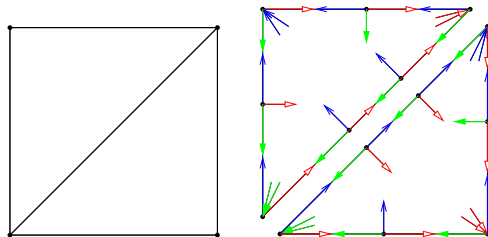


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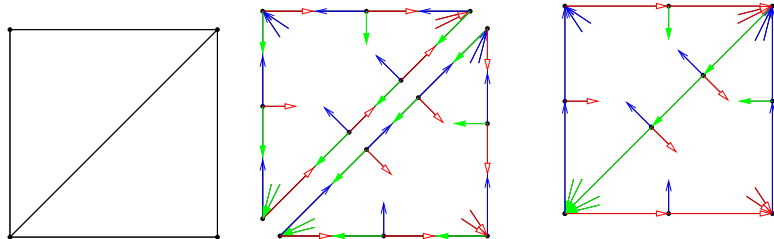


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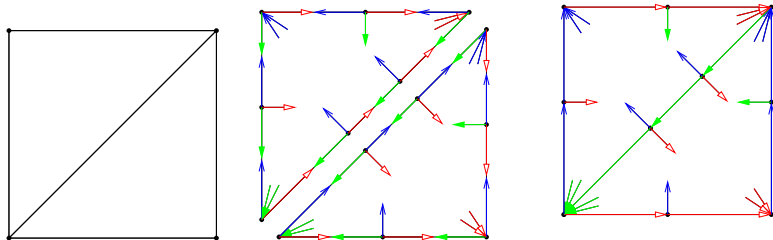


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$\implies$  one-point-crossing Schnyder wood



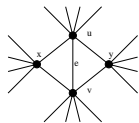
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By edge contraction

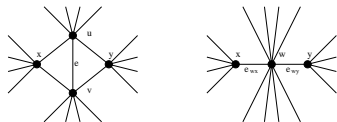
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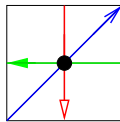
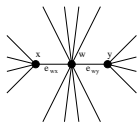
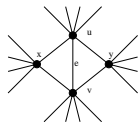
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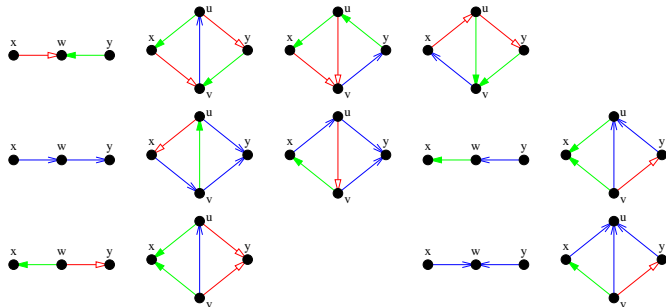
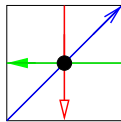
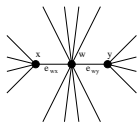
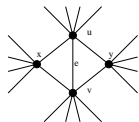
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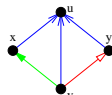
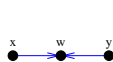
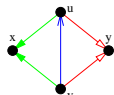
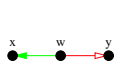
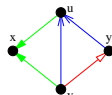
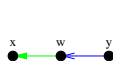
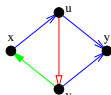
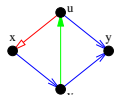
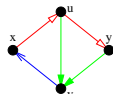
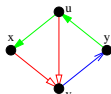
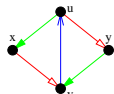
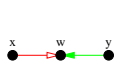
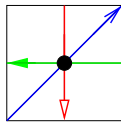
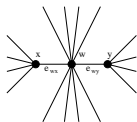
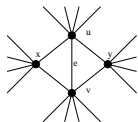
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**Theorem** [Goncalves, Lévêque \(2014\)](#) A toroidal map admits a **crossing** Schnyder wood if and only if it is essentially 3-connected.

## Torus : 3rd proof of existence



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Start with a 3-orientation

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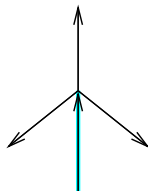
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**Lemme** A middle walk ends on a middle cycle.

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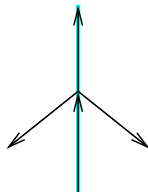
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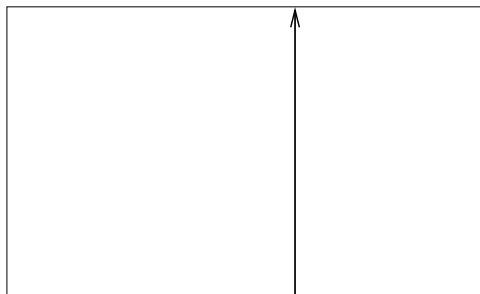
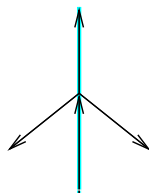
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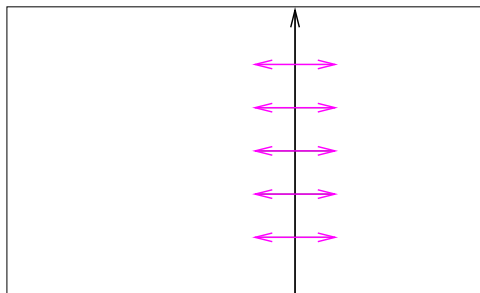
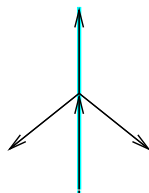
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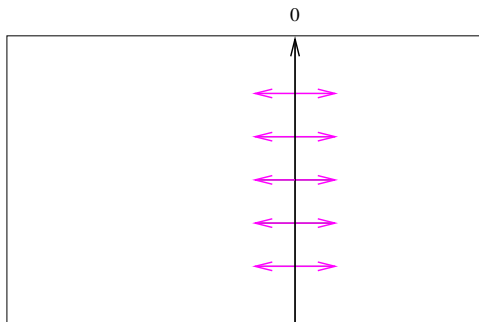
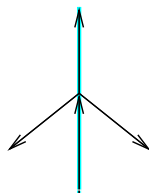
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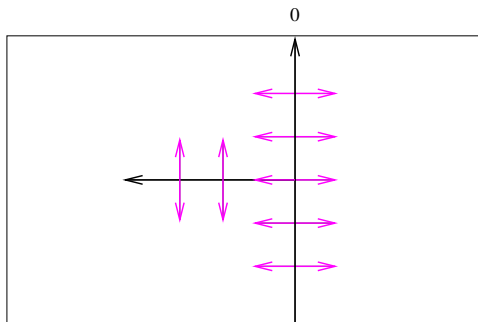
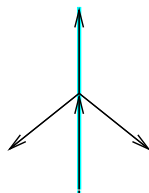
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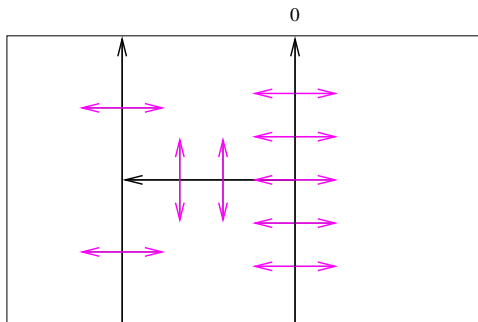
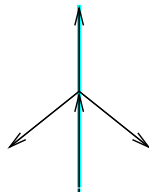




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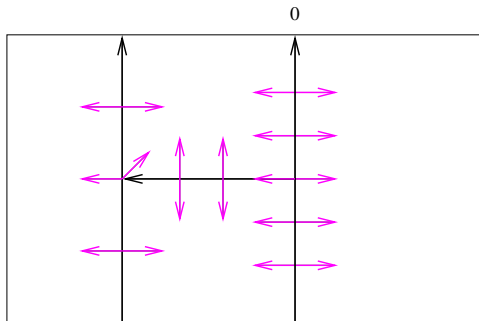
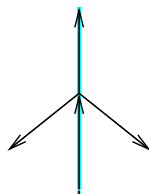
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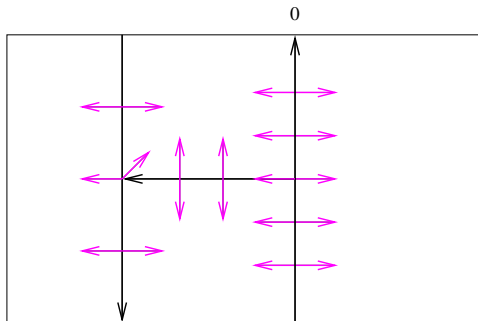
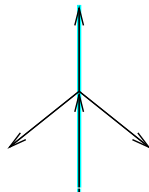
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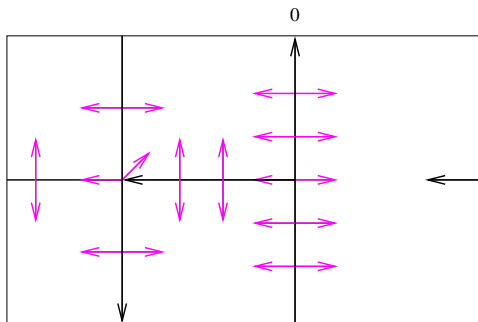
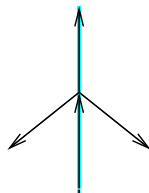
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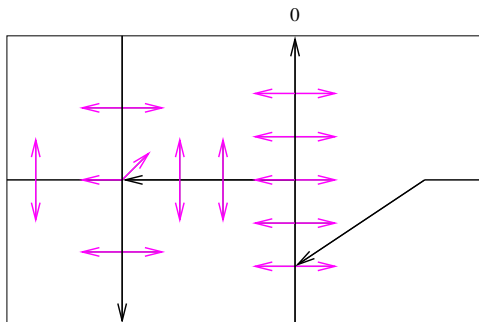
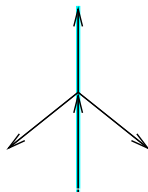
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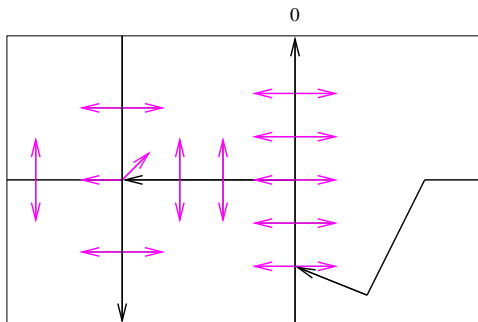
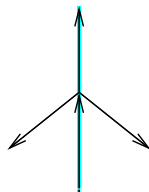
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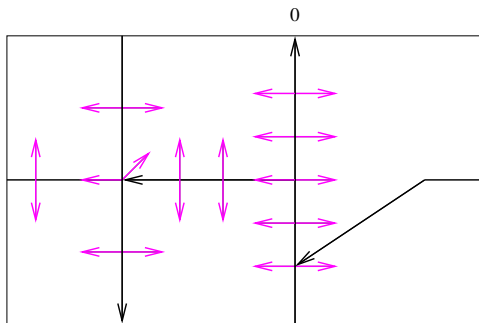
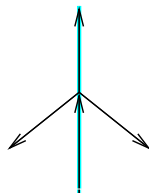
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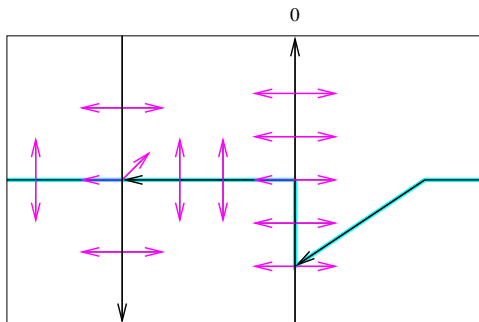
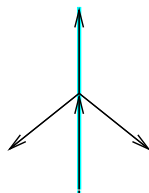
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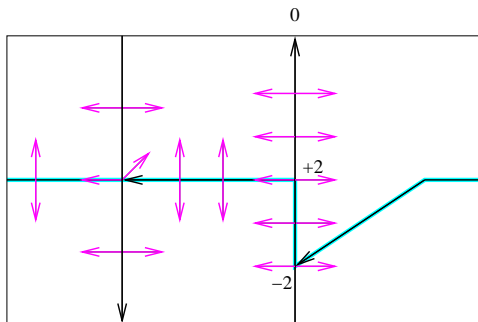
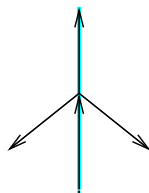




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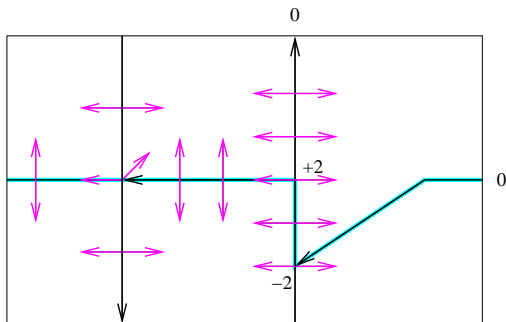
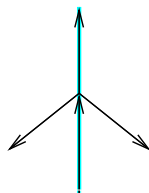
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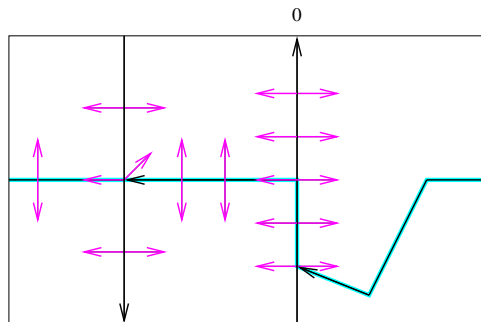
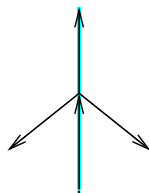
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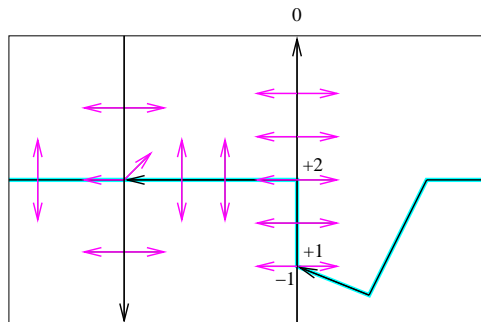
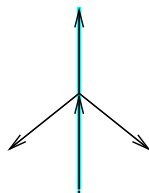
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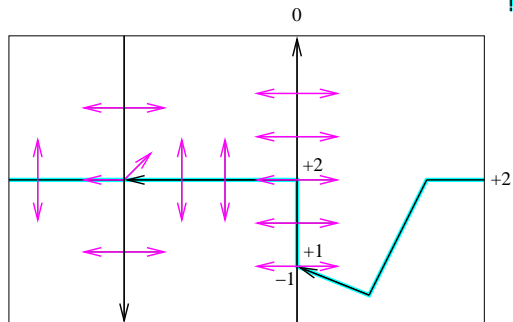
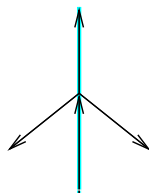
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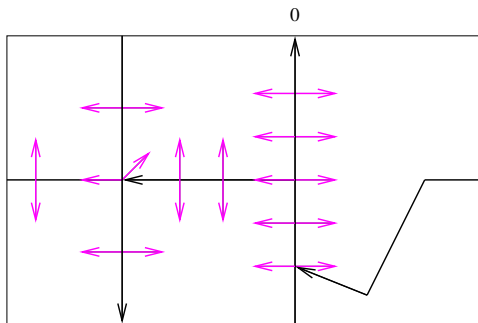
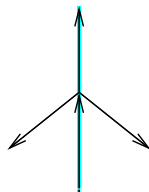
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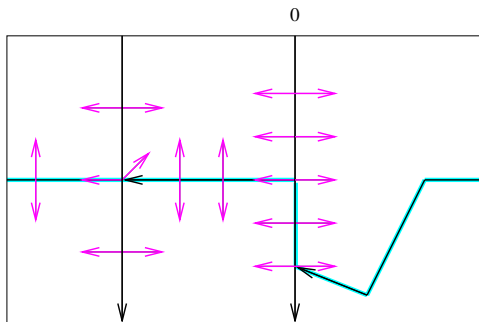
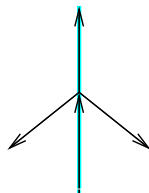




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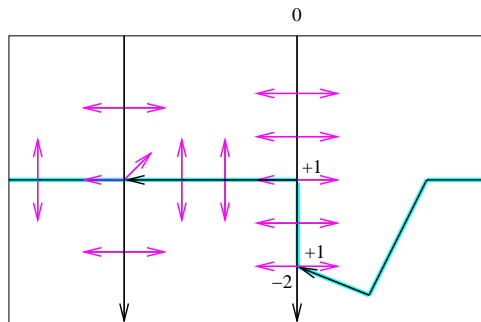
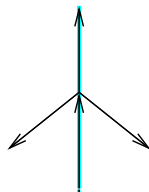




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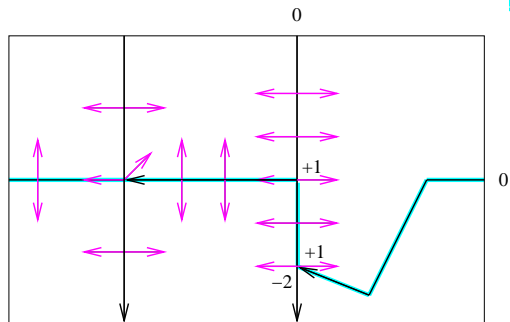
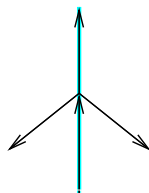
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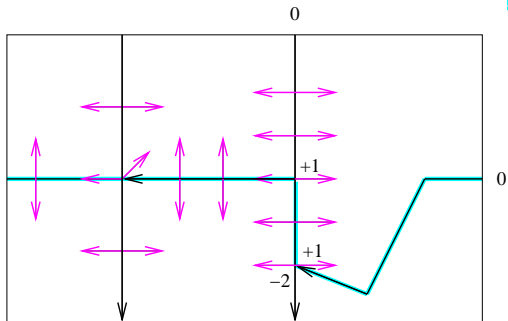
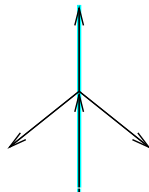
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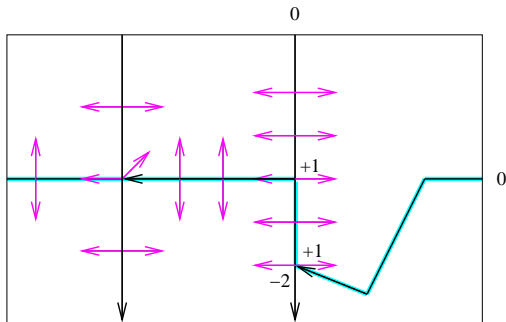
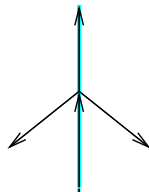


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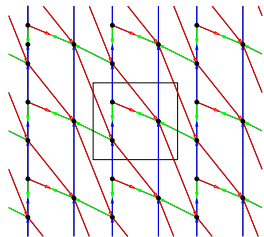
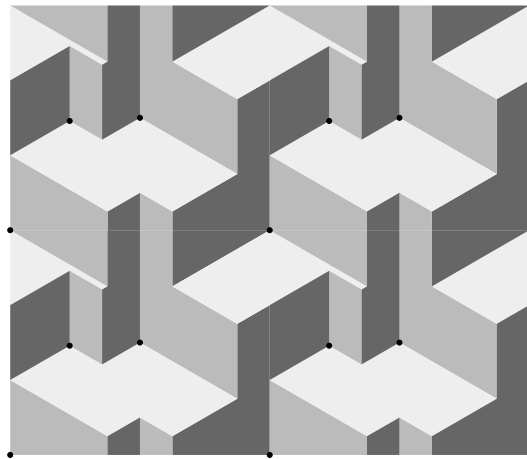
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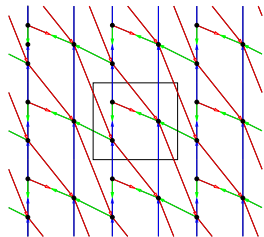
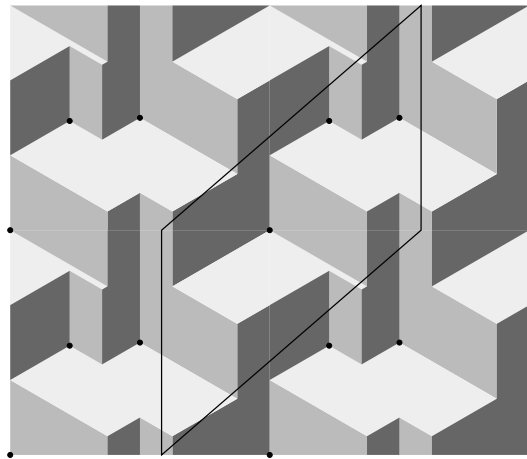
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$\implies$  Schnyder wood

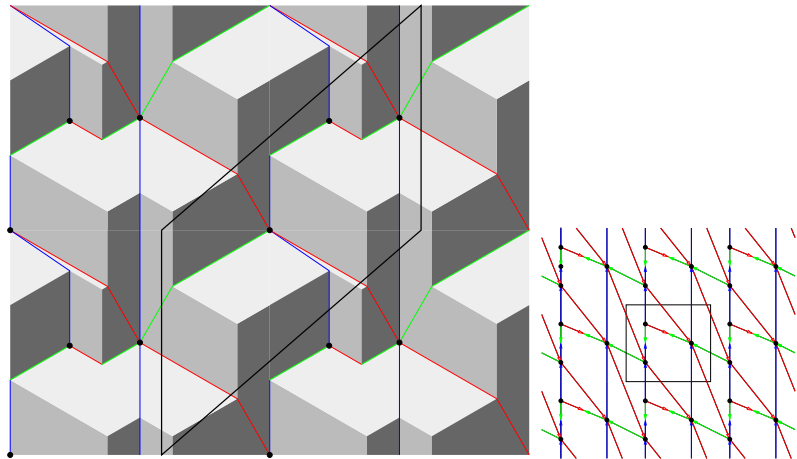
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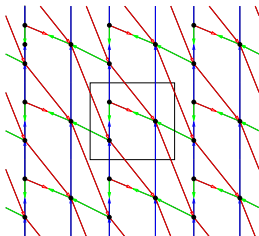
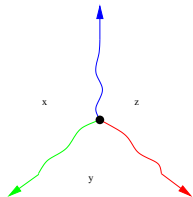
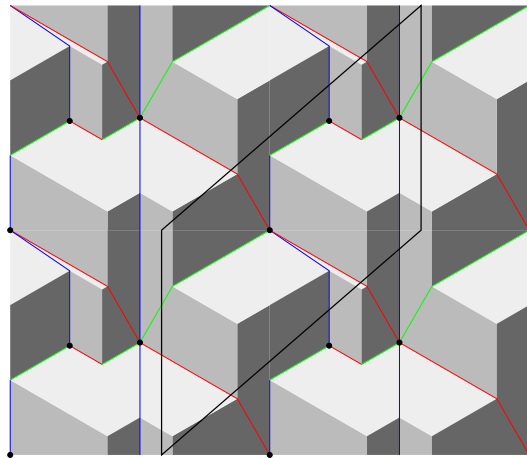
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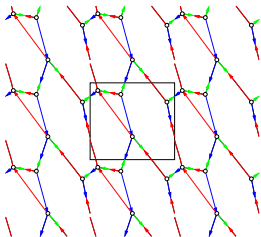
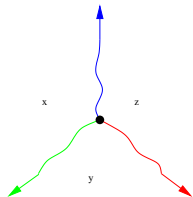
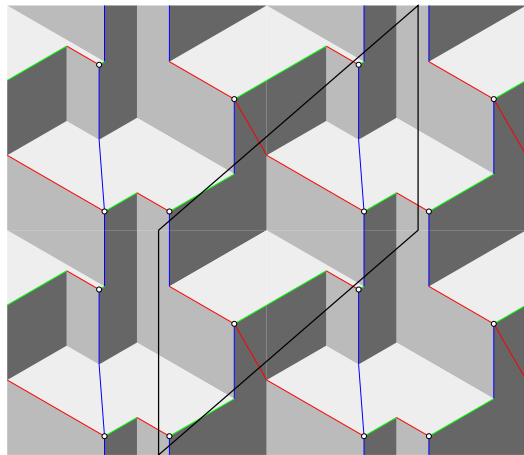


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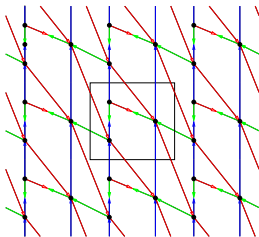
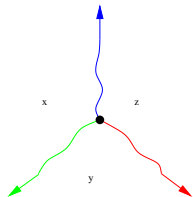
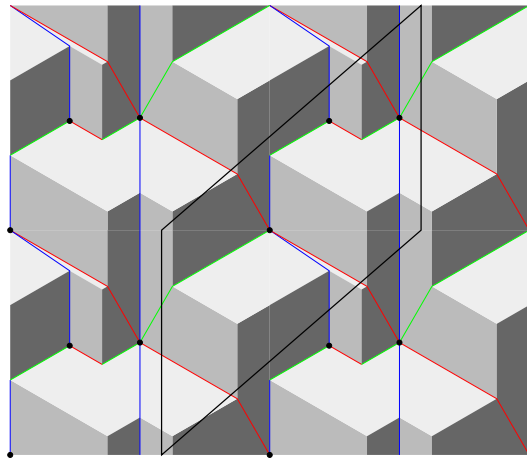




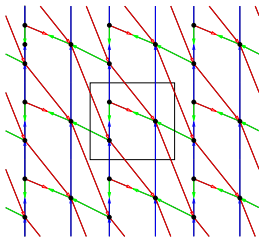
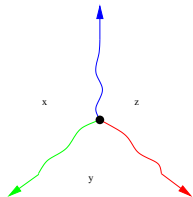
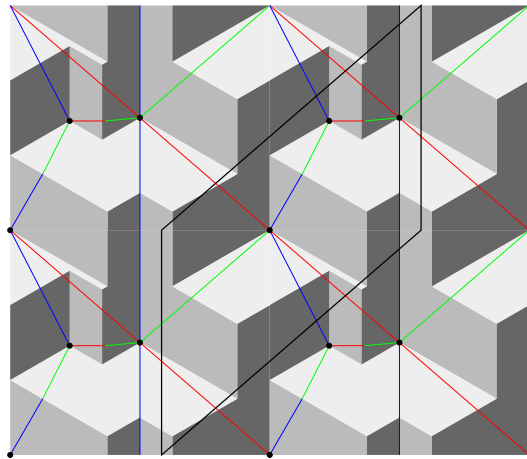
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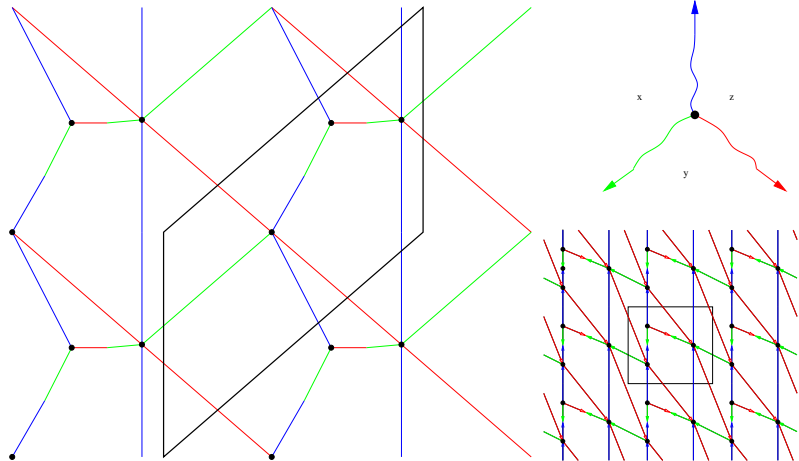
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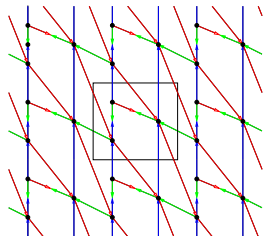
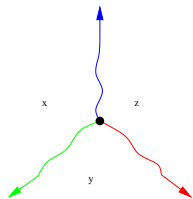
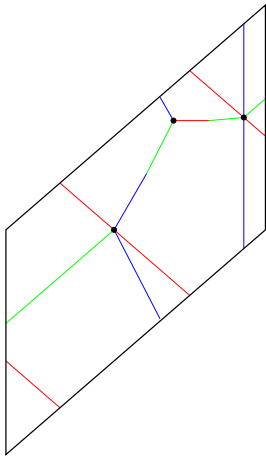
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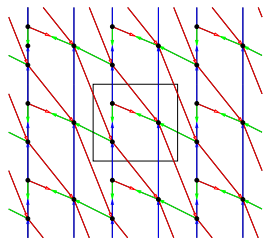
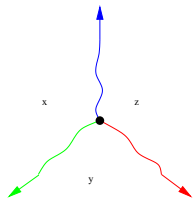
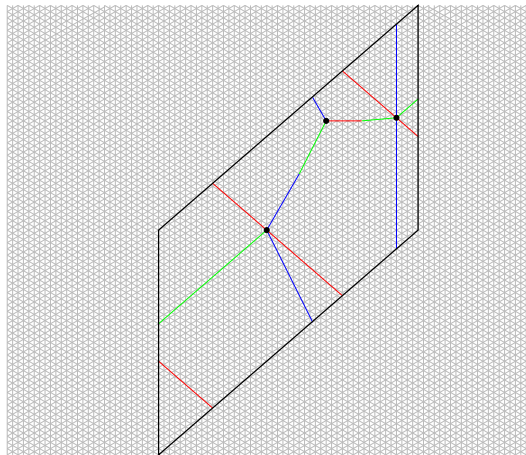
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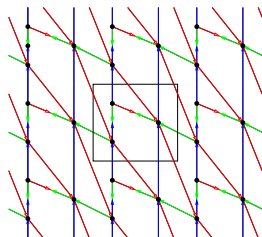
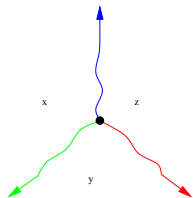
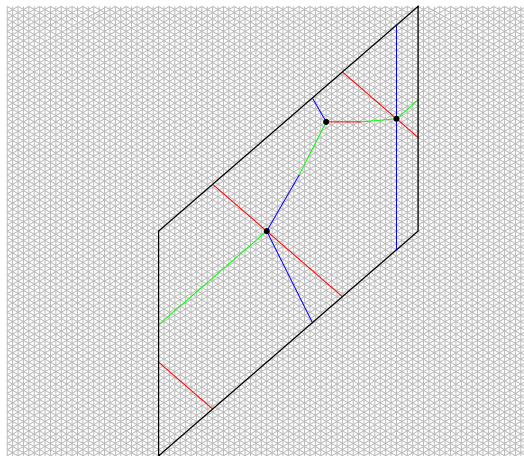
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**Theorem** A simple toroidal map admits a straight line representation in a grid of size  $\mathcal{O}(n^2) \times \mathcal{O}(n^2)$

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Enumeration, Compression, Spanners, . . .