

# Coverings of the sphere

## Algorithms and Algebra

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Happy birthday, Alexander!

January 28, 2016

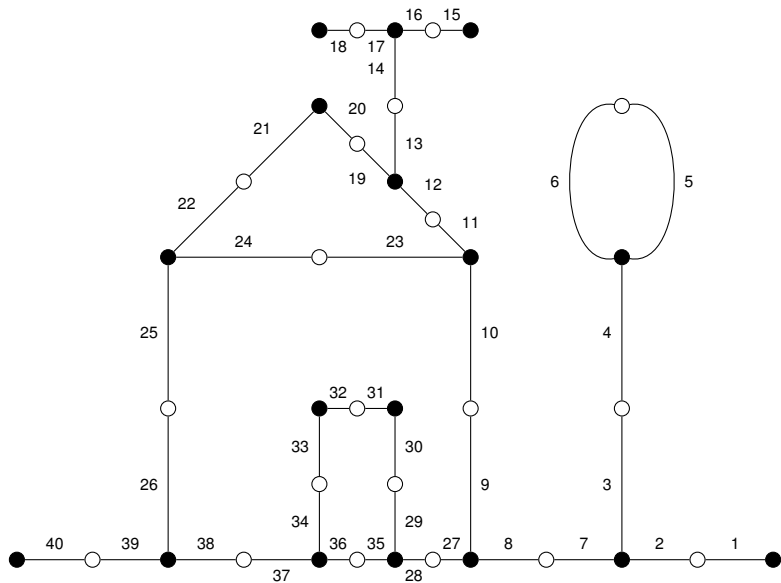
Grothendieck's house

Cui's map

Pilgrim's map



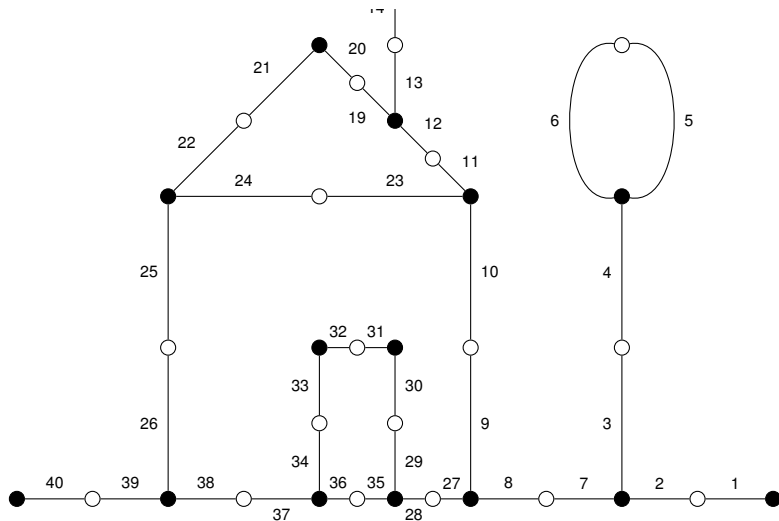
# Dessins d'enfant



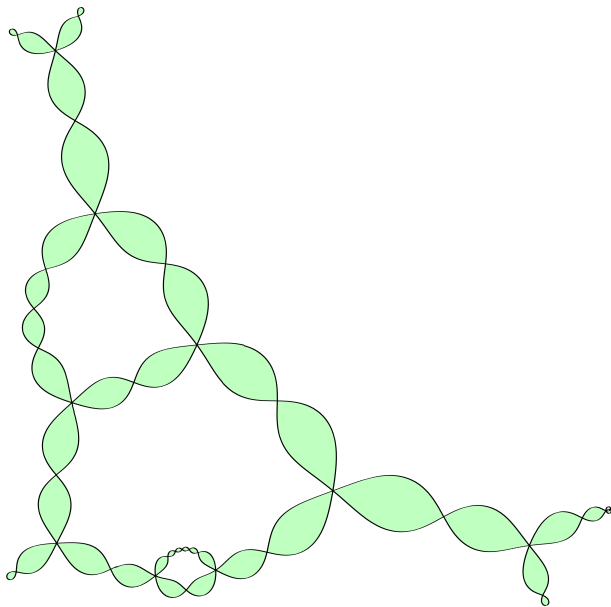
# Dessins d'enfant

$$\sigma = (1, 2)(3, 4) \cdots (39, 40)$$

$$\tau = (1)(2, 3, 7)(4, 5, 6)(8, 9, 27) \cdots (26, 37, 38)(40)$$

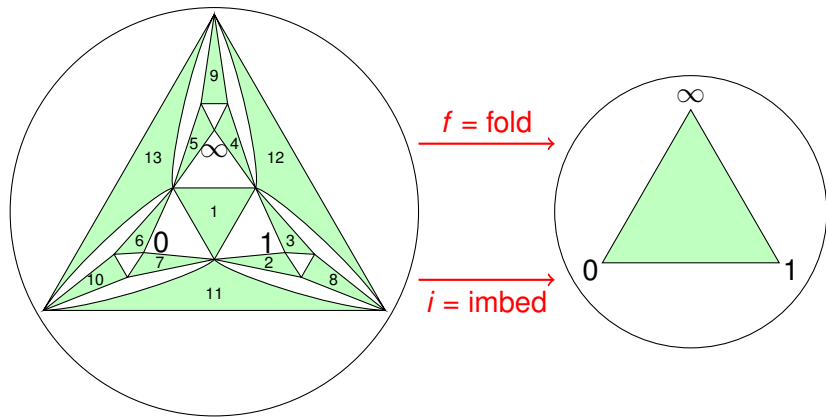


# Dessins d'enfant – Grothendieck's house





# Thurston maps – triangulations

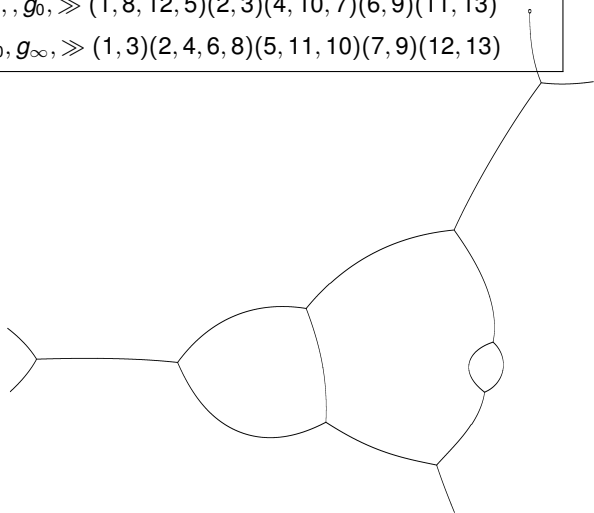


## Cui's map – biset and dessin d'enfant

$$g_0 = \langle\langle g_0, \dots, g_1^{-1}, g_0^{-1}, \dots, g_\infty^{-1} \rangle\rangle (1, 2)(3, 5, 4)(6, 7)(8, 9, 10, 13)(11, 12)$$

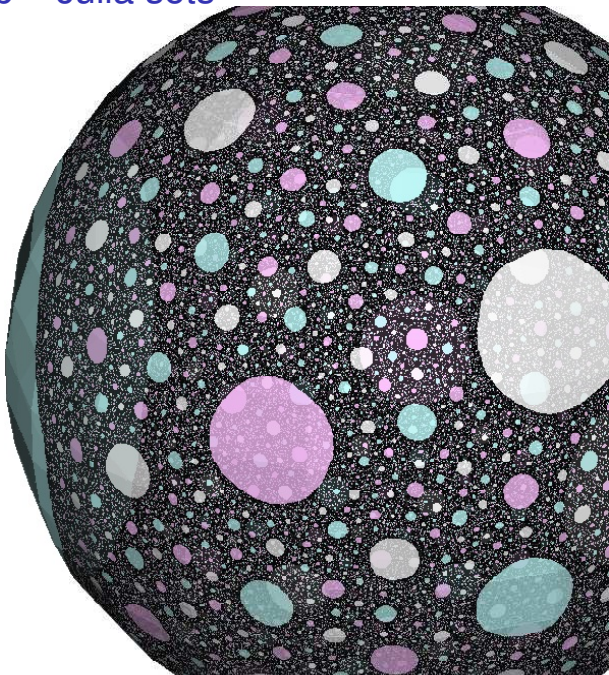
$$g_1 = \langle\langle g_0^{-1}, \dots, g_1, g_0 \rangle\rangle (1, 8, 12, 5)(2, 3)(4, 10, 7)(6, 9)(11, 13)$$

$$g_\infty = \langle\langle g_0^{-1}, \dots, g_0, g_\infty \rangle\rangle (1, 3)(2, 4, 6, 8)(5, 11, 10)(7, 9)(12, 13)$$



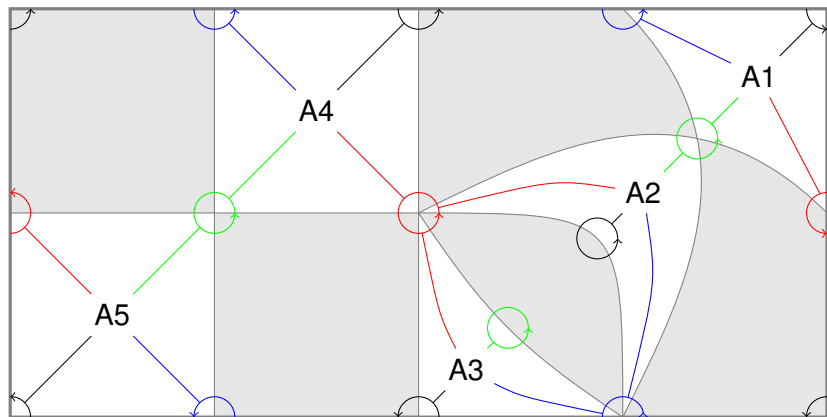


## Cui's map – Julia sets

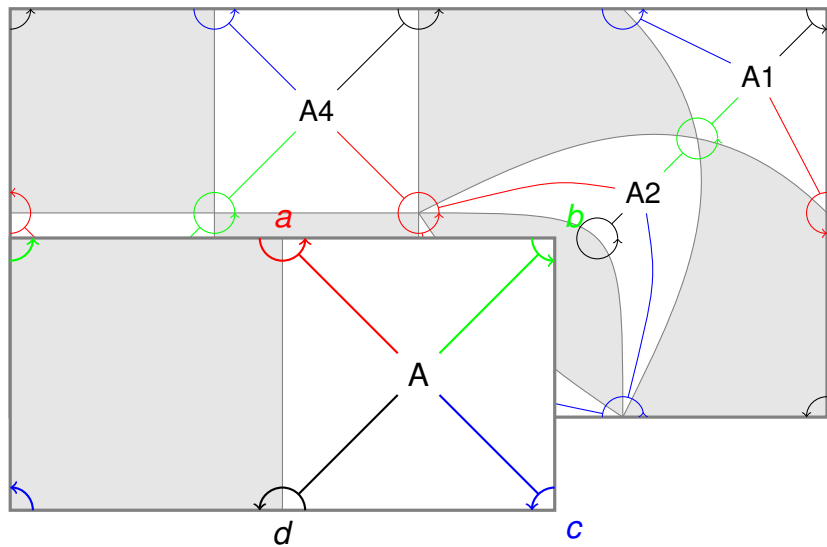




# Pilgrim's map – before



# Pilgrim's map – before



## Pilgrim's map – biset

$$a = \lll c^{-1}, 1, 1, 1, c \ggg (1, 5)(2, 4, 3)$$

$$b = \lll 1, 1, 1, d, d^{-1} \ggg (1, 2)(4, 5)$$

$$c = \lll a, 1, 1, a^{-1}, 1 \ggg (1, 4)(2, 3, 5)$$

$$d = \lll b, 1, d, a, c \ggg ()$$

## Pilgrim's map – biset

$$a = \lll c^{-1}, 1, 1, 1, c \ggg (1, 5)(2, 4, 3)$$

$$b = \lll 1, 1, 1, d, d^{-1} \ggg (1, 2)(4, 5)$$

$$c = \lll a, 1, 1, a^{-1}, 1 \ggg (1, 4)(2, 3, 5)$$

$$d = \lll b, 1, d, a, c \ggg ()$$

$$x = ac = \lll c^{-1}, cx^{-1}, 1, 1, x^{c^{-1}} \ggg (1, 2)(4, 5)$$

# Pilgrim's map – after

