

Two models dedicated to image colorization.
Second Workshop on Mathematical Analysis of Images in
Bordeaux

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Tuesday, April 8, 2014.

Joint work with Jean-François Aujol, Aurélie Bugeau, Nicolas Papadakis, Vinh-Thong Ta.

Plan

Colorization problem.

Starting point.

Luminance-chrominance model.

Primal-dual like algorithm.

Collaborative colorization.

A *RGB* model.

Conclusion.

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Luminance-chrominance space

Luminance :

$$Y = 0.299R + 0.587G + 0.114B$$

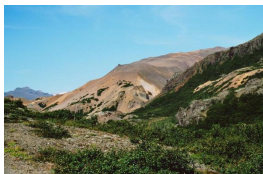
(U, V) are defined such that:

$$[0, 255]^3 \rightarrow [a1, a2] \times [b1, b2] \times [c1, c2]$$

$$(R, B, G) \mapsto (Y, U, V)$$

is a bijection.

- ▶ The human eye is sensitive to Y ;
- ▶ If Y is a gray-scale image to colorize, the computation of U and V gives the color image.



Color image.



Gray-scale version (Y).

Manual colorization.

- ▶ State-of-the-art: Levin *et al.* 2004, Sapiro 2005, Horiuchi *et al.* 2004 ;
- ▶ Need some tedious work for the user;
- ▶ Regular results;
- ▶ Most of methods work in luminance-chrominance spaces.

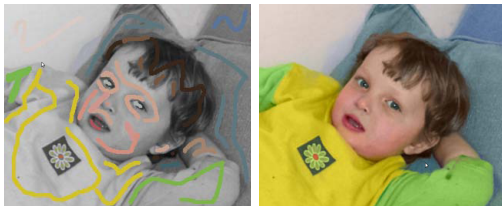


Figure: Manual colorization of Levin *et al.*

The problem of exemplar-based colorization.



Source.

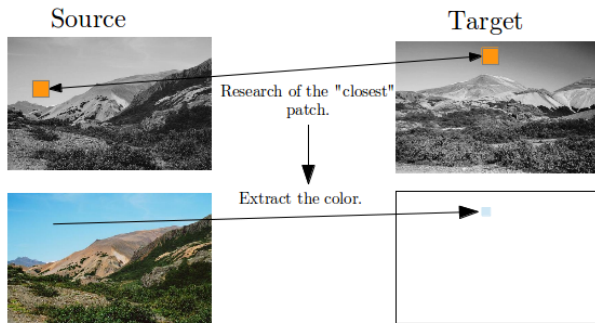
Target.

Result.

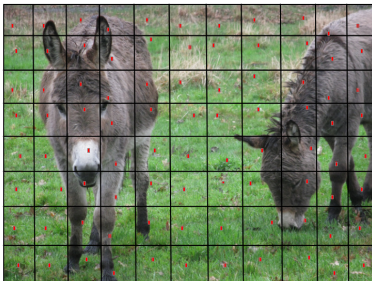
- ▶ Correspondence between textures and colors;
- ▶ Regularization of the result for a realistic image;
- ▶ A new difficulty: find an appropriate source image.

Add of a prior with textures.

- ▶ State-of-the-art: Welsh *et al.* 2002;
- ▶ No regularization;
- ▶ Problem of the choice of metric between patches;
- ▶ Method of Welsh *et al.* works in the $l\alpha\beta$ color-space.



Speed up the search.



- ▶ Sub-sampling of 200 pixels of the image on a grid.
- ▶ If the algorithm chooses 8 candidates over this set of 200 pixels it can take at least two same candidates with probability 13.22%.

Add of a prior with textures.

- ▶ State-of-the-art: Charpiat *et al.* (*Lab*) 2008, Bugeau *et al.* 2013 (*YUV*), Gupta *et al.* 2013 (*YUV*).
- ▶ Two steps:
 - ▶ Search of information: candidates extraction from textures criteria;
 - ▶ Spatial regularization of colors.

Proposed metrics by Bugeau, Ta and Papadakis (*YUV*).

Used criteria:

- ▶ standard-deviation: $\rho_1(p, q, P) := |\sigma^2(P_p) - \sigma^2(P_q)|$, where $\sigma(P_p)$ stands for the standard-deviation of the patch around the pixel p ;
- ▶ the DFT: $\rho_2(p, q, P) := \sum_{\xi} \left| \|\hat{P}_p(\xi)\|_2 - \|\hat{P}_q(\xi)\|_2 \right|$, where \hat{P}_p is the DFT of the patch around the pixel p ;
- ▶ cumulative histogram: $\rho_3(p, q, P) := \sum_i |H_{P_p}(i) - H_{P_q}(i)|$, where H is the cumulative histogram of the patch around the pixel p .

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Model of Bugeau, Ta and Papadakis (*YUV*).

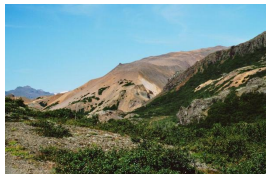
Computing of a local minimum of the following functional:

$$\begin{aligned} F(u, W) := & TV(u) + \frac{\lambda}{2} \int_{\Omega} \sum_{i=1}^N w_i \|u - c_i\|^2 \\ & + \alpha \int_{\Omega} \sum_{i=1}^N w_i (1 - w_i) \\ & + \chi_{u \in [0,255]^2} + \chi_{W \in \Delta}. \end{aligned}$$

u : chrominance vector, to be computed.

Weights w_i are such that $0 \leq w_i \leq 1$, $\forall i \in [1..N]$ et $\sum_{i=1}^N w_i = 1$.
 c_i are chrominance candidates.

Images are drab.



Source.



Target.



Result of Bugeau *et al.*

- ▶ Contours are damaged due to the lack of coupling;
- ▶ Over regularization to avoid this problem ;
- ▶ Drab images: need a post-processing.

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Luminance-chrominance coupled model.

A new energy-based model:

$$F(u, W) := TV_c(u) + \frac{\lambda}{2} \int_{\Omega} \sum_{i=1}^8 w_i \|u - c_i\|^2 \\ + \chi_{u \in [0,255]^2} + \chi_{W \in \Delta}.$$

u : chrominance vector, to be computed.

$$\Delta = \left\{ w \in \mathbb{R}^N \text{ s.t. } 0 \leq w_i \leq 1, \forall i \in [1..8] \text{ and } \sum_{i=1}^8 w_i = 1 \right\}.$$

c_i are chrominance candidates.

$$TV_c(u) = \int_{\Omega} \sqrt{\gamma \partial_x Y^2 + \gamma \partial_y Y^2 + \partial_x U^2 + \partial_y U^2 + \partial_x V^2 + \partial_y V^2}$$

Intuitions about coupling.

Consider the following model:

$$F(u, W) := TV_c(u) + \frac{\lambda}{2} \int_{\Omega} \|M(u - c)\|^2$$

With M a mask and c seeds of color put by the user.



Scribbles



Without coupling



With coupling

Intuitions about coupling.

Different regularizations with different coupling:



Small γ : contours of low perimeter for chrominance channels.

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Technical recalls.

Primal problem:

$$\min_{x \in X} [F(Kx) + G(x)] ,$$

where $G : X \rightarrow [0, +\infty]$; $F^* : Y \rightarrow [0, +\infty]$ are convex, proper, lower semi-continuous; K is a continuous linear operator.

Associated dual problem:

$$\min_{u \in X} \max_{p \in Y} [\langle Ku | p \rangle_X + G(u) - F^*(p)] .$$

Minimization algorithm.

Convergence of the primal-dual algorithm to a saddle-point of the functional.

Algorithm 1 Primal-dual algorithm of Chambolle and Pock 2011.

- 1: Initialization $\tau, \sigma > 0, \theta \in [0, 1]$, $(x^0, y^0) \in X \times Y$ et $\bar{x}^0 = x^0$.
 - 2: **for** $n \geq 0$ **do**
 - 3: $y^{n+1} = \text{prox}_{\sigma \partial F^*}(y^n + \sigma K \bar{x}^n)$
 - 4: $x^{n+1} = \text{prox}_{\tau \partial G}(x^n - \tau K^* y^{n+1})$
 - 5: $\bar{x}^n = x^{n+1} + \theta(x^{n+1} - x^n)$.
 - 6: **end for**
-

where $\text{prox}_f(\tilde{u}) = \operatorname{argmin}_u \frac{\|\tilde{u} - u\|_2^2}{2} + f(u)$.

New model.

Non-convex model:

$$\min_{x \in X} \min_{w \in W} \max_{y \in Y} \langle Kx|y \rangle - F^*(y) + G(x) + h(x, w) + H(w), \quad (1)$$

where $G : X \rightarrow [0, +\infty)$, $F^* : Y \rightarrow [0, +\infty)$, $H : W \rightarrow [0, +\infty)$ and $h : (X \times W) \rightarrow [0, +\infty)$ are proper, lower semi-continuous, F^* , G , H are convex, h is convex with respect to each variable. Moreover $\forall w \in \mathbb{R}^n$,

$$G + h(\cdot, w)$$

and $\forall x \in \mathbb{R}^n$,

$$H + h(x, \cdot)$$

are proper.

Minimization algorithm.

Algorithm 2 Computing a solution.

```
1: for  $n \geq 0$  do  
2:    $y^{n+1} \leftarrow \text{prox}_{\sigma F^*}(y^n + \sigma K \bar{x}^n)$   
3:    $w^{n+1} \leftarrow \text{prox}_{\rho H + \rho h(\bar{x}^n, \cdot)}(w^n)$   
4:    $x^{n+1} \leftarrow \text{prox}_{\tau G + \tau h(\cdot, w^{n+1})}(x^n - \tau K^* y^{n+1})$   
5:    $\bar{x}^{n+1} \leftarrow 2x^{n+1} - x^n$   
6: end for
```

Theorem (Pierre *et al.* 2014)

This algorithm converges to a fix-point.

Minimization algorithm.

Algorithm 3 Applied to colorization.

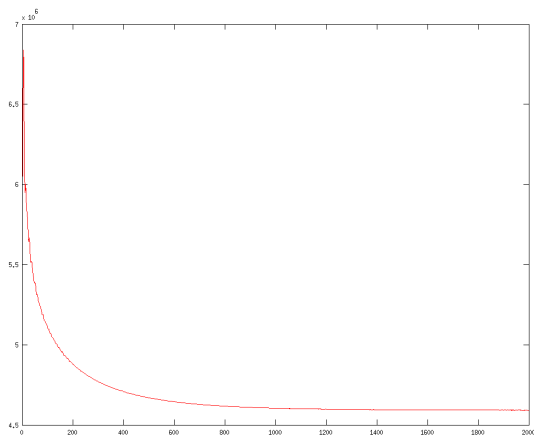
- 1: $W \leftarrow 1/N$
 - 2: $u^0 \leftarrow \sum_i w_i c_i$
 - 3: $Z^0 \leftarrow \nabla u^0$
 - 4: **for** $n \geq 1$ **do**
 - 5: $Z^{n+1} \leftarrow PB(Z^n + \sigma \nabla \bar{u}^n)$
 - 6: $W^{n+1} \leftarrow P_{\Delta} \left(W^n - \tau_w \left(\left(\|\bar{u}^n - c_i\|^2 \right)_i \right) \right)$
 - 7: $u^{n+1} \leftarrow PG \left(\frac{u^n + \tau (\operatorname{div}(Z^{n+1}) + \lambda \sum_i w_i^{n+1} c_i)}{1 - \delta \lambda} \right)$
 - 8: $\bar{u}^{n+1} \leftarrow 2u^{n+1} - u^n$
 - 9: **end for**
-

- ▶ P_{Δ} projection onto the simplex (Chen et al. 2011.);
- ▶ PG is a projection onto a rectangle.

Results in exemplar-based case.



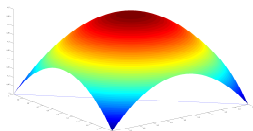
Numerical convergence.



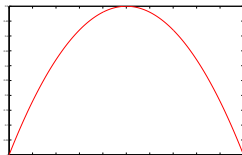
The value of the functional decreases during the convergence of the algorithm.

Ideas about the non-convexity.

$$\int_{\Omega} \sum_{i=1}^N w_i(1 - w_i) + \chi_{W \in \Delta}.$$



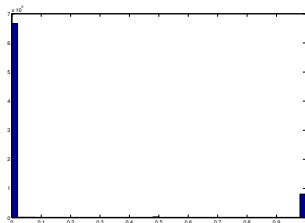
First term, $N = 2$.



Global energy, $N = 2$.

- ▶ With the non-convex term, weights are forced to be close to 1 and 0.
- ▶ Without, weights go to 1 or 0 just by taking account of the regularization of the image.

Ideas about the non-convexity.



The histogram shows that the non-convex term is unused. Notice that about 14% of weights do not go to 0 and 1.

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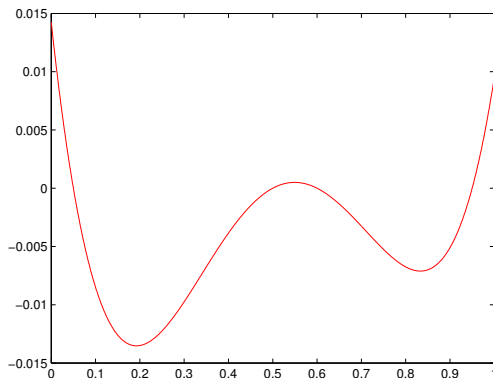
Conclusion.

Issues of the collaborative colorization.

No current method is effective:

- ▶ The selection of an image for the exemplar-based method is complex.
- ▶ The manual colorization is long and tedious.
- ▶ The exemplar-based method often provides good result, but small defects are very common.

Use of the non-convexity.



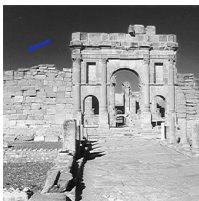
Gradient descent algorithm:

$$x_{n+1} \leftarrow x_n - \gamma \nabla f(x_n).$$

The result depends of the initialization.

Choice of the initialization.

W is initialized as the inverse of the geodesic distance for the candidate corresponding to the scribble.



Initial scribble

Geodesic distance.

Diffusion.

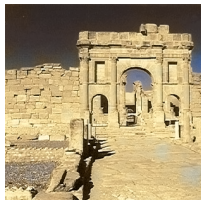
Example of diffusion of color with geodesic distance.

Choice of the initialization.

W does the initialization of u .



Source.



Exemplar-based.



With one scribble.



3 scribbles.



Final result.

Choice of the initialization.



Source.

Target.

With exam-
ple.



Scribbles.

Scribbles.

Scribbles.

Scribbles.



Results.



Target

Source

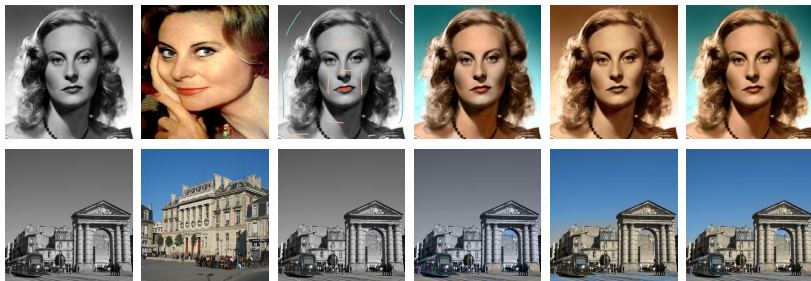
Scribbles

Manual

Exemplar

Both

Results.



Target

Source

Scribbles

Manual

Exemplar

Both

Victoire !



Plan.

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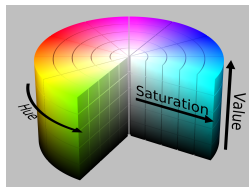
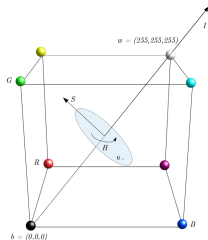
Primal-dual like algorithm.

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Conclusion.

Cylindrical coordinates for a perceptual representation of colors.



- ▶ Easy description of colors;
- ▶ Coordinates linked to human perception;
- ▶ *RGB* cube strictly included in cylinder (gamut).

RGB model.

A new energy-based model:

$$F(u, W) := TV(u) + \frac{\lambda}{2} \int_{\Omega} \sum_{i=1}^8 w_i \|u - c_i\|^2$$

$$+ \chi_{u \in [0,255]^3} + \chi_{u \text{ s.t. } Y(u)=g} + \chi_{W \in \Delta}.$$

u : RGB vector, to be computed.

$$\Delta = \left\{ w \in \mathbb{R}^8 \text{ s.t. } 0 \leq w_i \leq 1, \forall i \in [1..8] \text{ and } \sum_{i=1}^8 w_i = 1 \right\}.$$

c_i are RGB candidates.

$$TV(u) = \int_{\Omega} \sqrt{\partial_x R^2 + \partial_y R^2 + \partial_x G^2 + \partial_y G^2 + \partial_x B^2 + \partial_y B^2}$$

Algorithm

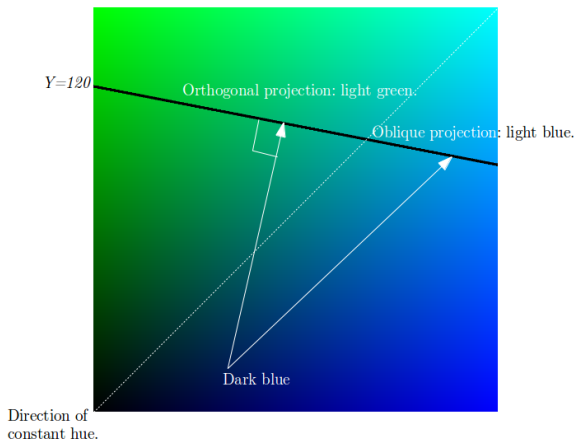
Algorithm 4 Applied to colorization.

1: $Z^0 \leftarrow 0$
2: **for** $n \geq 1$ **do**
3: $Z^{n+1} \leftarrow PB(Z^n + \sigma \nabla \bar{u}^n)$
4: $W^{n+1} \leftarrow P_{\Delta} \left(W^n - \tau_w \left(\left(\|\bar{u}^n - c_i\|^2 \right)_i \right) \right)$
5: $u^{n+1} \leftarrow \tilde{P}G \left(\frac{u^n + \tau (\operatorname{div}(Z^{n+1}) + \lambda \sum_i w_i^{n+1} c_i)}{1 - \delta \lambda} \right)$
6: $\bar{u}^{n+1} \leftarrow 2u^{n+1} - u^n$
7: **end for**

- ▶ P_{Δ} projection onto the simplex (Chen et al. 2011.);
- ▶ $\tilde{P}G$ is a projection onto the intersection

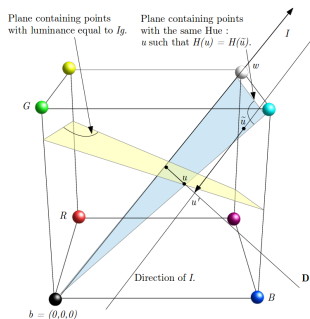
$$[0, 255]^2 + \{u \text{ s.t. } Y(u) = g\}. \quad (2)$$

Oblique projection



- ▶ The orthogonal projection is not able to preserve the hue;
- ▶ An oblique projection, in the direction of constant hue is used.

Oblique projection



The oblique projection is computed in two steps:

- ▶ Projection onto D in the direction of \vec{wb} .
- ▶ Projection onto the yellow plan, in the direction of D .

Oblique projection vs orthogonal projection.



Source image.



Target image.

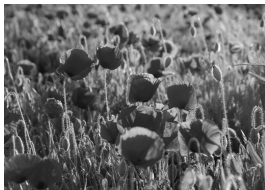
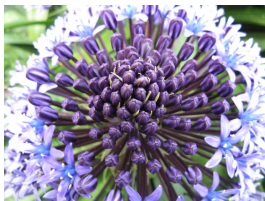


Colorization with orthogonal projection.



Colorization with respect of the hue.

Numerical results



Source

Target

Result

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Conclusion.

- ▶ Variational model efficient for colorization;
- ▶ New concept mixing both manual colorization and exemplar-based;
- ▶ Convergence of an algorithm for a non-convex model.

Future works

Study of interest:

- ▶ Adaptation to video;
- ▶ Colorization with Y channel damaged;
- ▶ Acceleration with discrete algorithms.

Thanks...

For listening !