Audio inpainting by sparse regularization methods

Journées bordelaises d'analyse mathématique des images

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April 8, 2014

OUTLINE

General introduction

The audio declipping inverse problem

Sparse and cosparse regularization

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1
AUDIO INPAINTING
General introduction

Inpainting problems in audio signal processing

Recovery of audio signals corrupted by:

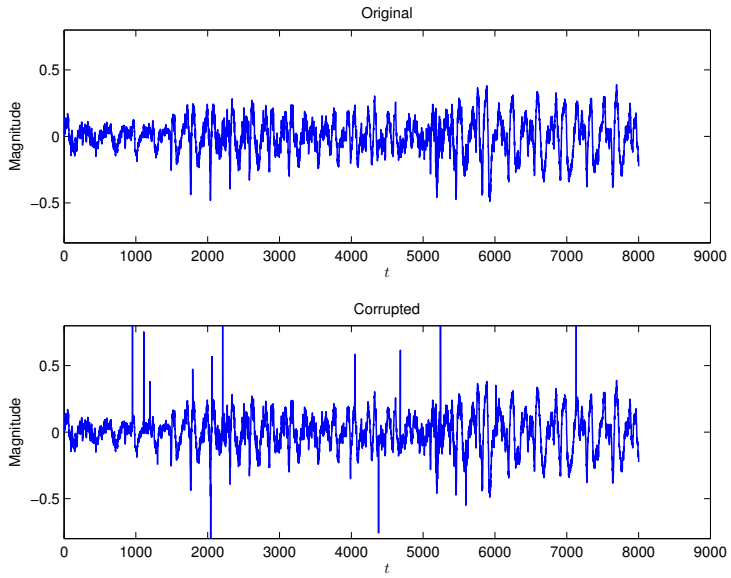
- Impulsive noise / clicks,
- Clipping / magnitude saturation,
- Packet loss,
- CD/DVD scratches,
- Source separation and more.

Different approaches, depending on the context:

- AR modeling [JVV86],
- Bayesian estimation [GR95, MG14],
- Neural networks [Unc03, Czy97],
- Bandwidth replication [LA05],
- Sparse recovery [PBD$^+$10, MG14].

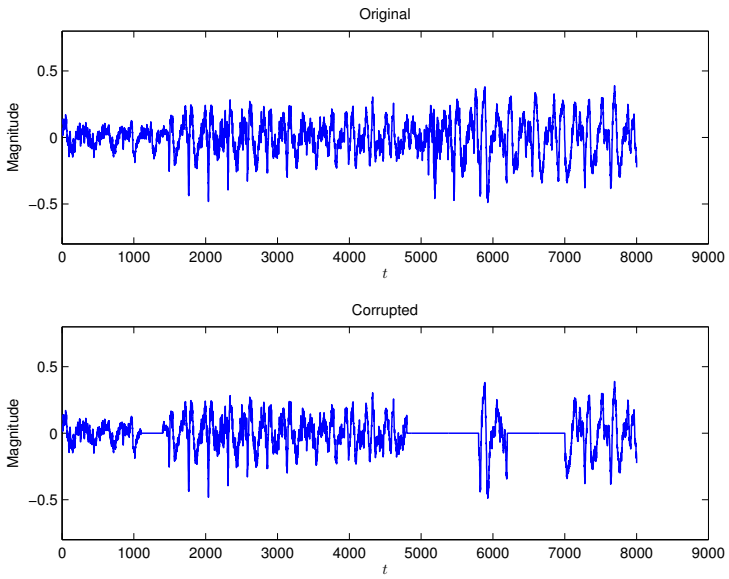
Impulsive noise

- A localized, impulsive degradation, at random position in the signal.
- Duration of the degradation is between 20μs and 4ms.
- Many interpolation approaches, such as median filtering, ”splicing“ etc.
- The most effective is a model-based approach based on AR-process.



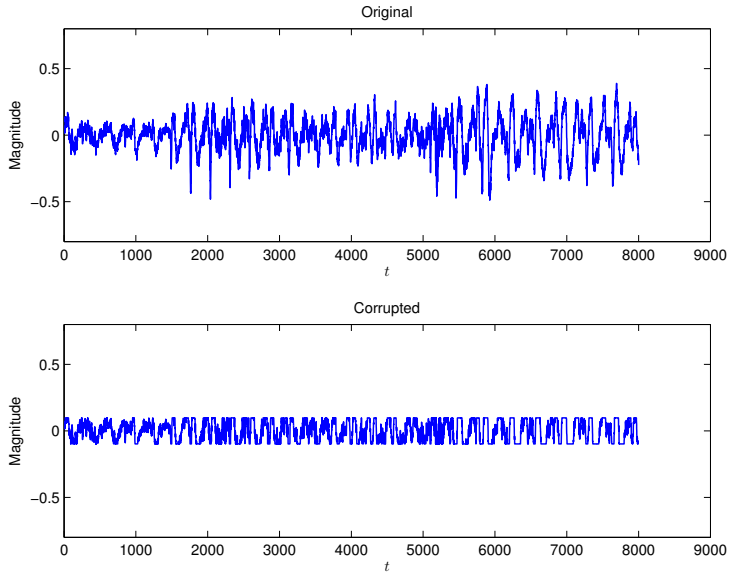
Packet loss

- Very difficult scenario - entire blocks of data are completely lost.
- Duration of the "gap" depends on the packet size and may be over 100ms.
- Packet Loss Concealment (PLC) techniques [WSL00] based on insertion, waveform substitution and model-based methods.
- Typically, speech signals can be recovered if the gap is smaller than a phoneme duration (less than 40ms).



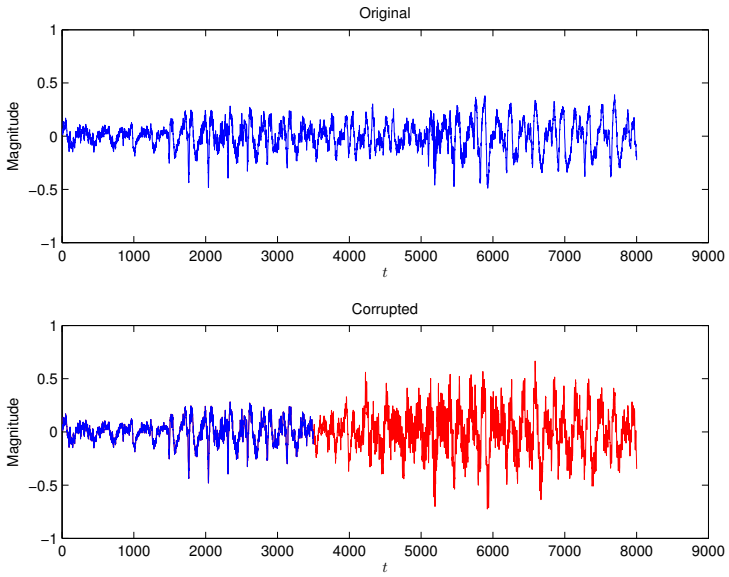
Clipping

- Another difficult scenario - magnitude information above the threshold is completely lost.
- Duration of the ”gap“ depends on the threshold.
- Declipping techniques based on interpolation and signal models.
- Recovery performance depends on the clipping threshold and the audio content.



Source separation

- Specific case where one source is desired and the rest are considered as noise.
- Duration of the ”gap“ depends on the period during which only the desired source is active.
- Standard separation methods based on ICA [CJ10], for example.
- In the case of multichannel audio, pattern matching techniques [SLOVB14] may be effective.



2
AUDIO DECLIPPING
The inverse problem

Mathematical formulation of audio clipping

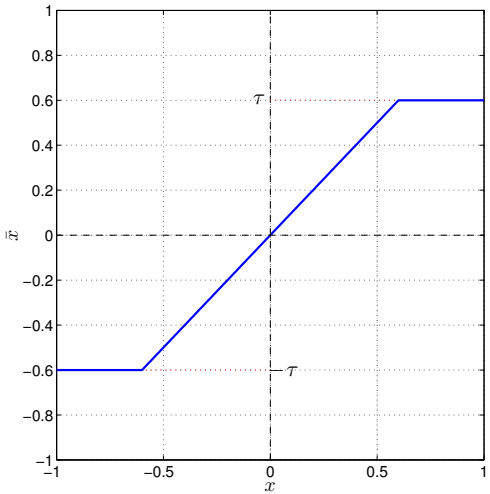
Let $x \in \mathbb{R}^n$ be the single channel, discrete time audio signal and $\mathcal{C}_\tau(x) = \bar{x} \in \mathbb{R}^n$ its clipped version.

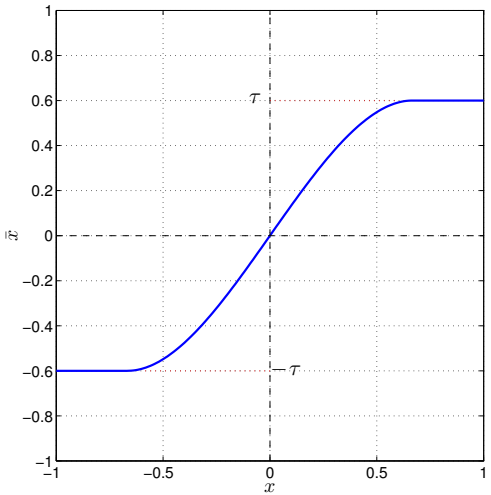
Hard clipping:

$$\bar{x}_i = \begin{cases} x_i & \text{if } |x_i| < \tau, \\ \operatorname{sgn}(x_i)\tau & \text{otherwise.} \end{cases}$$

Soft clipping (by cubic nonlinearity):

$$\bar{x}_i = \begin{cases} \left(\frac{9x_i}{4} - \frac{27x_i^3}{16}\right)\tau & \text{if } |x_i| < \frac{2}{3}\tau, \\ \operatorname{sgn}(x_i)\tau & \text{otherwise.} \end{cases}$$

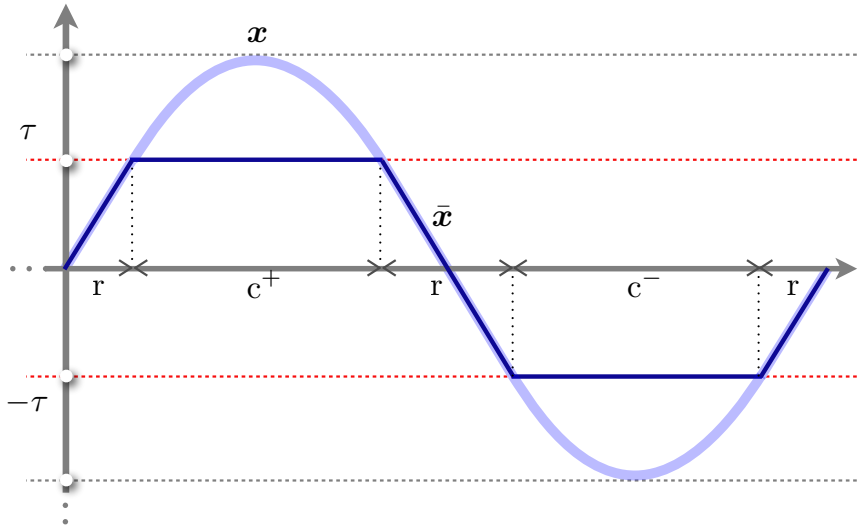




Declipping *hard*-clipped signal

- $\bar{x} = \mathcal{C}_\tau(x)$, a hard-clipped signal.
- $\mathsf{M}_\mathsf{r}, \mathsf{M}_\mathsf{c}^+, \mathsf{M}_\mathsf{c}^-$ extract "reliable", clipped-positive and clipped-negative samples.
- The goal is to find an estimate $\hat{x}$ such that:

$$\mathsf{M}_\mathsf{r}\,\hat{x} = \mathsf{M}_\mathsf{r}\,\bar{x}$$
$$\mathsf{M}_\mathsf{c}^+\,\hat{x} \geq \mathsf{M}_\mathsf{c}^+\,\bar{x}$$
$$\mathsf{M}_\mathsf{c}^-\,\hat{x} \leq \mathsf{M}_\mathsf{c}^-\,\bar{x}$$



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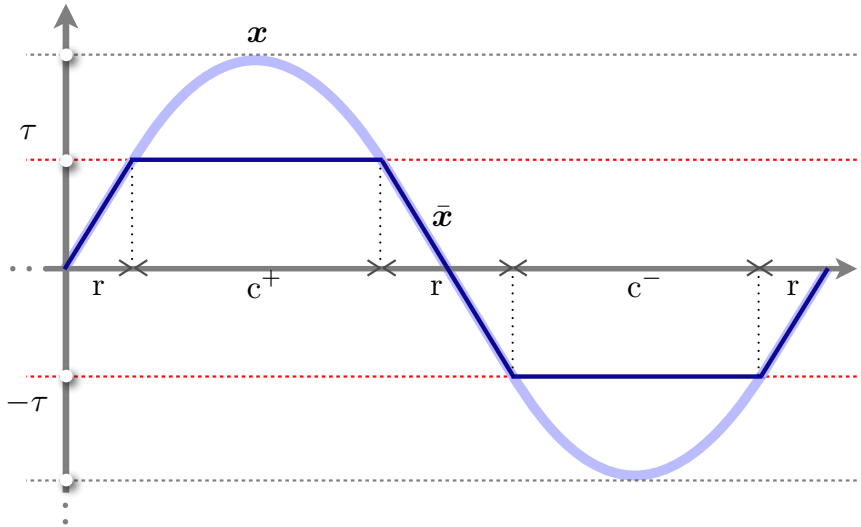
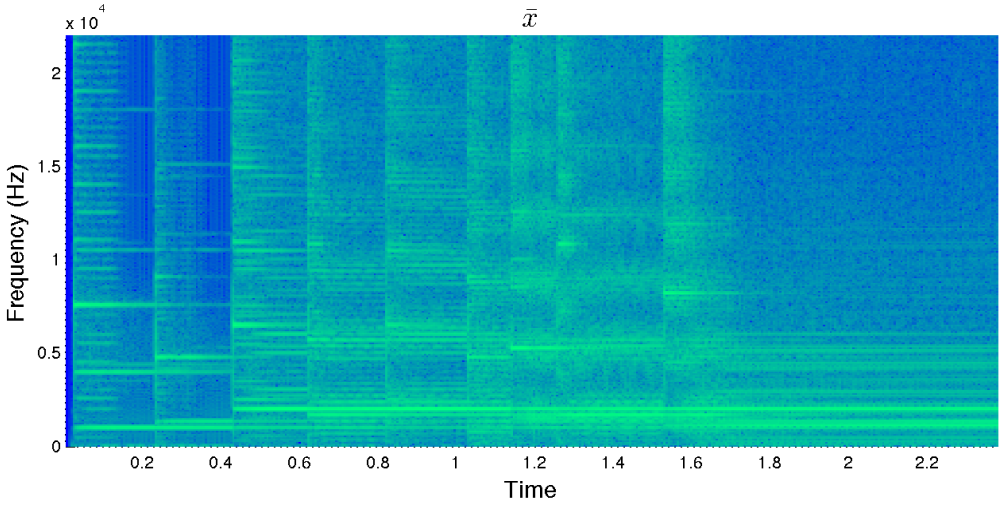
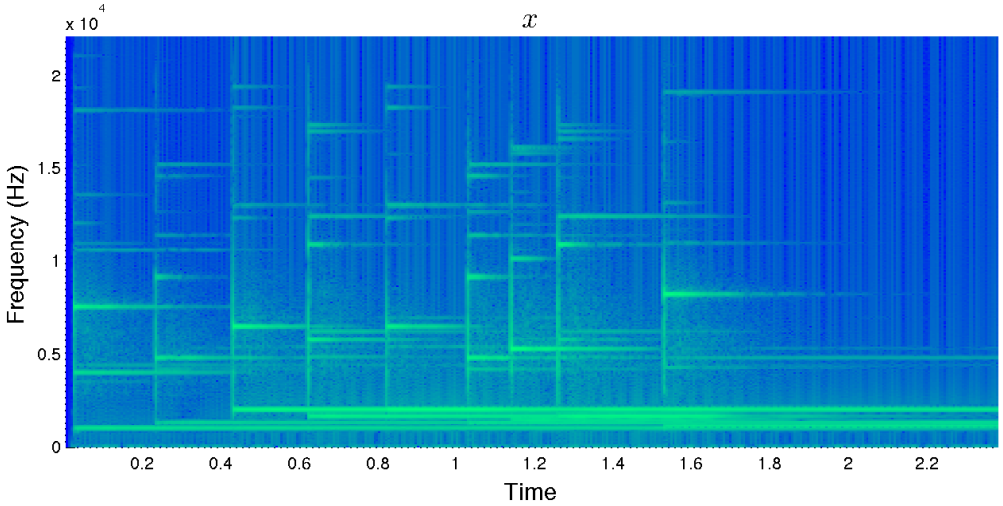


An ill-posed problem!

Time-frequency visualization

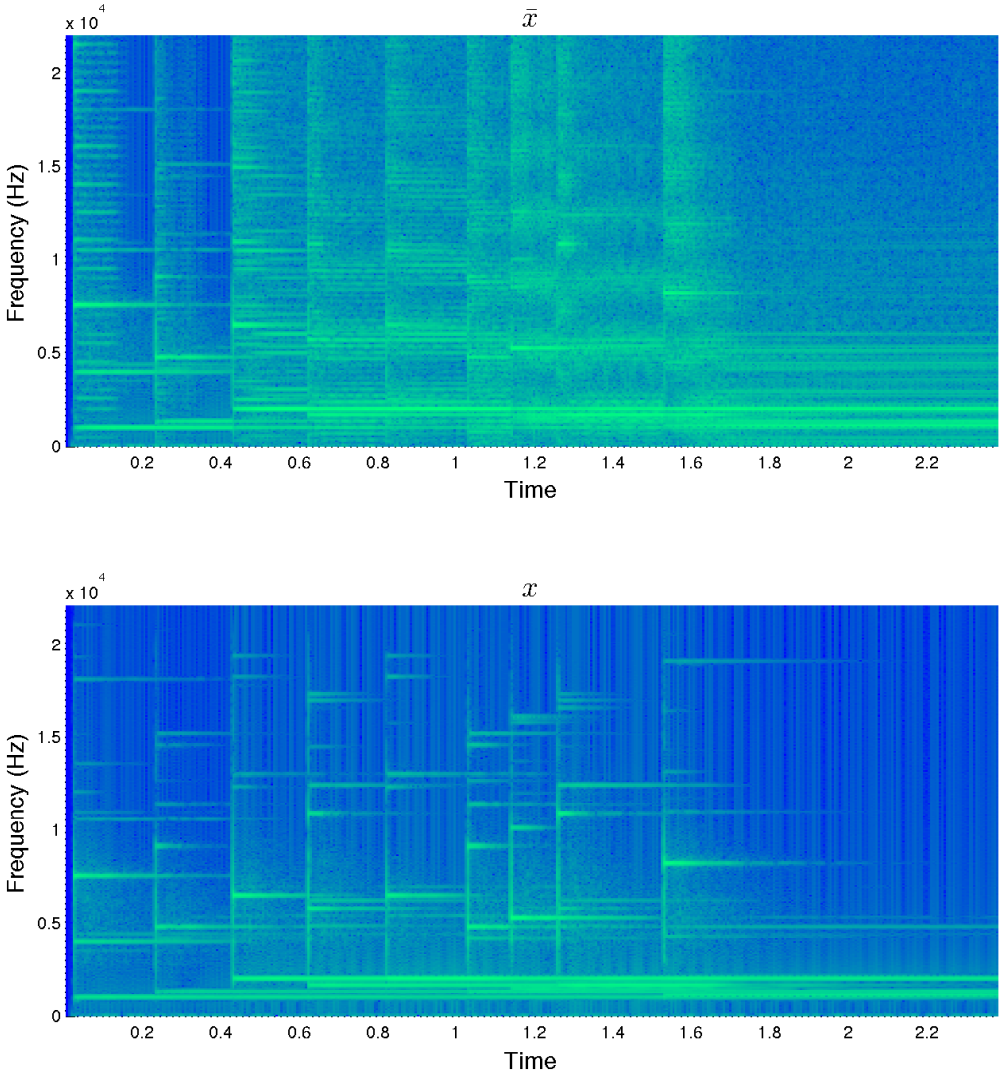


Clipped: spectrum spreading due to introduced discontinuities in the waveform.



Original: in the time-frequency plane, the energy of audio signals is *mostly* concentrated!

Time-frequency visualization



The idea: regularize the ill-posed declipping problem by enforcing the energy compactness in an estimate.

3
SPARSE REGULARIZATION

Synthesis *vs* analysis approach

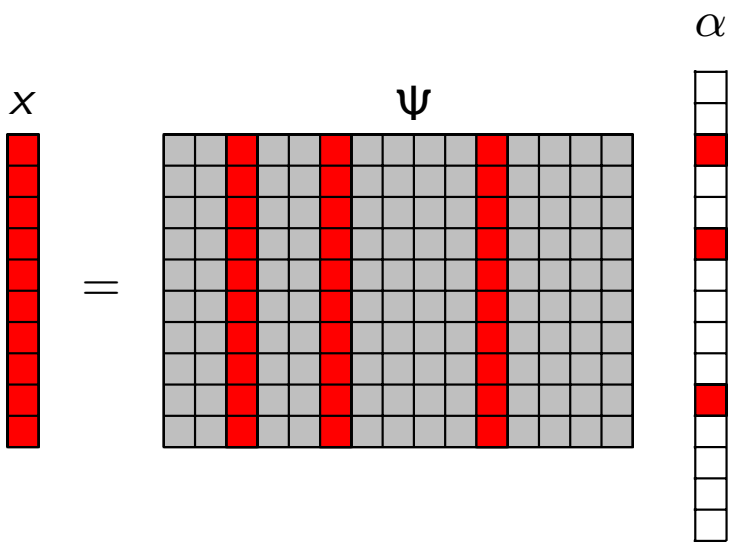
Sparse synthesis framework

The assumption: signal $x \in \mathbb{R}^n$ can be approximated by a linear combination of *atoms* taken from a *dictionary* $\Psi \in \mathbb{R}^{n \times m}$, $n \leq m$:

$$x = \Psi\alpha$$

The number of atoms (eq. non-zero weights in $\alpha \in \mathbb{R}^m$) k needed for the approximation is relatively small compared to N:

$$\#\{\alpha\} = \|\alpha\|_0 = k \ll n, m$$



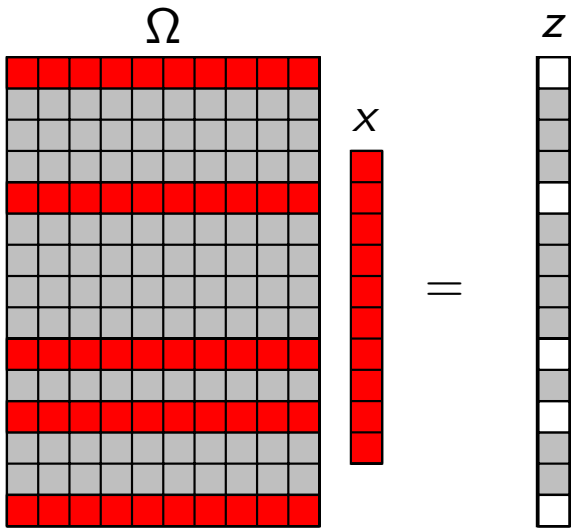
Sparse analysis framework

The assumption [NDEG13]: signal $x \in \mathbb{R}^n$ can be sparsified by applying a suitable *analysis operator* $\Omega \in \mathbb{R}^{p \times n}$, $p \geq n$:

$$z = \Omega x$$

The number of zero-elements ℓ in the product $z \in \mathbb{R}^p$ is relatively large:

$$p - \|\Omega x\|_0 = \ell$$



Comparison of the two methodologies

Sparse synthesis

- Dictionary: $\Psi \in \mathbb{R}^{n \times m}$.
- Constructive model, atomic composition.
- Support: column vectors of Ψ corresponding to non-zeros in α.
- Non-unique representation.
- Number of subspaces: $\binom{m}{k}$, dimension: k.

Sparse analysis

- Operator: $\Omega \in \mathbb{R}^{p \times n}$.
- Descriptive model, constrained decomposition.
- Cosupport: row vectors of Ω orthogonal to x.
- Unique representation.
- Number of subspaces: $\binom{p}{\ell}$, dimension $n - \ell$.

Nominal equivalence: only if $\Psi = \Omega^{-1}$.

Regularized declipping problem

Sparse synthesis

$$\begin{aligned} \text{minimize } & {}_{\alpha}\|\alpha\|_0 \\ \text{subject to } & \mathsf{M}_\mathsf{r}\,\Psi\alpha = \mathsf{M}_\mathsf{r}\,\bar{x} \\ & \mathsf{M}_\mathsf{c}^{+}\,\Psi\alpha \geq \mathsf{M}_\mathsf{c}^{+}\,\bar{x} \\ & \mathsf{M}_\mathsf{c}^{-}\,\Psi\alpha \leq \mathsf{M}_\mathsf{c}^{-}\,\bar{x} \end{aligned}$$

Sparse analysis

$$\begin{aligned} \text{minimize } & {}_{x}\|\Omega x\|_0 \\ \text{subject to } & \mathsf{M}_\mathsf{r}\,x = \mathsf{M}_\mathsf{r}\,\bar{x} \\ & \mathsf{M}_\mathsf{c}^{+}\,x \geq \mathsf{M}_\mathsf{c}^{+}\,\bar{x} \\ & \mathsf{M}_\mathsf{c}^{-}\,x \leq \mathsf{M}_\mathsf{c}^{-}\,\bar{x} \end{aligned}$$

Ω and Ψ are some *overcomplete* transform matrices known for compacting the energy of audio signals.

Choice of the dictionary and the operator

- Modulated complex lapped transform (MCLT) [DD06]:

$$\Psi = \begin{bmatrix} \psi_0^c & \psi_1^c & \dots & \psi_{m/2-1}^c & \psi_0^s & \psi_1^s & \dots & \psi_{m/2-1}^s \end{bmatrix},$$

$$\psi_j^c(t) = \cos\left(\frac{\pi}{m}(t+1/2)(j+1/2)\right), \psi_j^s(t) = \sin\left(\frac{\pi}{m}(t+1/2)(j+1/2)\right),$$

  where $t = [0, n-1]$ and $j = [0, m-1]$.

- Two-times redundant ($m = 2n$) DCT-DST dictionary.
- The atoms are chosen according to recommendations in [Gri01] to use the transform as Gabor-like dictionary.
- The analysis operator is the transpose $\Omega = \Psi^T$, ($p = m$), for consistency.

Computational perspective

Minimizing either $\|\alpha\|_0$ or $\|\Omega x\|_0$ is NP-hard!

Sparse synthesis

- Convex relaxation: minimize $\|\alpha\|_1$ or some other convex objective, if applicable.
- Greedy: MP, OMP, IHT, HTP etc.

Sparse analysis

- Convex relaxation: minimize $\|\Omega x\|_1$ or some other convex objective, if applicable.
- Greedy: GAP, analysis IHT/HTP etc.

Important: model is rarely a perfect reflection of reality
(assume "$\approx$" rather than "$=$")!

4
DECLIPPING ALGORITHMS
based on sparse and cosparse prior

Constrained Matching Pursuit for Audio Declipping

- Two-stage algorithm [AEJ+12]:
  1. Orthogonal Matching Pursuit for CS:
     - Initialize the support $\Lambda = \{\emptyset\}$ and residual $r^{(0)} = \mathrm{M_r}\,\bar{x}$.
     - Select atom: $j = \arg\max_j \langle r^{k-1}, \psi_j \rangle$,
     - Update support: $\Lambda \leftarrow \Lambda \cup j$; $\Psi_\Lambda = \left[\psi_i\right], \{\psi_i \in \Psi \mid i \in \Lambda\}$,
     - Compute the estimate: $\alpha^{(k)} = \arg\min_\alpha \| \mathrm{M_r}\,\bar{x} - \mathrm{M_r}\,\Psi_\Lambda \alpha \|_2^2$,
     - Compute new residual: $r^{(k)} = \mathrm{M_r}\,\bar{x} - \mathrm{M_r}\,\Psi_\Lambda \alpha^{(k)}$,
     - Termination criterion: $\|r^{(k)}\|_2 \leq \epsilon$.
  2. Refinement by clipping constraints:

$$\begin{aligned} \hat{\alpha} = \arg\min_\alpha & \| \mathrm{M_r}\,\bar{x} - \mathrm{M_r}\,\Psi_\Lambda \alpha \|_2^2 \\ \text{subject to } & \mathrm{M_c^+}\,\Psi_\Lambda \alpha \geq \mathrm{M_c^+}\,\bar{x} \\ & \mathrm{M_c^-}\,\Psi_\Lambda \alpha \leq \mathrm{M_c^-}\,\bar{x} \end{aligned}$$

- Final estimate: $\hat{x} = \Psi_\Lambda \hat{\alpha}$.

Consistent Iterative Hard Thresholding

- Algorithm [KJM+13] based on IHT by Blumensath et al. Objective:

  $\min_\alpha \| \mathrm{M_r}\,\Psi\alpha - \mathrm{M_r}\,\bar{x}\|_2^2 + \| \left(\mathrm{M_c^+}\,\bar{x} - \mathrm{M_c^+}\,\Psi\alpha\right)_+ \|_2^2 + \| \left(\mathrm{M_c^-}\,\bar{x} - \mathrm{M_c^-}\,\Psi\alpha\right)_- \|_2^2$

  subject to α being sparse and $(u_i)_\pm = \pm\max(0, \pm u_i)$.
- Define:

$$\mathcal{B}(u_i) = \begin{cases} u_i & \forall i \in S_r, \\ (u_i)_+ & \forall i \in S_p, \\ (u_i)_- & \forall i \in S_n. \end{cases}$$

- Iterative update: $\alpha^{(k+1)} = \mathcal{H}_{k+1}\left(\alpha^{(k)} + \mu\Psi^T\mathcal{B}(\bar{x} - \Psi\alpha^{(k)})\right)$.
- $\mathcal{H}_K(\cdot)$ is the hard-thresholding operator.
- $K \leftarrow k+1$: sparsity relaxed per iteration.
- Stepsize μ estimated through line-search.
- Termination criterion: $\|r^{(k)}\|_2 = \|\mathcal{B}(\bar{x} - \Psi\alpha^{(k)})\|_2 \leq \epsilon$.

Analysis Hard Thresholding for Audio Declipping

- Ideas from Consistent IHT cannot be readily applied, since:

  $$\text{minimize}_x \|y - x\|_2^2 \text{ subject to } \|\Omega x\|_0 \leq k \text{ is NP-hard [TGP14]!}$$

- Instead, we enforce *approximate* cosparsity through ADMM approach [KBG14].
- Reformulate the problem by splitting variables ($z \in \mathbb{R}^p$):

$$\begin{aligned} \text{minimize}_{\ z,x} & \|\Omega x - z\|_2^2 \\ \text{subject to } & \|z\|_0 \ll p, \\ & \mathrm{M_r}\, x = M_r \bar{x} \\ & \mathrm{M_c^+}\, x \geq \mathrm{M_c^+}\, \bar{x} \\ & \mathrm{M_c^-}\, x \leq \mathrm{M_c^-}\, \bar{x}. \end{aligned}$$

Analysis Hard Thresholding for Audio Declipping

1. Initialize: $x^{(0)} = \bar{x}, u^{(0)} = 0$.
2. $z^{(k)} = \arg\min_z \|z - \Omega x^{(k-1)} - u^{(k-1)}\|_2^2$ s. t. $\|z\|_0 \leq k = \mathcal{H}_k\left(\Omega x^{(k-1)} + u^{(k-1)}\right)$.
3. x-update:

$$x^{(k)} = \arg\min_x \|\Omega x - z^{(k)} + u^{(k-1)}\|_2^2$$
$$\text{subject to } \mathrm{M_r}\, x = \mathrm{M_r}\, \bar{x}$$
$$\mathrm{M_c^+}\, x \geq \mathrm{M_c^+}\, \bar{x}$$
$$\mathrm{M_c^-}\, x \leq \mathrm{M_c^-}\, \bar{x}$$

4. Lagrangian variable update: $u^{(k)} = u^{(k-1)} + \Omega x^{(k)} - z^{(k)}$.
5. Termination criterion: $\|r^{(k)}\|_\infty = \|\Omega x^{(k)} - z^{(k)} + u^{(k)}\|_\infty \leq \epsilon$.

Analysis Hard Thresholding for Audio Declipping

- Computing the exact x-update is expensive!
- Instead we first solve for the equality constraints only:

$$\begin{aligned}\text{Let: } \hat{x}^{(k)} &= (\mathrm{I} - \mathrm{M_r}^{\dagger}\,\mathrm{M_r})x_{\text{null}} + \mathrm{M_r}^{\dagger}\,\mathrm{M_r}\,\bar{x} = \Pi x_{\text{null}} + \mathrm{M_r}^{\dagger}\,\mathrm{M_r}\,\bar{x}.\\ \text{Solve: } x_{\text{null}} &= \arg\min_x \|\Omega(\Pi x + \mathrm{M_r}^{\dagger}\,\mathrm{M_r}\,\bar{x}) - z^{(k)} + u^{(k-1)}\|_2^2.\\ &= \arg\min_x \|\tilde{\Omega}_{\bar{r}} x - q\|_2^2.\end{aligned}$$

- Then we project the solution to clipping (box) constraints:

$$x^{(k)} = \hat{x}^{(k)} + \mathcal{B}(\bar{x} - \hat{x}^{(k)})$$

.

- Suboptimal, but sufficient for the convergence.

Linear prediction declipping

- Adaptation of the interpolation method proposed by Janssen [JVV86].
- The signals are modeled as autoregressive (AR) processes of finite order $r = 3c + 2$, where c is number of clipped samples.
- The objective is "whitening" the signal ($a \in \mathbb{R}^r$ are the filter coefficients, $a_0 = -1$):

$$Q(a, x) = \sum_{i=r}^{n-1} \left(\sum_{j=0}^{r} a_j x_{i-j} \right)^2 = \sum_{i=r}^{n-1} e_i^2$$

- Vectors a and x are estimated by alternating minimization of $Q(a, x)$ and projecting the estimate x to clipping constraints.
- Potential downfalls: sensitive to initialization, computational time proportional to filter order.

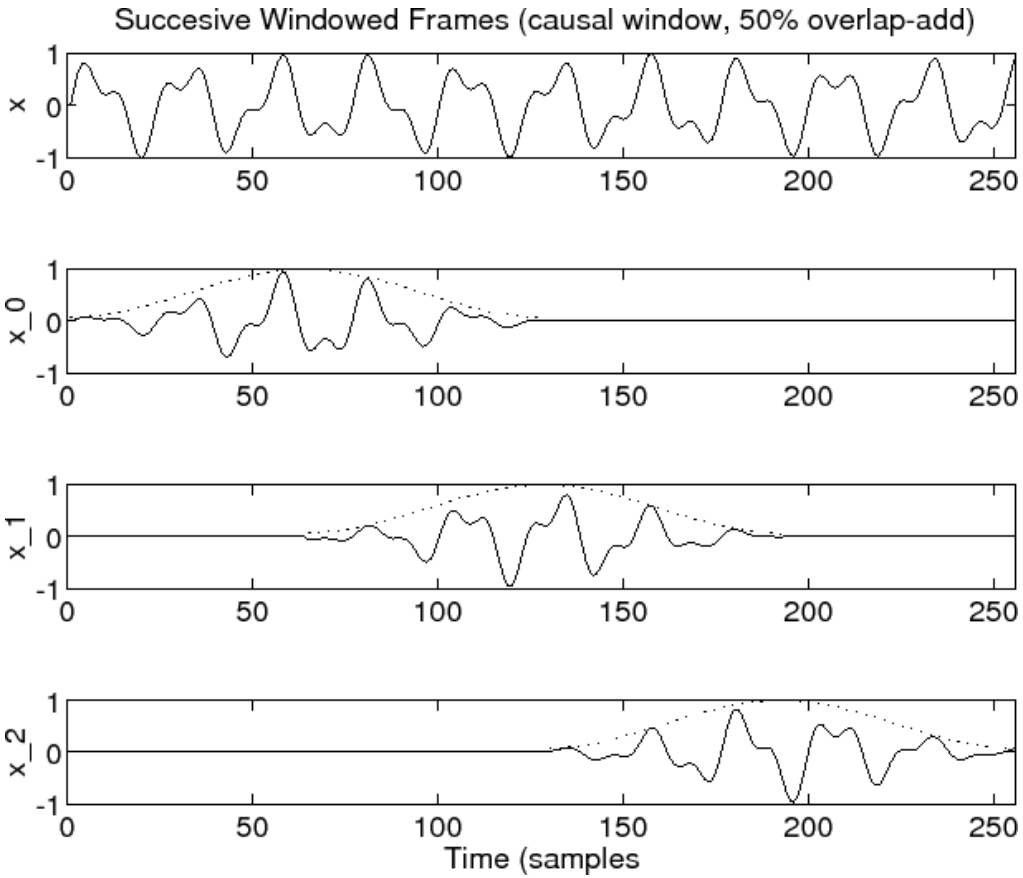
Conceptual analysis

- Constrained MP: support is chosen without clipped observations in $\bar{x}$!
- Constrained MP and Consistent IHT: sensitive trade-off between good fit and overfitting.
- Analysis HT and Constrained MP: potentially slow due to intermediate constrained minimization steps.
- AR declipping will be slow for severely clipped signals.
- Efficant computation of $\Psi(\cdot)$, $\Psi^T(\cdot)$, $\Omega(\cdot)$ and $\Omega^T(\cdot)$ is highly recommended!
- All algorithms are non-convex heuristics and only locally optimal.

5
AUDIO BENCHMARKS
Declipping wideband audio data

Frame-based processing

- Constant Overlap-Add (COLA) scheme.
- Overlap stepsize: 75%.
- Weighting function: Hamming (square rooted for the analysis and synthesis window).
- Frame duration: ∼20ms.



Benchmark

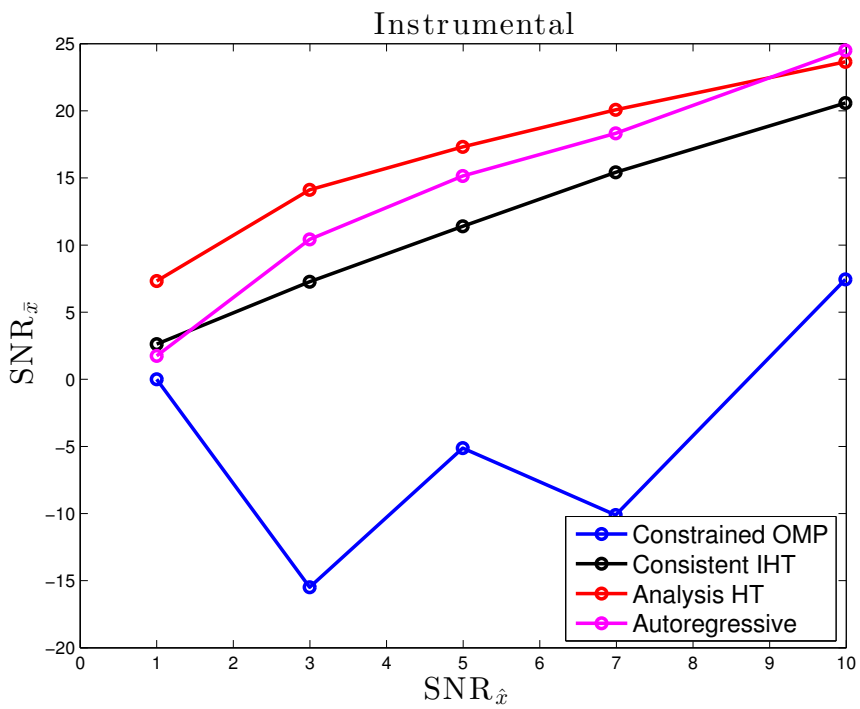
- Two wideband audio tracks (sampling: 16kHz, encoding: 16bit).
- DCT and Gabor for the dictionary/operator.
- Performance criterion: SNR difference between the input and post-processed data:

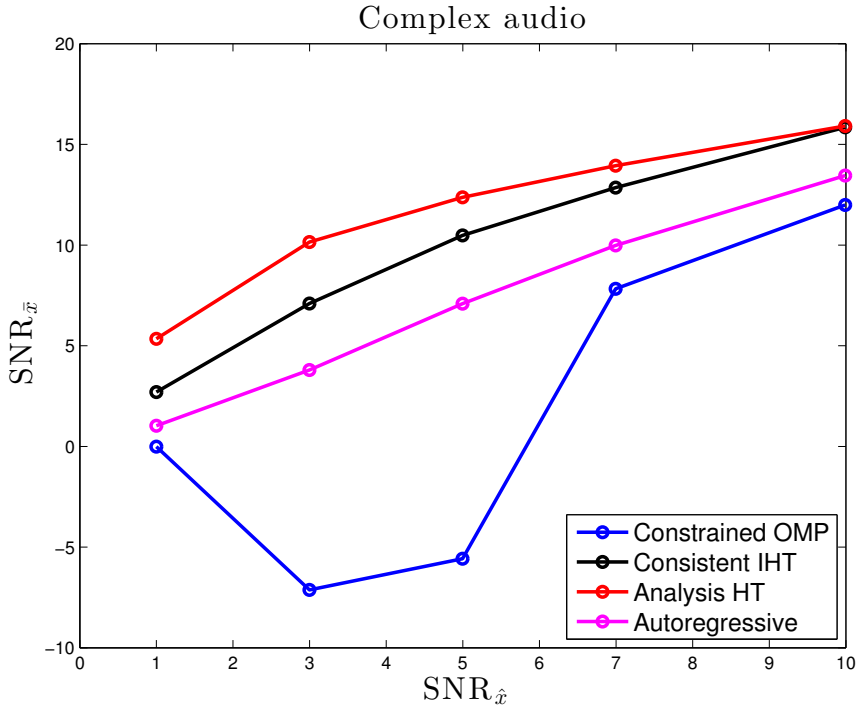
$$\mathrm{SNR}_{\bar{x}} = 20 \log_{10} \frac{\|x\|_2}{\|x - \bar{x}\|_2}$$

$$\mathrm{SNR}_{\hat{x}} = 20 \log_{10} \frac{\|x\|_2}{\|x - \hat{x}\|_2}$$

- Clipping range: from $\mathrm{SNR}_{\bar{x}} = 1\mathrm{dB}$ to $\mathrm{SNR}_{\bar{x}} = 10\mathrm{dB}$.

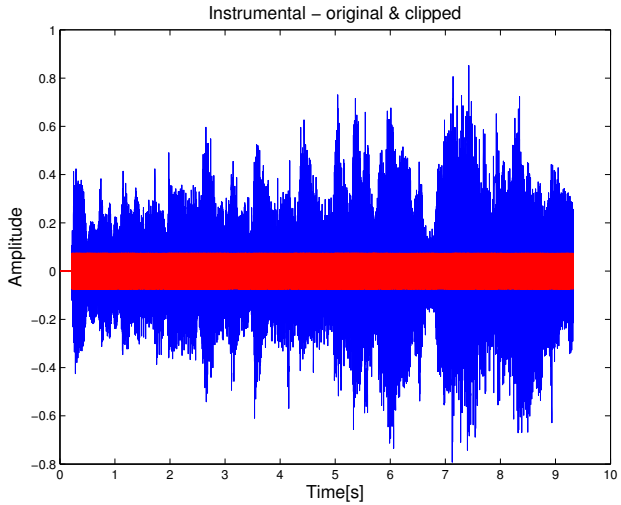
Results - recovery performance

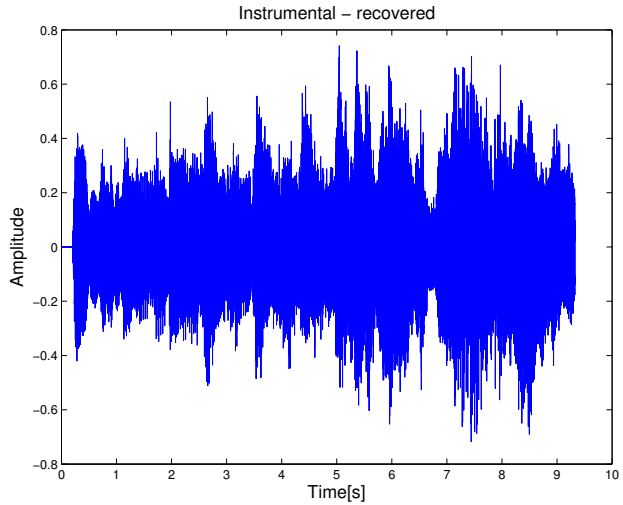


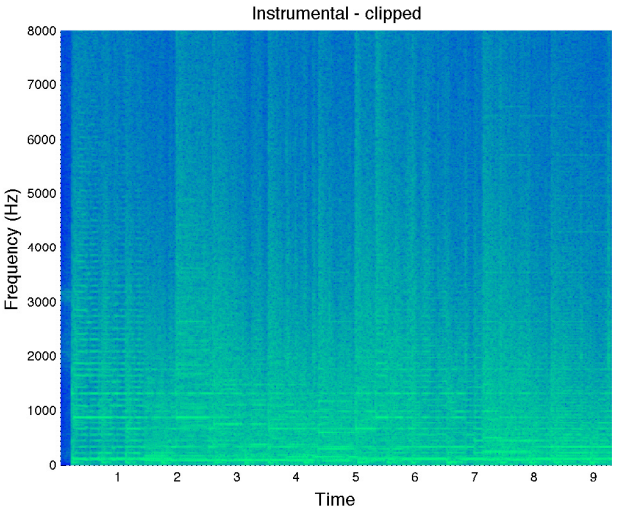


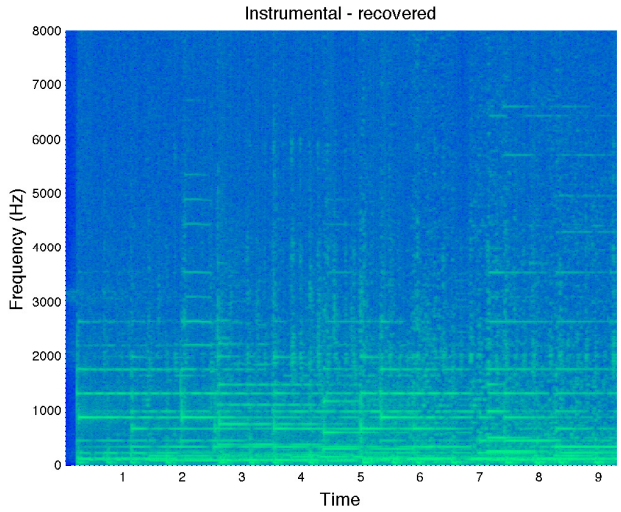
Output *vs* input SNR for the benchmarked algorithms.

Results - audio preview

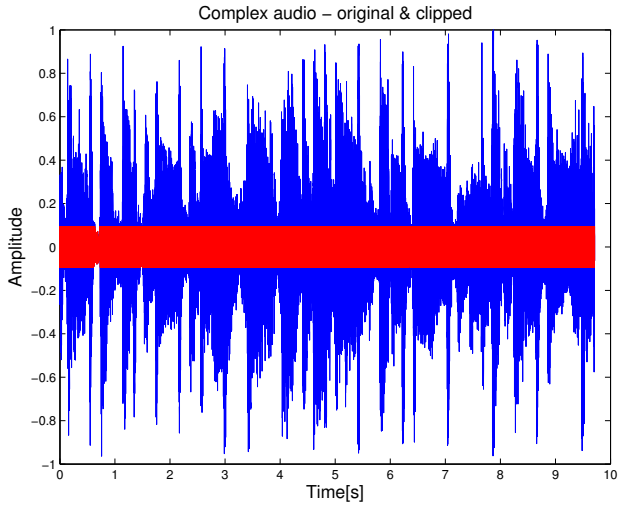


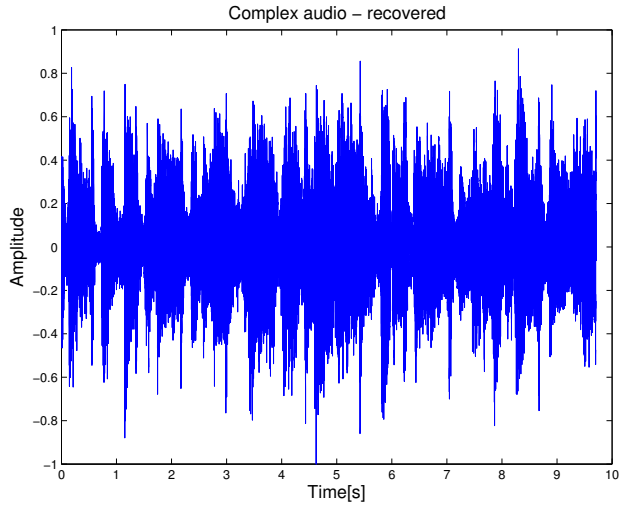


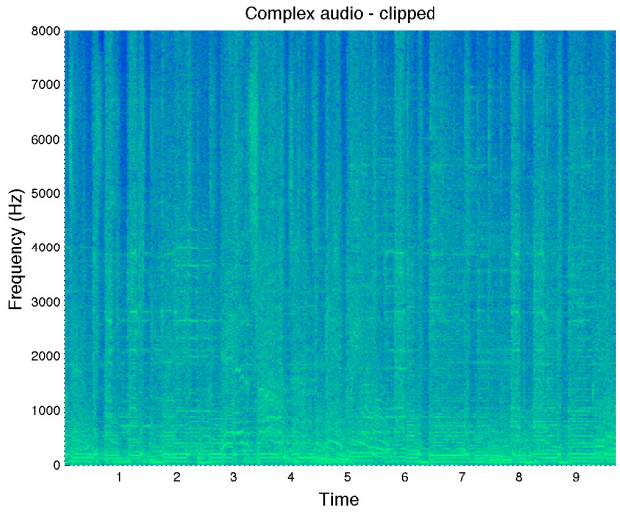


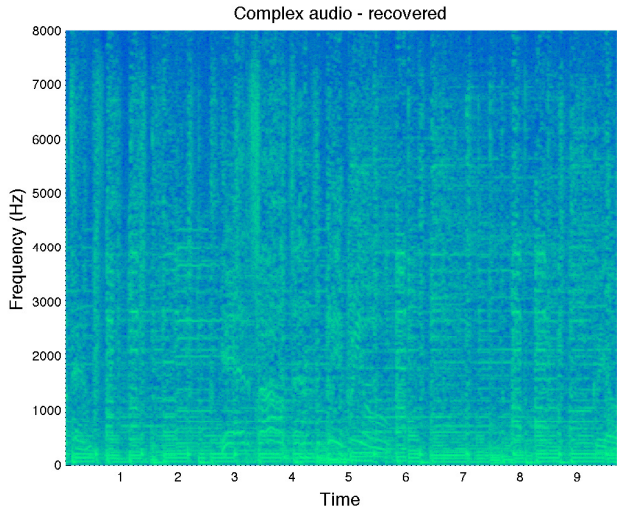


Results - audio preview





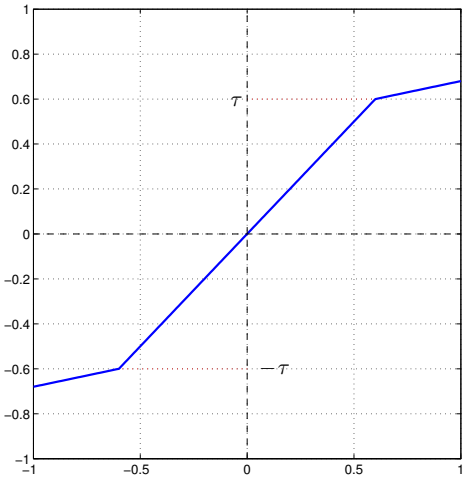




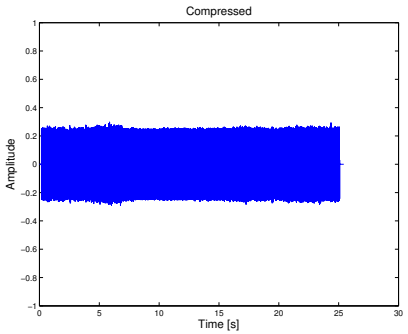
Blind decompression

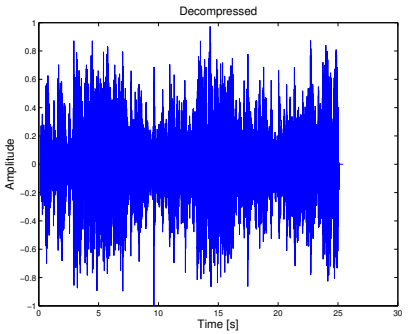
- More realistic scenario [MGS03]: data is not "perfectly" clipped; instead, high magnitudes are gradually compressed.
- The threshold τ and compression coefficient γ are unknown.

$$\bar{x}_i = \begin{cases} x_i & \text{if } |x_i| < \tau, \\ \operatorname{sgn}(x_i)\tau(1-\gamma) + \gamma x_i & \text{otherwise.} \end{cases}$$



- Arbitrary declipping is not entirely satisfactory:



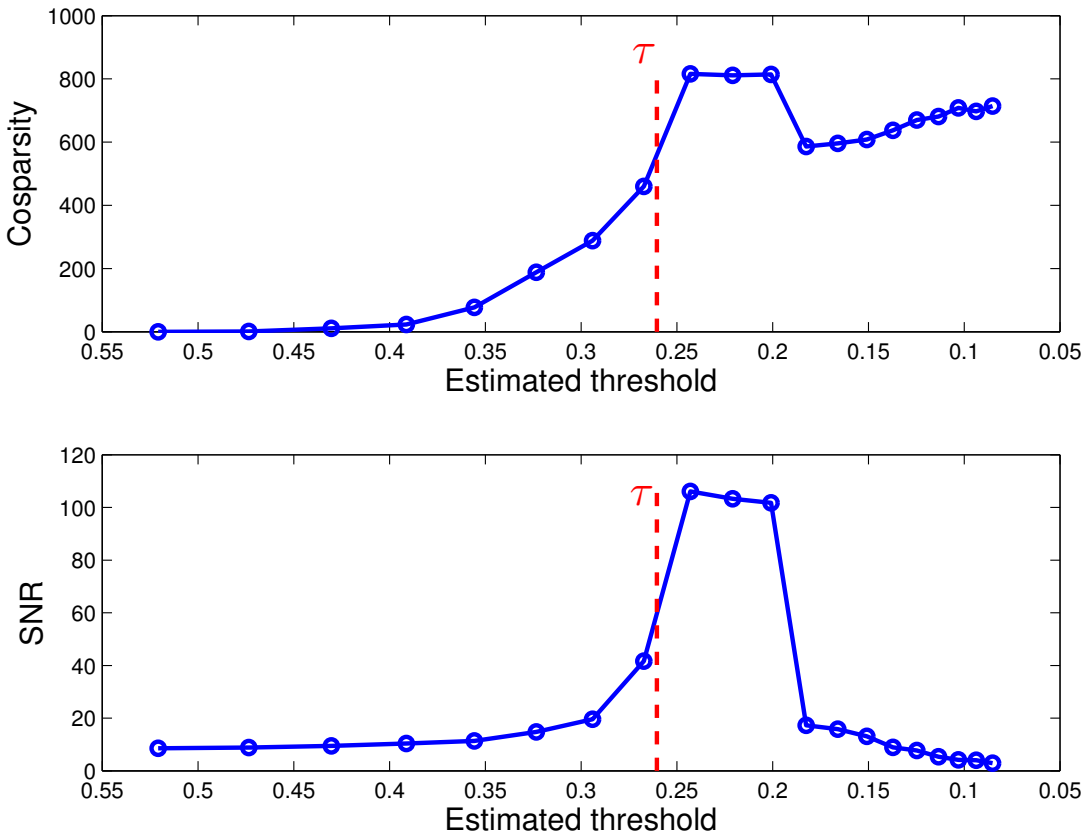


Blind decompression

More systematic approach:

1. Initialization: $\hat{\tau} = \|\bar{x}\|_\infty$.
2. Backtracking: $\hat{\tau} \leftarrow \frac{1}{b}\hat{\tau}, b > 1$.
3. Define the measurement matrices $\mathsf{M}_\mathsf{r}(\hat{\tau}), \mathsf{M}_\mathsf{c}^+(\hat{\tau}), \mathsf{M}_\mathsf{c}^-(\hat{\tau})$, assuming that samples $|\bar{x}_i| < \hat{\tau}$ are reliable.
4. Compute the estimate $\hat{x}^{(k)}$ by analysis HT declipping.
5. Evaluate approximate cosparsity of the estimate: $\hat{\ell}^{(k)} = p - \|\hat{z}^{(k)}\|_0$.
6. Stopping criterion based on the cosparsity decrease: $\hat{\ell}^{(k+1)} \leq \hat{\ell}^{(k)}$.

Blind decompression - results



Cosparsity of the estimate and decompressed SNR *vs* estimated threshold for the ℓ-cosparse signal x.

- Exhibits a "phase transition" behavior.
- Crude scheme - works, but computationally expensive.
- Impact of τ and γ on the performance?
- Performance on the real audio data?

Conclusions

- Analysis Hard Thresholding outperforms all the others for most of the given clipping range.
- Consistent IHT and AR declipping offer good trade-off between processing time and quality of reconstruction.
- Constrained OMP fails to recover severely clipped signals due to inaccurate support estimation.
- Blind decompression / declipping seems possible.
- Envisioned improvements: enforcing structure in sparse and cosparse estimation (some existing approaches - check [SKD14]).
- Coupling AR model with sparse/cosparse regularization?

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