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Nested Atomic Sections with Thread Escape: A Formal Definition

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18/11/2013

Journées GDR LTP



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- Definitions
- Particular traces

3 Serialisability



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Concurrent programming

- Multi-core trend
- Gap between programming languages and hardware
- No suitable abstraction to deal with shared data

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Locks : the most common pattern

- Map operations to protect to a lock
- Responsibility to the user to prevent interference
- Error prone : deadlocks, need to understand the whole program
- Dilemma of the granularity : coarse/inefficient or fine/complex

Example

count := 0; Lock I;
<pre>procedure inc_count(){</pre>
lock(I)
{
count := count+1;
}
unlock(I)

```
f
procedure main()
{
    a := fork(inc_count);
    b := fork(inc_count);
    print(count);
```

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Atomic sections : an alternative?

- Responsibility to the **run-time system** or the **compiler** to prevent interference
- The user only delimits the region to protect

Example

```
count := 0;
procedure inc_count(){
  atomic
  {
    count := count+1;
  }
}
```

```
procedure main()
{
    a := fork(inc_count);
    b := fork(inc_count);
    print(count);
}
```

Atomic sections : implementations

- Transaction
 - Inspired from a database managements systems
 - Optimistic approach : assume no interference
 - Cancelled, roll-back and re-execute if interference happens
- Locks inference
 - Pessimistic approach : prevent interference
 - Set of locks inferred by the compiler

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Nested atomic sections and inner parallelism

- nested for modularity
- distinctive instructions of spawn
- threads live either completely inside or outside a section
- simple, but poor expressivity

Example

```
procedure m1(x)
```

atomic

procedure main(){
x := 1;
atomic{
 y := fork (m1,x);
}

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No such constraints, nested and escaping thread. Need to define a new notion of $\ensuremath{\textbf{atomicity}}.$

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Language

- Simple imperative language
- fork/join and atomic primitives
- Nested atomic sections
- Thread can escape surrounding section

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Traces			

- Abstraction from program semantics.
- Sequence of events.
- events ::= (thread,action)
- Assume disjoint countable set of memory locations, thread identifiers and section names.

Actions

а	::=	$\mid \tau$
		\mid alloc ℓ $n \mid$ free ℓ
		$ \operatorname{read} \ell n v \operatorname{write} \ell n v$
		fork $t $ join t
		\mid open $p \mid$ close p

Definitions

Well-formed traces

- Set of properties on traces.
- Abstraction of the operational semantics.

Conditions

- Each section name, thread identifier is unique.
- A fork of a thread is done before the action this thread.
- A join on a thread *t* is done after the action this thread and after its fork.
- The opening and the closing of a section is done by the same thread, and the close matches the last open.

Others conditions, need to have some additional definitions.

Definitions

tribe_s

- for a trace *s* and a section name, gives the thread identifiers which are allowed to interfere with each other while the section is opened.
- defined as the least set containing :
 - the thread owner of the section
 - the threads forked as a side effect of the execution of the section (tribe children)



FIGURE : $tribe_s(p_0) = \{t_0, t_1, t_2\}$

Definitions

subsection

 $p' \Subset_s p$

- sections opened by the thread owner of p while p is opened
- sections opened by tribe children of p

concurrent section

$$p \smile_s p' \triangleq p \not \equiv_s p' \land p' \not \equiv_s p$$



FIGURE : $p_1, p_2 \Subset_s p_0, p_0 \smile_s p_3$

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Definitions

- pending section : a section is pending if there is no closing
- projection : $\pi_s(i) i^{th}$ element of the trace s
- projection : $\pi_s^{act}(i) i^{th}$ action of the trace s



$\ensuremath{\mathbf{Figure}}$: pending section

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Well-formed trace(2)

Conditions

- A terminated thread don't have any pending section.
- A thread to join another thread *t*, must have received explicitly its identifier.
- Two concurrent sections are in mutual exclusion, i.e. the closing of one must precede the opening of the other.

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Well-synchronized

We restraint to well-synchronized trace.

Well-synchronisation

A trace is well-synchronized if all of its conflicting accesses are synchronized

conflicting accesses : two actions conflict if they are both memory accesses over the same location and at least one is a write.

$$(\text{read } \ell \ n \ v) \ \bowtie \ (\text{write } \ell \ n \ v') \\ (\text{write } \ell \ n \ v) \ \bowtie \ (\text{read } \ell \ n \ v') \\ (\text{write } \ell \ n \ v) \ \bowtie \ (\text{write } \ell \ n \ v')$$

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synchronisation

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synchronisation of two actions of the same thread

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synchronisation of a fork of a thread and the action done by it

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synchronisation of a fork of a thread and the action done by it

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synchronisation of the action done by a thread and the join

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synchronisation of the action done by a thread and the join

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synchronisation of the closing, opening of two concurrent sections

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transitive closure

Atomicity

- Weak atomicity :
 - Atomic section isolated from concurrent atomic sections
 - Ensure by our condition of well-formedness wf
- Strong atomicity :
 - Atomic sections isolated from outside.
 - We want to prove that well-synchronised traces ensure this property of serialisability

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Theorem

Every well-formed and well-synchronised trace is serialisable.



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Every well-formed and well-synchronised trace is serialisable.



Serialisable

A trace s is serialisable if there exists a serial trace s' such that s is equivalent to s'.



Serialisable

A trace s is serialisable if there exists a **serial** trace s' such that s is equivalent to s'.

Serial trace

A trace s is serial if for all section p only threads members of tribes p run when the section is opened.



Serialisable

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Serial trace

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Equivalence

Two traces s and s' are (schedule-)equivalent^a, noted s = s', if there exists a bijection γ between positions of s and s' such that

(1)
$$\pi_s(i) = \pi_{s'}(\gamma(j))$$
 for all $i < |s|$
(2) $sw_s \ i \ j \Leftrightarrow sw_{s'} \ \gamma(i) \ \gamma(j)$ for all $i, j < |s|$

a. This equivalence relation implies the more classical (conflict-)equivalence relation

Serialisability

Sketch of the proof

Theorem

Every well-formed and well-synchronised trace is serialisable.

Sketch of the proof

induction on trace s.

- base case : trivial.
- $s \cdot (t, a)$ with s = s' where s' is serial
 - $s \cdot (t, a) = s' \cdot (t, a)$
 - $s' \cdot (t, a)$ serialisable.

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- Formalisation in the proof assistant Coq
- 15 000 lines :
 - 30% definitions
 - 70% proofs

Ongoing work

Properties on traces can be viewed as a specification.

Atomic Fork Join

- Imperative language with atomic sections
- Operational semantics
- Proof that it verifies the specification
- Source language for compilation

Lock Unlock Fork Join

- No atomic sections
- Lock-based implementation
- One lock by level
- Target language for compilation

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Results

- Language supporting nesting and escaping thread
- Definition of well-synchronised and serialisability

Perspectives

- Verification of implementation
- Efficient implementation

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