

Hypertree-width and related invariants

To motivate the study of hypertree-width, consider the following algorithmic problem HOM, the *homomorphism problem*: Given two finite relational structures A and B , is there a homomorphism from A to B ? The homomorphism problem naturally arises in different areas of computer science. It is equivalent to the problem of evaluating conjunctive queries on databases and to the constraint satisfaction problem. Since HOM is NP-complete in general, considerable efforts have been made to find tractable restrictions.

For a class \mathcal{C} of finite relational structures, let $\text{HOM}(\mathcal{C})$ denote the restriction of the homomorphism problem to instances $A \in \mathcal{C}$, while B may still be arbitrary. This kind of restriction is known as ‘left-hand side restriction’. Martin Grohe proved a beautiful classification theorem for left-hand side restriction in the case that \mathcal{C} is a class of structures of bounded arity. Under some complexity theoretic assumption the theorem essentially says that $\text{HOM}(\mathcal{C})$ is solvable in polynomial time if, and only if, \mathcal{C} has *bounded tree-width modulo homomorphic equivalence*. This theorem is particularly nice because it characterises the tractable classes in terms of graph theory and algebra.

Nevertheless, the theorem fails if the arity of the structures in \mathcal{C} is unbounded - and this is where hypertree-width comes into play: Georg Gottlob, Nicola Leone and Francesco Scarcello proved that there are classes \mathcal{C} of structures of unbounded tree-width modulo homomorphic equivalence, such that $\text{HOM}(\mathcal{C})$ is solvable in polynomial time: namely, classes of structures whose *underlying hypergraph* has bounded *hypertree-width*.

A *hypergraph* is a pair $H = (V, E)$ consisting of a finite set V (the *vertices* of H), and a set E of subsets of V (the *hyperedges* of H). Hypertree-width generalises the notion of acyclicity. A hypergraph H is *acyclic*, if its hyperedges can be arranged as nodes of a tree T so that for every $v \in V$, the subgraph of T defined by the nodes containing v is connected.

My talk is about a specific aspect of hypertree-width: The hypertree-width of a hypergraph H can be characterised by the number of cops necessary to catch a robber on the underlying hypergraph of H . In this game the robber’s escape space must decrease in a monotone way. I show that (in contrast to Seymour’s and Thomas’ game characterising tree-width) the number of cops necessary to win the game may increase due to this restriction, but at most by a factor of three. This last result is joint work with Georg Gottlob and Martin Grohe.