

Facial Circuits of Planar Graphs and Context-Free Languages ^{*}

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Abstract. It is known that a language is context-free iff it is the set of borders of the trees of recognizable set, where the border of a (labelled) tree is the word consisting of its leaf labels read from left to right.

We give a generalization of this result in terms of planar graphs of bounded tree-width. Here the border of a planar graph is the word of edge labels of a path which borders a face for some planar embedding. We prove that a language is context-free iff it is the set of borders of the graphs of a set of (labelled) planar graphs of bounded tree-width which is definable by a formula of monadic second-order logic.

Thatcher and Wright [12] (see also Doner [5]) characterize context-free languages as the images of the recognizable sets of finite trees under a mapping *border* that produces for each given tree the sequence of symbols labeling its leaves, read from left to right.

Our aim is to extend such a characterization to Monadic Second Order definable sets of graphs. Here, the border mapping concerns *special graphs*, that have a unique path of labeled edges. Such a mapping, whose study is investigated by Engelfriet and Heyker [6], associates with every special graph the word read on the path. We know that the language generated by a MS-definable set of graphs is in general not context-free (see Example 1).

Example 1. The noncontext-free language $\{a^n b^n c^n \mid n \geq 1\}$ is defined by the MS-definable set of special graphs $\{G_n \mid n \geq 1\}$ described in Fig. 1.

Thus, we impose two restrictions. First, we consider *special planar* graphs, that admit a planar embedding in which the labeled path borders a face (see Example 2). The path is actually made from a circuit by means of a special edge marked $\#$. Secondly, we consider sets of graphs of uniformly bounded *tree-width*. The tree-width is a measure of complexity for graphs introduced by Robertson and Seymour [11].

Example 2. The context-free language $\{a^n b^n \mid n \geq 1\}$ is defined by the MS-definable set of special-planar graphs $\{H_n \mid n \geq 1\}$ described in Fig. 2.

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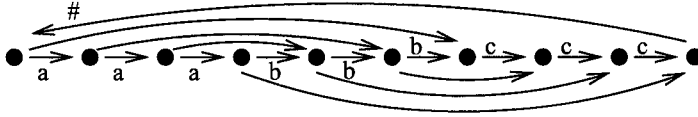


Fig. 1. A special graph G_3 that defines the word $a^3b^3c^3$.



Fig. 2. A special-planar graph H_3 that defines the word a^3b^3 .

Our main result is to characterize the context-free languages as the sets of borders of MS-definable sets of special-planar graphs of bounded tree-width.

1 Special Graphs

Definition 3. A *hypergraph* G is a tuple (V, E, f) , denoted by $(\mathbf{V}_G, \mathbf{E}_G, \mathbf{vert}_G)$, where V is the finite set of *vertices*, where E is the finite set of *hyperedges* which is supposed disjoint with V and where f associates with every $e \in E$ the sequence of its *extremities* $f(e)$, supposed pairwise distinct (there is no loop) and denoted by $(f(e, 1), \dots, f(e, n))$. The *initial* (resp. *terminal*) extremity of e is $f(e, 1)$ (resp. $f(e, n)$). A *graph* is a hypergraph such that each hyperedge is an *edge*, that has two extremities. A vertex x and an edge e are *incident* if x is an extremity of e .

A *path* of some graph G is a sequence $p = (o_1, \dots, o_m) \in (\mathbf{V}_G \cup \mathbf{E}_G)^+$ for some $m \geq 1$ where o_i and o_{i+1} are incident for every $i \in [m - 1]$ and where o_1 (resp. o_m) is a vertex, its *initial* (resp. *terminal*) extremity. All vertices and edges of a path, except eventually its extremities, are supposed pairwise distinct. A path is a *cycle* if $o_1 = o_m$. It is said *oriented* if for every arc o_i with $i \in [2, m - 1]$, we have $o_{i-1} = \mathbf{vert}_G(o_i, 1)$ and $o_{i+1} = \mathbf{vert}_G(o_i, 2)$. A *circular path* is an oriented path having identical extremities. To simplify, an oriented path is represented by a sequence of edges. The graph G is *connected* if all two vertices are the extremities of some path of G .

Let H and K be two hypergraphs. H is a *subhypergraph* of K if $\mathbf{V}_H \subseteq \mathbf{V}_K$, $\mathbf{E}_H \subseteq \mathbf{E}_K$ and $\mathbf{vert}_H \subseteq \mathbf{vert}_K$. If $\mathbf{vert}_H(d) = \mathbf{vert}_K(d)$ for each $d \in \mathbf{E}_H \cap \mathbf{E}_K$, the *union* of H and K , denoted by $H \cup K$, is the hypergraph $(\mathbf{V}_H \cup \mathbf{V}_K, \mathbf{E}_H \cup \mathbf{E}_K, \mathbf{vert}_H \cup \mathbf{vert}_K)$.

Definition 4. A *tree* is a connected graph that contains no cycle. A vertex (resp. edge) of a tree T is called a *node* (resp. *arc*). Their set is denoted by \mathbf{N}_T (resp. \mathbf{A}_T). If T is a tree, if $f \in \mathbf{A}_T$ and s is an extremity of f , we let $T(f, s)$ denote the subtree of T consisting of s and all paths in T containing s but not f . If a tree T is given with a root r , we direct its arcs from the root

towards the leaves. Every node except the root has then a unique father and zero, one or several sons. A leaf has no son. If $x \in \mathbf{N}_T$, we let $T/x = T$ if $x = r$, $T/x = T(f, x)$ if f is the edge between x and its father. The root of T/x is x .

In this article, $\#$ denotes a special label.

Definition 5 (Special graphs). Let X be an alphabet that does not contain $\#$. A *special graph* over X is a graph G with a unique edge labeled $\#$, denoted by $\#_G$, and with a family $(E_a)_{a \in X}$ where $E_a \subseteq \mathbf{E}_G$, the sets E_a are pairwise disjoint and $E_X = \bigcup \{E_a \mid a \in X\} \cup \{\#_G\}$ is the set of edges of a unique circular path $\gamma(G)$ of G having as last edge $\#_G$; moreover, we suppose that no two distinct edges having same sequence of extremities and same label (resp. no label). We shall say that $e \in \mathbf{E}_G$ has *label* a iff $e \in E_a$. Some edges of G may have no label.

For each special graph G , we let $\underline{bd}(G) \in X^+$ be the nonempty word $a_1 \dots a_n$ where $(e_1, \dots, e_n, \#_G)$ is the circular path $\gamma(G)$ and where for each $i \in [n]$, e_i is labeled a_i . We call it the *border* of G . If $\pi = (e_i, e_{i+1}, \dots, e_j)$ is a path in $\gamma(G)$, we denote by $\underline{bd}(\pi)$ the word $a_i a_{i+1}, \dots, a_j$ which is a factor of $\gamma(G)$.

Such a graph will be represented by the *relational structure* $|G|_2 = \langle \mathbf{V}_G \cup \mathbf{E}_G, \text{inc}_G, (lab_{aG})_{a \in X \cup \{\#\}} \rangle$ where $lab_{aG}(e)$ holds iff $e \in E_a$ (or $e = \#_G$) and $\text{inc}_G = \{(e, x, y) \mid e \in \mathbf{E}_G, e \text{ links } x \text{ to } y\}$. Hence, sets of graphs can be *defined* by formulas in Monadic Second Order logic or in *Counting MS logic*, a refinement of MS logic using special predicate expressing cardinality of sets modulo fixed integers; see [2,3,4]. Such sets are said *MS-definable* (resp. *CMS-definable*), for short. MS logic is the extension of First-Order logic with set variables. For words and binary trees, MS-definability equals recognizability.

Definition 6 (Tree-width). A *tree-decomposition* is a pair (T, g) where T is a tree and where g associates with every node t of T a graph $g(t)$ such that:

- $\mathbf{E}_{g(s)} \cap \mathbf{E}_{g(t)} = \emptyset$, for all distinct nodes s, t of T .
- for all nodes s, u of T and every node t of the path of T from s to u , we have: $\mathbf{V}_{g(s)} \cap \mathbf{V}_{g(u)} \subseteq \mathbf{V}_{g(t)}$.

The *width* of (T, g) is the maximum of $\text{card}(\mathbf{V}_{g(t)}) - 1$ taken over all $t \in \mathbf{N}_T$. The *tree-width* of a hypergraph G , denoted by $\underline{twd}(G)$, is the minimum width of all tree-decompositions (T, g) of G (such that $\bigcup_{t \in \mathbf{N}_T} g(t) = G$).

Definition 7. Let (T, g) be a tree-decomposition of a graph G . If T' is a subtree of T , we denote by $G[T', g]$ the subgraph of G defined as $\bigcup \{g(s) \mid s \in \mathbf{N}_{T'}\}$. Thus $(T', g/T')$ is a tree-decomposition of $G[T', g]$. Let now G be special. A tree-decomposition (T, g) of G is *special* if:

1. T has a root r which verifies $\#_G \in \mathbf{E}_{g(r)}$.
2. for every $s \in \mathbf{N}_T$ and incident edge f , the set of edges $E_X \cap \bigcup \{\mathbf{E}_{g(u)} \mid u \in T(f, s)\}$ is either empty, or is E_X or forms a noncircular path, that we shall denote by $\gamma(G, f, s)$.

We say that such a tree-decomposition is *compact* if for all f and s , $\gamma(G, f, s)$ is a (nonempty) noncircular path. Let t be a node of some compact tree-decomposition (T, g) , d the eventual edge linking t to its parent. Its sons t_1, \dots, t_n are linked to t by d_1, \dots, d_n . Clearly, if t is (resp. is not) the root, $\gamma(G)$ (resp. $\gamma(G, d, t)$) admits a unique expression of the form $w_0 \gamma(G, d_{\pi(1)}, t_{\pi(1)}) w_1 \dots \gamma(G, d_{\pi(n)}, t_{\pi(n)}) w_n$ for some permutation π on $[n]$. We denote by \hat{t} the sequence of words $(\underline{bd}(w_0), \dots, \underline{bd}(w_n))$.

Here is some intuition: a path in a graph given with a tree-decomposition yields a traversal of a portion of the tree of the tree-decomposition. Special means that the distinguished path yields a single traversal of each subtree. Compact means that every box of the tree-decompositions contains one edge of the distinguished path.

Example 8. Figures 3 and 4 represents two tree-decompositions (T, g) and (T, h) of a same graph G (in the left part of Fig. 4) and having a same tree T (in the left part of Fig. 3). The node 1 is the root of T . (T, g) is not special: the graph $\gamma(G) \cap G[T/5, g]$ consists of two paths with corresponding words bb and dd .

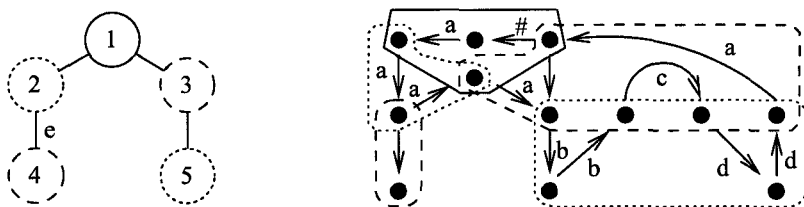


Fig. 3. A non special tree-decomposition.

(T, h) is special: for example, $\gamma(G) \cap G[T/5, h]$ consists of a path with corresponding word $bcbdd$. (T, h) is not compact, because $\gamma(G, e, 4)$ is empty.

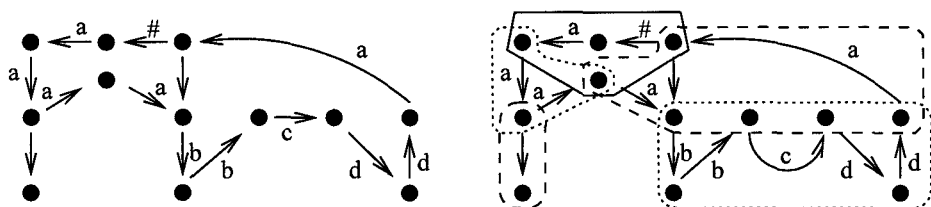


Fig. 4. A special but non compact tree-decomposition.

Definition 9. Let F be a finite signature and $\mathbf{T}(F)$ be the set of finite terms built over F . Let X be a finite alphabet. For every $f \in F$ of arity k , let be given a sequence of words $\hat{f} = (u_0, \dots, u_k)$, $u_0, \dots, u_k \in X^*$. We define a mapping $\alpha : \mathbf{T}(F) \rightarrow X^*$ by letting: $\alpha(f(t_1, \dots, t_k)) = u_0\alpha(t_1)u_1\alpha(t_2)\dots\alpha(t_k)u_k$ where $(u_0, \dots, u_k) = \hat{f}$, $f \in F$, $t_1, \dots, t_k \in \mathbf{T}(F)$. If $k = 0$ then $\hat{f} = (u_0)$ and we let $\alpha(f) = u_0$. Such a mapping α is called a *homomorphism*: $\mathbf{T}(F) \rightarrow X^*$.

Proposition 10. If $K \subseteq \mathbf{T}(F)$ is a recognizable set of terms and $\alpha : \mathbf{T}(F) \rightarrow X^*$ is a homomorphism, then $\alpha(K)$ is a context-free language.

Proof. One constructs a context-free grammar the derivation trees of which are the terms in K . The result is well known (see [5,12]). \square

2 Tree-Decompositions as Algebraic Terms

In this section, k denotes a fixed integer and X a finite alphabet.

Definition 11 (Hyperedge-substitution). We denote by \mathcal{G} the set of all hypergraphs G labeled over $X \cup \{\#\}$ equipped with a sequence of pairwise distinct vertices, called the sequence of *sources* and denoted by \underline{src}_G . Let $H, K \in \mathcal{G}$ with $\mathbf{E}_H \cap \mathbf{E}_K = \emptyset$ and let e be an edge of H such that $\mathbf{vert}_G(e)$ equals \underline{src}_K and contains every vertex of $\mathbf{V}_H \cap \mathbf{V}_K$, we denote by $H\{K/e\}$ the hypergraph obtained from $H \cup K$ by forgetting the edge e and having as sequence of sources \underline{src}_H . It results from the *substitution* in H of K for e .

Notation 12 We denote by \mathcal{T}_k the set of all compact tree-decompositions (T, g) of width at most $k - 1$ such that every $s \in \mathbf{N}_T$ has maximum degree k . Let $<$ be a linear ordering on $\mathbf{V}_G \cup \mathbf{E}_G$ such that $e_1 < e_2 < e_3 < \dots < e_p$ where $(e_1, e_2, \dots, e_p, \#_G) = \gamma(G)$. For every s in \mathbf{N}_T , we order its set of sons as a sequence (t_1, \dots, t_n) , in such a way that the \leq -smallest edge in $G[T/t_i, g]$ is strictly smaller with respect to \leq than the \leq -smallest one in $G[T/t_{i+1}, g]$ and we make $g(s)$ into a hypergraph with sources, denoted by $H(s)$, as follows:

- a) its sequence of sources is \underline{src}_G if $s = r$, and, otherwise, consists of $\mathbf{V}_{g(s)} \cap \mathbf{V}_{g(t_0)}$ in increasing order for \leq with t_0 the father of s .
- b) for each $i = 1, \dots, n$, we add to $g(s)$ a hyperedge h_i with sequence of vertices $\mathbf{V}_{g(s)} \cap \mathbf{V}_{g(t_i)}$ in increasing order for \leq .

For each isomorphism class of hypergraphs of the form $H(s)$ with n hyperedges and for each $n + 1$ -tuple of the form \hat{s} (see Definition 7), we get a function f for which $\hat{f} = \hat{s}$ and that maps an n -tuple (G_1, \dots, G_n) of graphs with sources into a graph with or without sources (If $n = 0$ then this function takes no argument and is a constant denoting a graph with sources). We denote by F_k such a signature and by α the homomorphism $\mathbf{T}(F_k) \rightarrow X^*$ induced by $\{(\hat{f}, f) \mid f \in F_k\}$.

It follows that a tree-decomposition (T, g) of a graph as above can be represented by a term $t \in \mathbf{T}(F_k)$. We shall denote by $\mathbf{val} : \mathbf{T}(F_k) \rightarrow \{\text{graphs}\}$ the mapping from terms to isomorphism classes of graphs.

With these notations, we have:

Claim 13 *For every node s of some $(T, g) \in \mathcal{T}_k$, we have $G[T/s, g] = (H(s)\{G[T/t_1, g]/h_1\} \dots \{G[T/t_n, g]/h_n\})$, where for each $i = 1, \dots, n$, $G[T/t_i, g]$ is equipped with a sequence of sources consisting of $\mathbf{V}_{g(t_i)} \cap \mathbf{V}_{g(s)}$ ordered in increasing way w.r.t \leq .*

Proof. Obvious from Definition 11 and the above construction. \square

Since X is finite, since multiple edges of any graph have distinct labels, and by the limitations given by the bound k , there are finitely many isomorphism classes of hypergraphs $H(s)$. Hence, F_k is finite.

The following result will allow us to represent compact tree-decompositions by terms in $\mathbf{T}(F_k)$.

Lemma 14. *The degree of a compact tree-decomposition of width at most $k - 1$ is at most k .*

Proof. Easy. \square

Proposition 15. *Let L be a CMS-definable set of special graphs, let L' be the set of graphs in L having a compact tree-decomposition of width at most $k - 1$. Then $\underline{bd}(L')$ is a context-free language.*

Proof. Let (T, g) be a compact tree-decomposition of some graph G and let \leq be the total order on $\mathbf{V}_G \cup \mathbf{E}_G$ defined in Notation 12. A term $t \in \mathbf{T}(F_k)$ represents (T, g) if it is constructed from (T, g) as in Notation 12 and if for every occurrence t of a symbol $f \in F_k$ associated with a node u of T , we have: $\hat{f} = \hat{u}$. For every arc e from s to u in T , we have $\alpha(t/u) = \underline{bd}(\gamma(G, e, u))$. This is straightforward by induction on the depth of t/u from the definitions of \hat{f} and \hat{u} . It follows that $\alpha(t) = \underline{bd}(\gamma(G)) = \underline{bd}(G)$.

Let L be a CMS-definable set of special graphs, L' be the subset of those having a compact tree-decomposition of width at most $k - 1$. Let $K \subseteq \mathbf{T}(F_k)$ be the set of terms representing all these tree-decompositions. Thus, every term $t \in \mathbf{T}(F_k)$ belongs to K iff

1. $\mathbf{val}(t) \in L$.
2. t actually specifies a compact tree-decomposition of width at most $k - 1$ (some terms may specify tree-decompositions that are even not special or define graphs that are not special).

Condition (1) is CMS-definable (because \mathbf{val}^{-1} preserves CMS-definability, see Courcelle [3]); condition (2) is easy to express in MS-logic. It follows that K is CMS-definable, hence, recognizable by the theorem of Doner et al. [5,2]. As a consequence of Lemma 14, each compact (T, g) is represented by some term $t \in \mathbf{T}(F_k)$ which verifies $\alpha(t) = \underline{bd}(\gamma(G))$. It follows that $\underline{bd}(L') = \underline{bd}(\mathbf{val}(t)) = \alpha(K)$, hence is a context-free language by Proposition 10. \square

Our aim is now to extend Proposition 15 to sets of graphs having special tree-decompositions of bounded width (as opposed to compact ones). We need some preliminary results from Courcelle (see [2] or the survey [4]).

Notation 16 (Parallel-composition) Let $n \in \mathbb{N}$, let H and K be two edge-disjoint graphs having the same sequence of n sources. If every common vertex of H and K is a source, then we denote by $H \square_n K$ the graph with no source obtained from the union $H \cup K$ by forgetting the sources. We denote by \sim the isomorphism of graphs with and without sources (the isomorphism must preserve the sequence of sources in an obvious way).

Let L be a CMS-definable set of graphs. It is recognizable (see [2]), hence we have the following:

Fact 17 *For every n , there exists a finite set $\{K_1, \dots, K_p\}$ of graphs with a sequence of n sources such that for every graph K with n sources, there exists i such that for every graph G if $G = H \square_n K$ then:*

$$G \in L \Leftrightarrow H \square_n K'_i \in L \text{ for some } K'_i \sim K_i$$

(This means that in a graph G , a "factor" K can be replaced by one of bounded size, which is "syntactically" equivalent w.r.t. L). For later reference, we let $\kappa(L, n)$ denote $\max\{\text{card}(\mathbf{V}_{K_i}) - n \mid i = 1, \dots, p\}$.

Proposition 18. *Let $m \in \mathbb{N}$ and L be a CMS-definable set of special graphs, each of them having a special tree-decomposition of width at most m . There exists an integer k and a subset L' of L such that $\underline{bd}(L') = \underline{bd}(L)$ and every graph in L' has a compact tree-decomposition of width at most k .*

Proof. Let $G \in L$ and (T, g) be a special tree-decomposition of G of width at most m which is not compact. We let $N_0 \subseteq N_T$ be the set of nodes s of T such that $g(s)$ contains at least one edge of $\gamma(G)$. We let T' be the subgraph of T consisting of the (undirected) paths in T linking two nodes of N_0 . It is clear by the properties of tree-decompositions that T' is connected hence is a tree. Note that $r \in T'$. We shall define $G' \in L$ such that $\underline{bd}(G') = \underline{bd}(G)$ and having a compact tree-decomposition of width at most k , where k is large enough, but fixed and defined from m and the CMS-formula φ which characterizes L .

Let $u \in N_{T'}$ which has neighbors in T which do not belong to T' ; we call them s_1, \dots, s_l (since $r \in N_{T'}$, the father of u in T is also in T'). We let $K = G[T/s_1, g] \cup \dots \cup G[T/s_l, g]$ and H be the unique subgraph of G such that $G = H \square K$. Note that $S = \mathbf{V}_H \cap \mathbf{V}_K \subseteq \mathbf{V}_{g(u)}$ hence has cardinality n at most $m + 1$. The circular path $\gamma(G)$ is included in H .

Let K' be the graph with n sources "syntactically equivalent" to K w.r.t. L obtained by Fact 17. Consider now $G' = H \square_n K'$; it belongs to L , hence is a special graph; the circular path $\gamma(G)$ being in H is also in G' . We obtain a tree-decomposition for G' from (T, g) by deleting $T/s_1, \dots, T/s_l$ and replacing

$g(u)$ by $g(u) \square_n K'$. We repeat this step for each node u of T' having neighbors in T that are not in $N_{T'}$, and we eliminate these neighbors. Doing this, we reduce the size of G , we restrict T to T' but we increase by the constant additive factor $\max\{\kappa(L, n) \mid n \leq m + 1\}$ (see Fact 17). Hence, we get at the end a graph $G' \in L$ such that $\underline{bd}(G') = \underline{bd}(G)$, G' has a compact tree-decomposition of width at most $k = m + \max\{\kappa(L, n) \mid n \leq m + 1\}$. \square

Theorem 19. *Let $m \in \mathbb{N}$ and L be a CMS-definable set of special graphs, each of them having a special tree-decomposition of width at most m . Then, $\underline{bd}(L)$ is a context-free language.*

Proof. Immediate consequence of Propositions 15 and 18. \square

3 Special-Planar Graphs

A special graph G is *special-planar* if it is planar and if $\gamma(G)$ borders a face in some planar embedding of G . Such graphs verify the following property, which is a consequence of a more general result of [8] or [7]. We have not enough space to include the proof.

Proposition 20. *Every special-planar graph G admits a special tree-decomposition of width $\underline{twd}(G)$.*

Proposition 21. *Every context free language $L \subseteq X^+$ is $\underline{bd}(K)$ for some MS-definable set K of special-planar graphs of $\underline{twd} \leq 2$.*

Proof. The proof is available in a more complete version at URL <http://dept-info.labri.u-bordeaux.fr/~courcell/ActSci.html>.

Now, we can establish our main theorem.

Theorem 22. *A language $L \subseteq X^+$ is context-free if and only if it is $\underline{bd}(K)$ for some CMS-definable set K of special-planar graphs of bounded tree-width.*

Proof. As a consequence of Propositions 20, 21 and Theorem 19. \square

As an immediate consequence of Theorem 22, a very simple proof of a well-known result on context-free languages (see [1]). Two words v, w are said to be *conjugates* if there exist words t, u such that $v = t \cdot u$ and $w = u \cdot t$. If L is a language, its conjugacy closure, denoted by L^\sim , is the set L augmented with all conjugates of all its words.

Corollary 23. *The class of context-free languages is closed under conjugacy closure.*

Proof. Let L be a context-free language. By Theorem 22, there is a CMS-definable set K of special-planar graphs of bounded tree-width such that $L = \underline{bd}(K)$. Two special graphs G and H are said *conjugate* if there is $p \in [n]$ such that H is obtained from G by relabelling every e_i with the label of $e_{f(i)}$ with $\gamma(G) = (e_1, \dots, e_n)$ and where f is defined by $f(i) = i$ (resp. $= n, = i - 1$) for each $i \in [p - 1]$ (resp. $i = p, i \in [p + 1, n]$). Let K^\sim be the conjugate closure of K . Clearly, K^\sim is CMS-definable, hence, verifies the conditions of Theorem 22 and $\underline{bd}(K^\sim) = L^\sim$. Then, L^\sim is context-free. \square

Discussion:

In Theorems 19 and 22, one can replace “CMS-definable” by “recognizable” since it is proved in [9] that CMS-definability equals recognizability for sets of graphs of bounded tree-width (see [2] or [3] or [4] for recognizable sets of graphs).

The limitation to “bounded tree-width” cannot be eliminated since it is proved in [10] that context-sensitive languages can be defined similarly as borders of grids.

With the limitation of $\gamma(G)$ to border a face, one can obtain noncontext-free languages as shown in Example 1.

One may ask about a similar characterization of linear languages in terms of special-planar graphs of bounded parh-with.

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