
SUJET EN DEUX PARTIES/ *This exam has two parts*
Partie 2 : 2 exercices / *Part 2 : 2 exercises*
Answers may be written in French or in English

ATTENTION : It is important that you deal with part of both subjects, so remind to switch subject after 45 minutes.

1 Matching number by atomic operations

Recall that a matching in a graph is a set of pairwise non adjacent edges (i.e. no two edges in the set share an extremity). We denote by $\nu(G)$ the maximum size of a matching in G .

Question 1.1 *Propose upper and lower bound in terms of $\nu(G)$ for*

- *the matching number $\nu(G - v)$ after deletion of some vertex v .*
- *the matching number $\nu(G \setminus e)$ after deletion of some edge e .*
- *the matching number $\nu(G/e)$ after contraction of some edge $e = uv$.*

For every case, you will justify your answer by a proof of the bound reached and an example of a graph that reach the bound. (This should give 6 bounds, 6 proofs and 6 examples).

Question 1.2 *What is the matching number of the disjoint union of two graphs? Prove your claims (upper and lower bound even if you give a close formula).*

Question 1.3 *What is the matching number of the joint of two graphs? You may have your answer depends on the parity of the size of the graphs, but it is possible to give a close formula.*

2 Product of graphs

Question 2.1 *For each of the graph products seen in the course, give the exact number of edges of the product of two graphs.*