Weighted codes in Lee Metrics

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Abstract Perfect weighted coverings of radius one have been often studied in the Hamming metric. In this paper, we study these codes in the Lee metric. To simplify the notation, we use a slightly different description, yet equivalent. Given two integers a and b, an (a, b)-code is a set of vertices such that vertices in the code have a neighbours in the code and other vertices have b neighbours in the code. An (a, b)-code is exactly a perfect weighted covering of radius one with weight $(\frac{b-a}{b}, \frac{1}{b})$. In this paper, we prove results of existence as well as of non-existence for (a, b)-codes on the multidimensional grid graphs.

 $\mathbf{Keywords}\ \mathrm{coverings}\ \cdot\ \mathrm{weighted}\ \mathrm{codes}\ \cdot\ \mathrm{Lee}\ \mathrm{metric}\ \cdot\ \mathrm{Manhattan}\ \mathrm{metric}$

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