

Monochromatic Hamiltonian t -tight Berge-cycles in hypergraphs

Paul Dorbec

Institut Fourier
100 rue des Maths, BP74
38402, Saint Martin d'Hères, France
paul.dorbec@ujf-grenoble.fr

Sylvain Gravier

Institut Fourier
100 rue des Maths, BP74
38402, Saint Martin d'Hères, France
sylvain.gravier@ujf-grenoble.fr

Gábor N. Sárközy*

Computer Science Department
Worcester Polytechnic Institute
Worcester, MA, USA 01609
gsarkozy@cs.wpi.edu

and

Computer and Automation Research Institute
Hungarian Academy of Sciences
Budapest, P.O. Box 63
Budapest, Hungary, H-1518

September 3, 2007

Abstract

In any r -uniform hypergraph \mathcal{H} for $2 \leq t \leq r$ we define an r -uniform t -tight Berge-cycle of length ℓ , denoted by $C_\ell^{(r,t)}$, as a sequence of distinct vertices v_1, v_2, \dots, v_ℓ , such that for each set $(v_i, v_{i+1}, \dots, v_{i+t-1})$ of t consecutive vertices on the cycle, there is an edge E_i of \mathcal{H} that contains these t vertices and the edges E_i are all distinct for $i, 1 \leq i \leq \ell$ where $\ell + j \equiv j$. For $t = 2$ we get the classical Berge-cycle and for $t = r$ we get the so-called tight cycle. In this note we formulate the following conjecture. For any fixed $2 \leq c, t \leq r$ satisfying $c+t \leq r+1$ and sufficiently large n , if we color the edges of $K_n^{(r)}$, the complete r -uniform hypergraph on n vertices, with c colors, then there is a monochromatic Hamiltonian t -tight Berge-cycle. We prove some partial results about this conjecture and we show that if true the conjecture is best possible.

*Research supported in part by the National Science Foundation under Grant No. DMS-0456401.