Monochromatic Hamiltonian $t$-tight Berge-cycles in hypergraphs

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Abstract

In any $r$-uniform hypergraph $\mathcal{H}$ for $2 \leq t \leq r$ we define an $r$-uniform $t$-tight Berge-cycle of length $\ell$, denoted by $C^{(r,t)}_\ell$, as a sequence of distinct vertices $v_1, v_2, \ldots, v_\ell$, such that for each set $(v_i, v_{i+1}, \ldots, v_{i+t-1})$ of $t$ consecutive vertices on the cycle, there is an edge $E_i$ of $\mathcal{H}$ that contains these $t$ vertices and the edges $E_i$ are all distinct for $1 \leq i \leq \ell$ where $\ell + j \equiv j$. For $t = 2$ we get the classical Berge-cycle and for $t = r$ we get the so-called tight cycle. In this note we formulate the following conjecture. For any fixed $2 \leq c, t \leq r$ satisfying $c + t \leq r + 1$ and sufficiently large $n$, if we color the edges of $K_n^{(r)}$, the complete $r$-uniform hypergraph on $n$ vertices, with $c$ colors, then there is a monochromatic Hamiltonian $t$-tight Berge-cycle. We prove some partial results about this conjecture and we show that if true the conjecture is best possible.

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