Domination game: effect of edge- and vertex-removal

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> > April 22, 2013

Abstract

The domination game is played on a graph G by two players, named Dominator and Staller. They alternatively select vertices of G such that each chosen vertex enlarges the set of vertices dominated so far. Dominator's goal is to finish the game as soon as possible, Staller wants the game to last as long as possible. It is assumed that both play optimally. Game 1 and Game 2 are variants of the game in which Dominator and Staller has the first move, respectively. The game domination number $\gamma_g(G)$ and the Staller-start game domination number $\gamma'_g(G)$ is the number of vertices chosen in Game 1 and Game 2, respectively. It is proved that if $e \in E(G)$, then $|\gamma_g(G) - \gamma_g(G - e)| \leq 2$ and $|\gamma'_g(G) - \gamma'_g(G - e)| \leq 2$, and that each of the possibilities here is realizable. It is also proved that if $v \in V(G)$, then $\gamma_g(G) - \gamma_g(G - v) \leq 2$ and $\gamma'_g(G) - \gamma'_g(G - v) \leq 2$, and that each of the possibilities is again possible.

Keywords: domination game; game domination number; edge-removed subgraph; vertex-