

Vertex-partitions of graphs into cographs and stars

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Abstract

A *cograph* is a graph that contains no path on four vertices as an induced subgraph. A *cograph k -partition* of a graph $G = (V, E)$ is a vertex-partition of G into k sets $V_1, \dots, V_k \subset V$ so that the graph induced by V_i is a cograph for $1 \leq i \leq k$. Gimbel and Nešetřil [5] studied the complexity aspects of the cograph k -partitions and raised the following questions: Does there exist a triangle-free planar graph that is not cograph 2-partitionable? If the answer is yes, what is the complexity of the associated decision problem? In this paper, we prove that such an example exists and that deciding whether a triangle-free planar graph admits a cograph 2-partition is NP-complete. We also show that every graph with maximum average degree at most $\frac{14}{5}$ admits a cograph 2-partition such that each component is a star on at most three vertices.

1 Introduction

In this paper we focus on vertex-partitions such that each partite set induces a graph with a given structure. Cographs form the minimal family of graphs containing K_1 that is closed with respect to complementation and disjoint union. Cographs are also characterized as the graphs containing no induced copy of P_4 , the path on four vertices (see for example [9]). A *star k -partition* (resp. *cograph k -partition*) of G is a vertex-partition of G in k sets V_1, \dots, V_k such that the graph induced by each V_i is a star forest (resp. a cograph). Moreover we call a *d -star k -partition* a star k -partition where every induced component has order at most d . A 1-star k -partition is a proper k -coloring.

Deciding whether a graph is cograph k -partitionable is linear time solvable when $k = 1$ [4] and is NP-complete for $k \geq 2$ [1]. In [5] Gimbel and Nešetřil focused on planar graphs and proved: