

The domination game played on unions of graphs

Paul Dorbec^{1,2} *Gašper Košmrlj³
Gabriel Renault^{1,2}

¹Univ. Bordeaux, LaBRI, UMR5800, F-33405 Talence

²CNRS, LaBRI, UMR5800, F-33405 Talence

Email: dorbec@labri.fr, gabriel.renault@labri.fr

³University of Ljubljana

Email: gasper.kosmrlj@student.fmf.uni-lj.si

December 2, 2013

Abstract

In a graph G , a vertex is said to dominate itself and its neighbors. The Domination game is a two player game played on a finite graph. Players alternate turns in choosing a vertex that dominates at least one new vertex. The game ends when no move is possible, that is when the set of chosen vertices forms a dominating set of the graph. One player (Dominator) aims to minimize the size of this set while the other (Staller) tries to maximize it. The game domination number, denoted by γ_g , is the number of moves when both players play optimally and Dominator starts. The Staller-start game domination number γ'_g is defined similarly when Staller starts. It is known that the difference between these two values is at most one [4, 9]. In this paper, we are interested in the possible values of the domination game parameters γ_g and γ'_g of the disjoint union of two graphs according to the values of these parameters in the initial graphs.

We first describe a family of graphs that we call no-minus graphs, for which no player gets advantage in passing a move. While it is known that forests are no-minus, we prove that tri-split graphs and dually chordal graphs also are no-minus. Then, we show that the domination game parameters of the union of two no-minus graphs can take only two values according to the domination game parameters of the initial graphs. In comparison, we also show that in the general case, up to four values may be possible.

Key words: domination game, game domination number, disjoint union

AMS subject classification: 05C57, 91A43, 05C69

1 Introduction

In a graph G , a vertex is said to dominate itself and its neighbors. The set of vertices dominated by v is called its closed neighborhood and is denoted by $N[v]$. A set of vertices is a dominating set if every vertex is dominated by some vertex in the set. The domination number is the

*financed in part by the European Union - European Social Fund, and by Ministry of Economic Development and Technology of Republic of Slovenia.