

Paired-domination in generalized claw-free graphs

Paul Dorbec,^{1,2} Sylvain Gravier,¹ *Michael A. Henning,²

¹ERTé “Maths à modeler”
Laboratoire Leibniz, 46 av. F. Viallet
38031 Grenoble Cedex, France
E-mail: paul.dorbec@imag.fr (UJF)
E-mail: sylvain.gravier@imag.fr (CNRS)

²School of Mathematical Sciences
University of KwaZulu-Natal
Pietermaritzburg, 3209 South Africa
E-mail: henning@ukzn.ac.za

Abstract

In this paper, we continue the study of paired-domination in graphs introduced by Haynes and Slater (Networks 32 (1998) 199–206). A set S of vertices in a graph G is a paired-dominating set of G if every vertex of G is adjacent to some vertex in S and if the subgraph induced by S contains a perfect matching. The paired-domination number of G , denoted by $\gamma_{\text{pr}}(G)$, is the minimum cardinality of a paired-dominating set of G . If G does not contain a graph F as an induced subgraph, then G is said to be F -free. Haynes and Slater (Networks 32 (1998) 199–206) showed that if G is a connected graph of order $n \geq 3$, then $\gamma_{\text{pr}}(G) \leq n - 1$ and this bound is sharp for graphs of arbitrarily large order. Every graph is $K_{1,a+2}$ -free for some integer $a \geq 0$. We show that for every integer $a \geq 0$, if G is a connected $K_{1,a+2}$ -free graph of order $n \geq 2$, then $\gamma_{\text{pr}}(G) \leq 2(an + 1)/(2a + 1)$ with infinitely many extremal graphs.

Keywords: bounds, generalized claw-free graphs, paired-domination

AMS subject classification: 05C69

*Research supported in part by the South African National Research Foundation and the University of KwaZulu-Natal.