## More results on total domination in direct products of graphs<sup>\*</sup>

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## Abstract

Upper and lower bounds on the total domination number of the direct product of graphs are given. The bounds involve the {2}-total domination number, the total 2-tuple domination number, and the open packing number of the factors. Using these relationships some exact total domination numbers are obtained. An infinite family of graphs is constructed showing that the bounds are best possible. The domination number of direct products of graphs is also bounded from below.

**Key words:** direct product, total domination, *k*-tuple domination, open packing, domination

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