

# Upper $k$ -tuple domination in graphs

Gerard Jennhwa Chang<sup>123\*</sup> Paul Dorbec<sup>4†‡</sup> Hye Kyung Kim<sup>5§</sup>  
André Raspaud<sup>4¶‡</sup> Haichao Wang<sup>6||</sup> Weiliang Zhao<sup>7\*\*</sup>

<sup>1</sup>Department of Mathematics, National Taiwan University, Taipei 10617, Taiwan

<sup>2</sup>Taida Institute for Mathematical Sciences, National Taiwan University, Taipei 10617, Taiwan

<sup>3</sup>National Center for Theoretical Sciences, Taipei Office, Taiwan

<sup>4</sup>Université de Bordeaux–CNRS, LaBRI, 351 cours de la Libération, 33405 Talence Cedex, France

<sup>5</sup>Department of Mathematics Education, Catholic University of Daegu, Kyongsan 712-702, Republic of Korea

<sup>6</sup>Department of Mathematics, Shanghai University of Electric Power, Shanghai 200090, China

<sup>7</sup>Zhejiang Industry Polytechnic College, Shaoxing 312000, China

December 27, 2011

## Abstract

For a positive integer  $k$ , a  $k$ -tuple dominating set of a graph  $G$  is a subset  $S$  of  $V(G)$  such that  $|N[v] \cap S| \geq k$  for every vertex  $v$ , where  $N[v] = \{v\} \cup \{u \in V(G) : uv \in E(G)\}$ . The upper  $k$ -tuple domination number of  $G$ , denoted by  $\Gamma_{\times k}(G)$ , is the maximum cardinality of a minimal  $k$ -tuple dominating set of  $G$ . In this paper we present an upper bound on  $\Gamma_{\times k}(G)$  for  $r$ -regular graphs  $G$  with  $r \geq k$ , and characterize extremal graphs achieving the upper bound. We also establish an upper bound on  $\Gamma_{\times 2}(G)$  for claw-free  $r$ -regular graphs. For the algorithmic aspect, we show that the upper  $k$ -tuple domination problem is NP-complete for bipartite graphs and for chordal graphs.

**Keywords:** Upper  $k$ -tuple domination,  $r$ -regular graph, bipartite graph, split graph, chordal graph, NP-completeness.

---

\*E-mail: gjchang@math.ntu.edu.tw. Supported in part by the National Science Council under grant NSC99-2923-M-002-007-MY3.

†E-mail: paul.dorbec@u-bordeaux1.fr.

‡Supported in part by Agence Nationale de la Recherche under grant ANR-09-blanc-0373-01.

§E-mail: hkkim@cu.ac.kr. Supported in part by the Basic Science Research Program, the National Research Foundation of Korea, the Ministry of Education, Science and Technology (2011-0025989).

¶E-mail: andre.raspaud@labri.fr.

||Email: whchao2000@163.com. Supported in part by the Foundation for distinguished Young Teachers, Shanghai Education Committee (No. sdl10023) and the Research Foundation of Shanghai University of Electric Power (No. K-2010-32).

\*\*Email: zwl@shu.edu.cn.