

Exercise 1

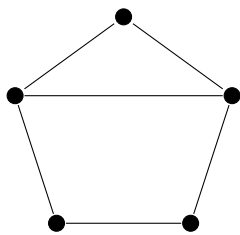
Find an example of a graph having 8 vertices, 12 edges, minimum degree equal to 1 and maximum degree equal to 5. Is your graph connected?

Exercise 2

Find an example of a disconnected graph having 8 vertices, 12 edges, minimum degree equal to 1 and maximum degree equal to 5.

Exercise 3

How many 3-colorings does the following graph have (up to permutation of the colors)?



Exercise 4

Prove that for every graph G , we have $\frac{n}{\alpha} \leq \chi$.

Exercise 5

For a fixed number of vertices n , find the largest number of edges of a disconnected graph G on n vertices. Is the extremal graph unique? If so, what is its structure?

Exercise 6

Prove Turán's theorem.

Hint. Prove the stronger version (including the characterization of the structure of extremal graphs) by induction on n :

Consider a largest clique in a graph without K_r . Observe that it is possible to add edges to your graph until it has a clique of size $r - 1$. Remove this maximal clique and apply induction.

Exercise 7

1. Find an example of a graph having 8 vertices, 12 edges, and girth equal to 3.
2. Find an example of a graph having 8 vertices, 12 edges, and girth equal to 4.
3. Find an example of a graph having 8 vertices, 12 edges, and girth equal to 5.