

Exercise 1

Find an example of a planar graph having 8 vertices, 12 edges, minimum degree equal to 1 and maximum degree equal to 5. Is your example outerplanar?

Exercise 2

Find an example of a planar graph having 8 vertices, 12 edges, minimum degree equal to 1 and maximum degree equal to 5, such that if you remove a vertex of degree 5, you obtain at least three disjoint components. Is your example outerplanar?

Exercise 3

Prove that a graph is planar if and only if each of its components is planar.

Exercise 4

Derive from the Four Color Theorem the following statement : Every outerplanar graph is 3-colorable.

Exercise 5

Prove that every outerplanar graph contains a vertex of degree at most two.

Exercise 6

Use the property from the previous exercise to prove that outerplanar graphs are 3-colorable without using the Four Color Theorem.

Exercise 7

Find an upper bound for the number of edges of a triangle-free planar graph. (Hint : Use Euler's formula and the fact, that every face of a triangle-free planar graph is of size at least 4.)

Exercise 8

Derive from the result of the previous exercise the following statement : Every triangle-free planar graph contains a vertex of degree 3.

Does this property imply directly that triangle-free planar graphs are 3-colorable?