

**Exercise 1**

Build a 5-degenerate graph that has all of its vertices but one of degree 5 and is not 5-colorable.

**Exercise 2**

Build by induction for all integer  $k$  a tree  $T_k$ , together with an ordering of the vertices of  $T_k$ , such that applying the greedy algorithm on  $T_k$  with the vertices in this order gives a  $k$ -coloring of  $T_k$ .

**Exercise 3**

For each of the following statements, either prove it or give a counter-example.

1. For every graph  $G$ , one can find a  $\chi(G)$ -coloring of  $G$  with a greedy algorithm.
2. For every graph class  $\mathcal{C}$  and every integer  $k$ , the complexity of deciding if a graph  $G \in \mathcal{C}$  is  $k$ -colorable is the same as the complexity of finding a  $k$ -coloring of  $G$ .

**Exercise 4**

Give a  $O(n^2)$ -algorithm to 3-color an outerplanar graph.

**Exercise 5**

Show that the two definitions of the degeneracy of a graph seen in course are equivalent : there exists an ordering of the vertices such that every vertex is adjacent to at most  $k$  vertices that come after it if and only if every subgraph of  $G$  has a vertex of degree at most  $k$ .