

# Proving in Coq (Pactole): A user perspective

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# Big (french) projects about DC in Coq

## PADEC

- Dedicated to **self-stabilizing algorithms**
- **Complex algorithms**
- Precise analyses, including **time complexities**
- K. Altisen, P. Corbineau, S. Devismes

## PACTOLE

- Dedicated to distributed computing by **mobile robots**
- **Large variety of models** and parameters
- **Positive** and **negative** results
- T. Balabonski, P. Courtieu, L. Rieg, S. Tixeuil, X. Urbain, ...

# Problem: Ring exploration with stop

## Model/context

- Team of robots
  - **sensing** the environment by **taking a snapshot** of it
  - that **do not communicate**
  - that are **anonymous and oblivious**
- **Anonymous unoriented rings.**

## Goal: exploration with stop

- **Each node must be visited** by at least one robot.
- All robots must **stop** after finite time.

# The Look-Compute-Move cycle

## Look

The robot takes an **instantaneous egocentric snapshot** of the network and its robots, **with multiplicity detection** (“zero”, “one”, or “more than one” robots).

## Compute

Based on this observation, it **decides to stay idle or to move to some neighbouring node**.

## Move

In the latter case it **instantaneously moves** towards its destination.

# Identical oblivious asynchronous robots

## Identical

Robots have **no IDs**. They execute the **same program**.

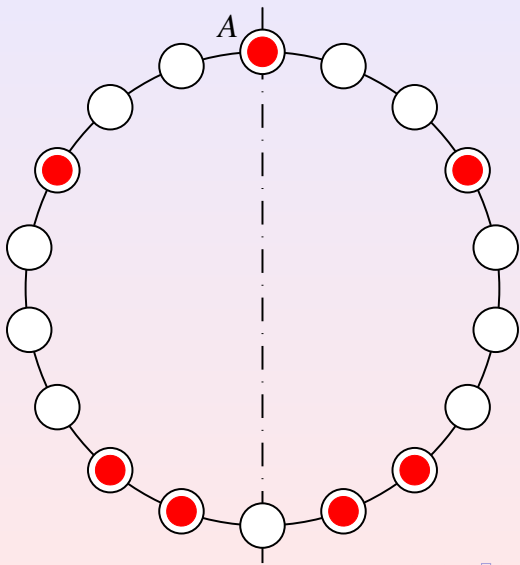
## Oblivious

The robots have **no memory** of observations, computations and moves made in previous cycles.

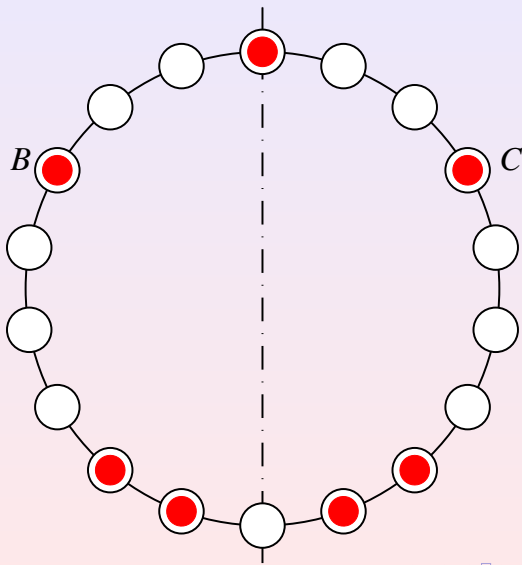
## Asynchronous

The time between Look, Compute, and Move operations is **finite but unbounded**.

# Some precisions



# Some precisions



# Problem definitions

Goal: exploration with stop

- Each node must be visited by at least one robot.
- All robots must stop after finite time.

Definition:  $\text{Explo}(k, n)$

We say that exploration of a  $n$ -node ring is possible with  $k$  robots if there exists an algorithm enabling the robots to perform exploration with stop starting from any initial configuration of the  $k$  robots without multiplicity (at most one robot per node).

More formal definition:  $\text{Explo}(k, n)$

exists Algo, forall Adv, Config,  
if is\_cycle( $n$ , Config) and has\_robots( $k$ , Config) and is\_flat(Config)  
then exploStop(Algo, Adv, Config)



# Results in PACTOLE

## Already in PACTOLE

- not  $\text{Explo}(\mathbf{1}, n)$
- not  $\text{Explo}(k, n)$  **if  $k$  divides  $n$**

## Our “new” result

for every positive integer  $m$ ,

$\text{not } \text{Explo}(k, n) \implies \text{not } \text{Explo}(k * m, n * m)$

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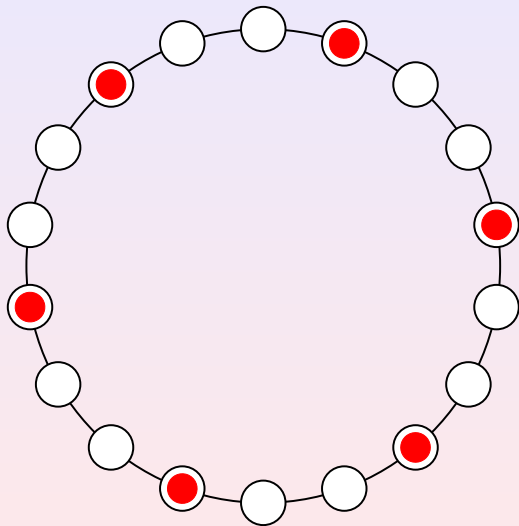
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# When $k$ divides $n$

## Lemma

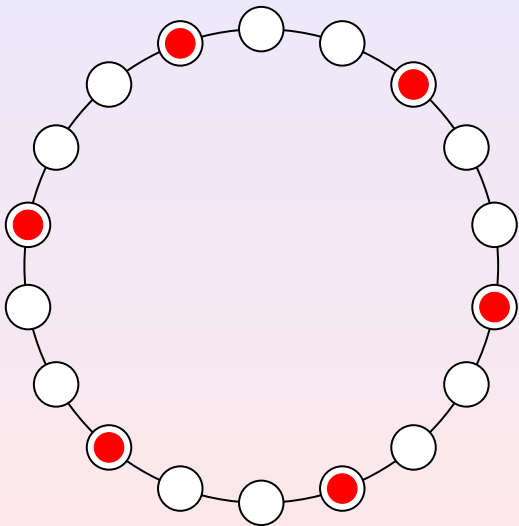
Impossible to stop  
(and sometimes to explore) when  $k|n$ .



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## Lemma

Impossible to stop  
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# Sketch of proof

Proved via:  $\text{Explo}(k * m, n * m) \implies \text{Explo}(k, n)$

Transformations from small  $_1$  to big  $_m$

- From  $_1$  to  $_m$ :  $i \rightarrow \{n \text{ or } k\} * j + i$ , for  $0 \leq j < m$
- From  $_m$  to  $_1$ :  $i \rightarrow i \text{ modulo } \{n \text{ or } k\}$

