

Graph Exploration by Oblivious Agents

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Problem

Model/context

- Team of mobile entities
 - **sensing** the environment by **taking a snapshot** of it
 - that **do not communicate**
 - that are **anonymous and oblivious**

• In a discrete environment: **anonymous graphs**

In this talk: exploration

- **Each node must be visited.**

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The Look-Compute-Move cycle

Look

The agent takes an **egocentric instantaneous snapshot** of the network and its agents

Compute

Based on this observation, it **decides to stay idle or to move to some neighboring node.**

Move

In the latter case it **instantaneously moves** towards its destination.

Identical oblivious asynchronous agents

Identical

Agents have **no IDs**. They execute the **same program**.

Oblivious

The agents have **no memory** of observations, computations and moves made in previous cycles.

Possibly asynchronous

The time between Look, Compute, and Move operations is **finite but unbounded**.

Reminder:

Non-communicating

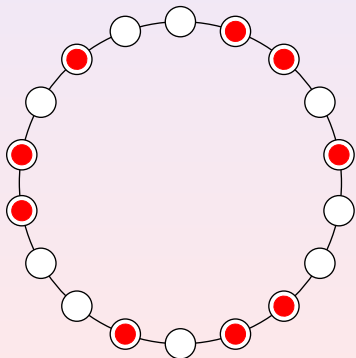
No communication mechanisms between agents, even locally.

Precisions concerning the model

Anonymous setting

- Agents and nodes have no IDs

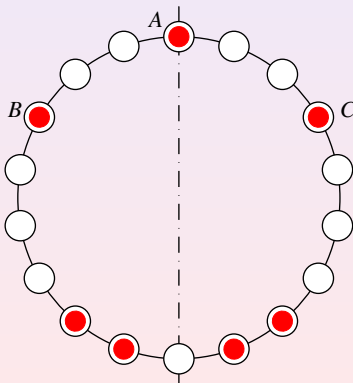
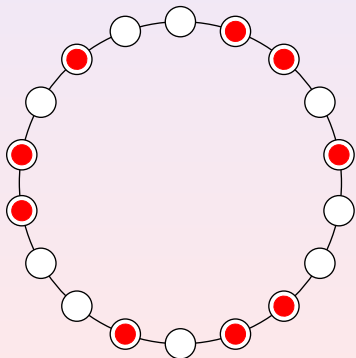
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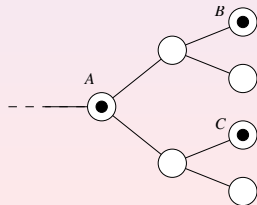
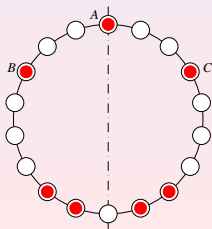
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In case of symmetry

- **Compute** : choice of an **equivalence class of neighbors**
- Actual choice: made by the adversary (i.e. worst case)



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Weak multiplicity detection

“zero”, “one”, or “**more than one**” agents

Two types of exploration

Exploration with stop

- Each node must be visited by at least one agent.
- All agents must stop after finite time.

Exclusive perpetual exploration

- Each node must be visited by every agent infinitely often.
- Agents must not collide (meet at a node or cross each other on an edge).

In both cases, the initial configurations are the configurations without multiplicities.

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Usual focus

n : number of nodes

Smallest exploring team

$\rho^-(n) = \text{Minimum}$ number of agents that can explore any n -node graph of a given family.

Largest exploring team

$\rho^+(n) = \text{Maximum}$ number of agents that can explore any n -node graph of a given family.

Families: Rings, (Partial) Grids, Trees, Tori, etc.

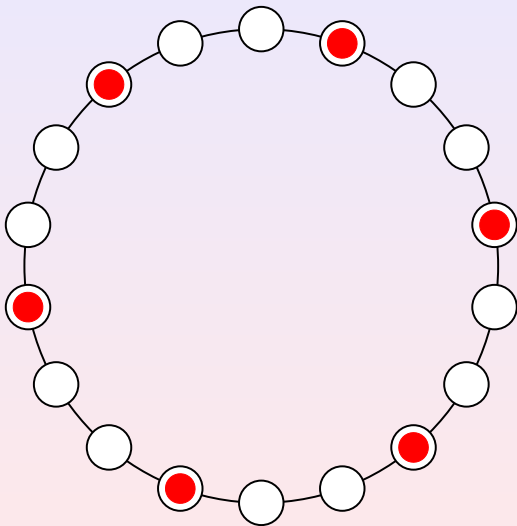
A simple impossibility result

Observation (in rings)

Impossible for both
types of exploration
when k divides n .

n : number of nodes

k : number of agents



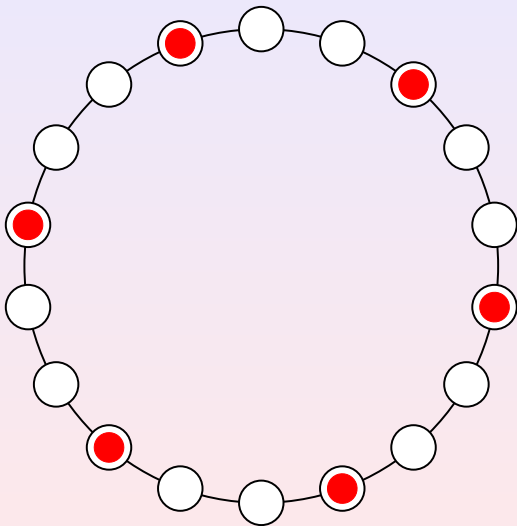
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Focus on Exploration with stop

Exploration with stop (reminder)

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Family: Rings

Additional assumption

n and k are coprime.

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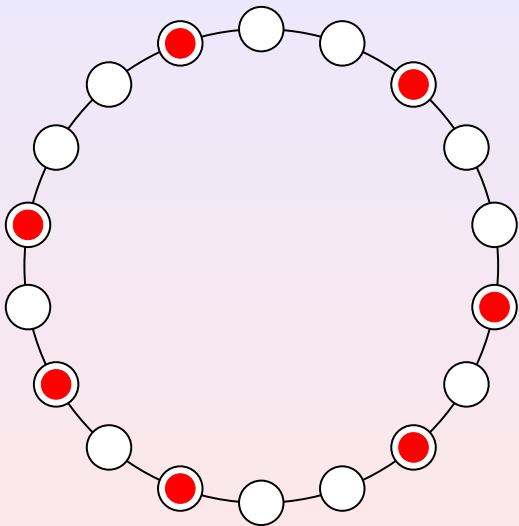
Interdistance

Minimum distance
taken over all pairs of
distinct agents.

Here interdistance=2.

Block

Maximal set of agents,
of size at least 2,
forming a line with an
agent every d nodes.
(d =interdistance)



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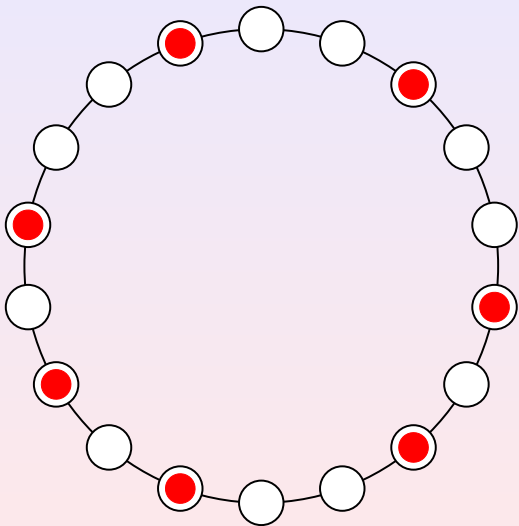
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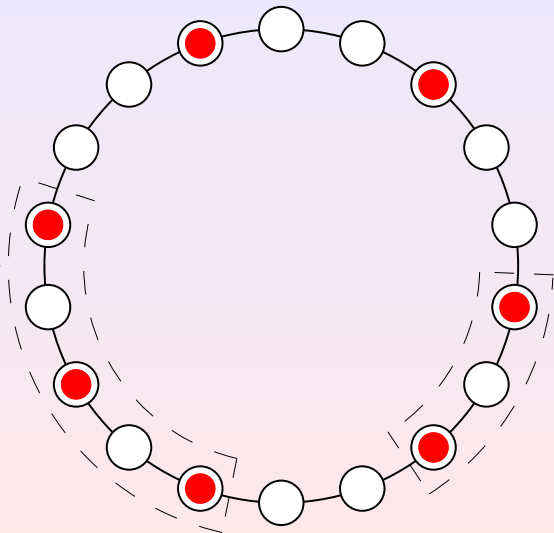
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Maximal set of agents, of size at least 2, forming a line with an agent every d nodes. (d =interdistance)



The algorithm

Set-Up Phase

Goal: to transform the (arbitrary) initial configuration into a configuration of interdistance 1 where there is a single block or two blocks of the same size.

Method: decrease the number of blocks whenever possible. Otherwise, decrease the interdistance.

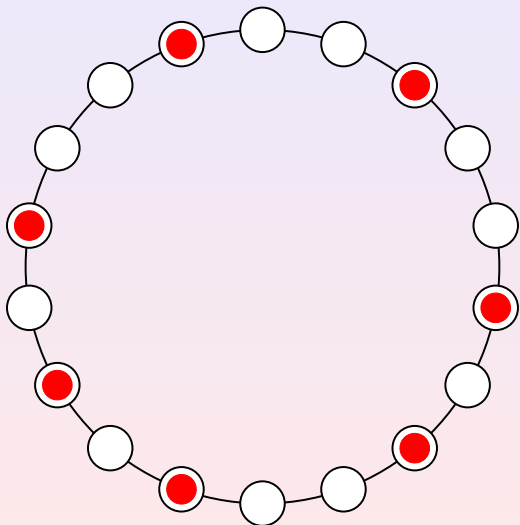
Tower-Creation Phase

Goal: to create one or two multiplicities (towers) inside each block; furthermore a number of agents become uniquely identified as explorers.

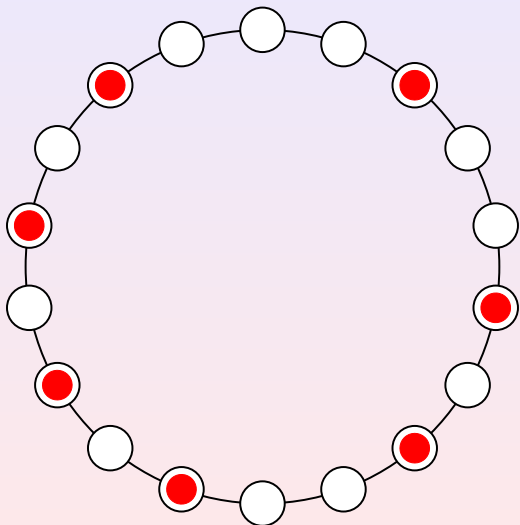
Exploration Phase

Goal: to perform exploration thanks to the explorers until reaching an identified final configuration.

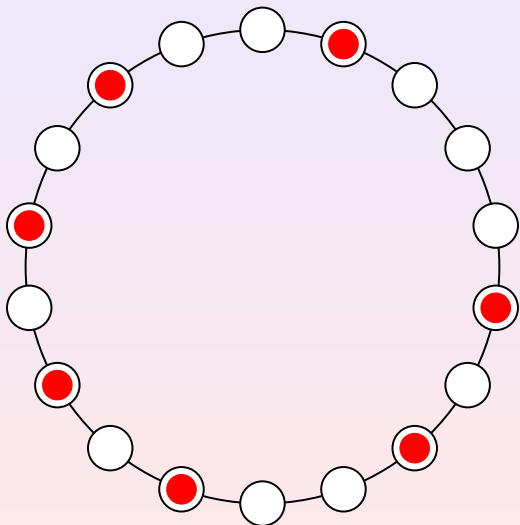
An example of execution



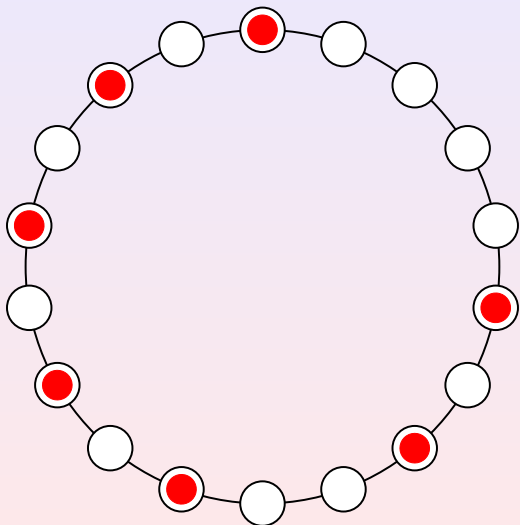
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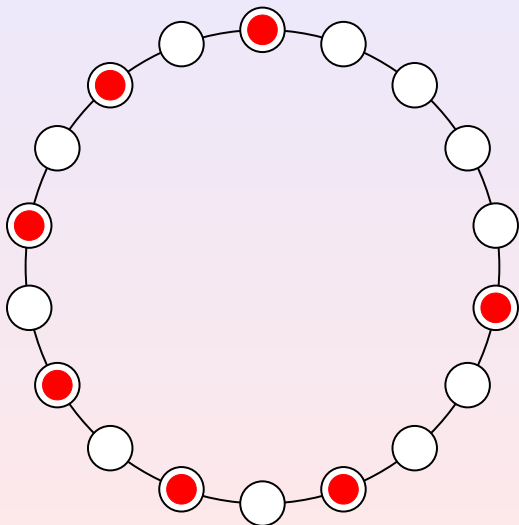
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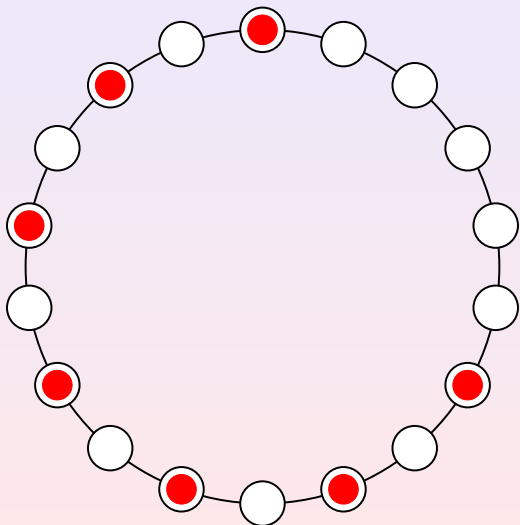
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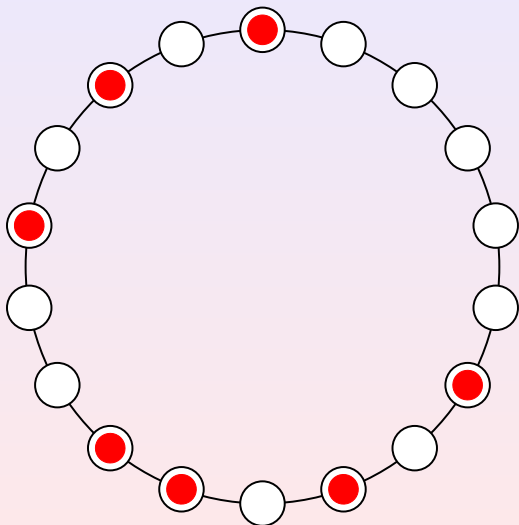
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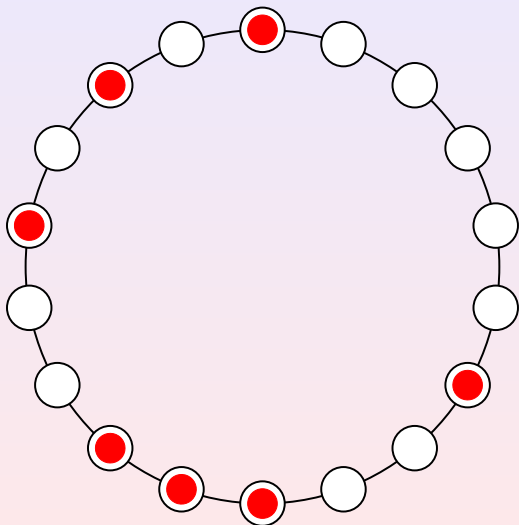
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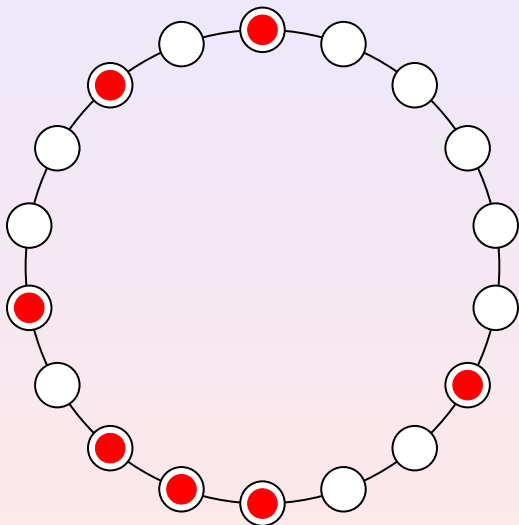
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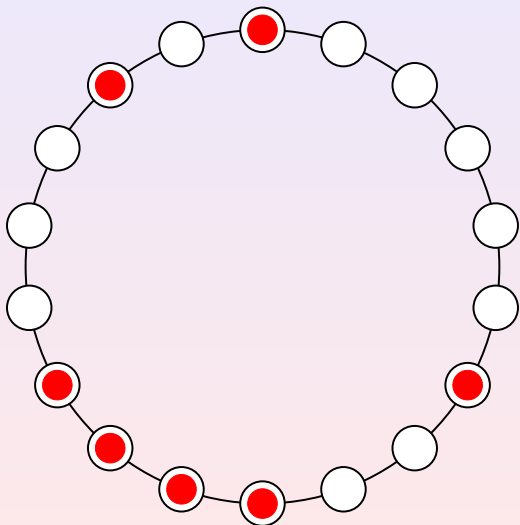
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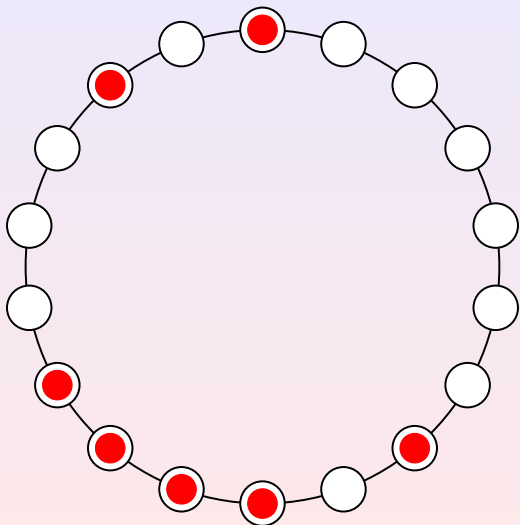
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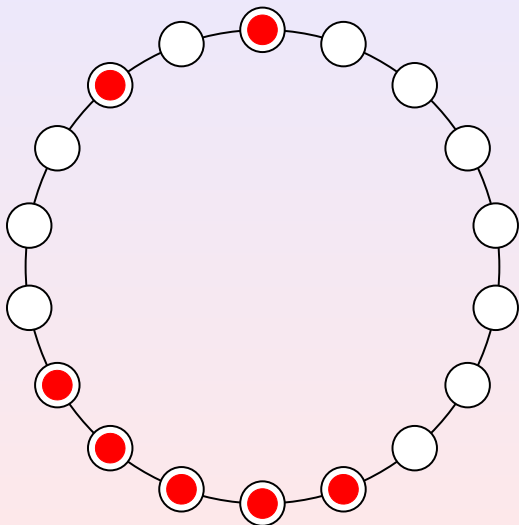
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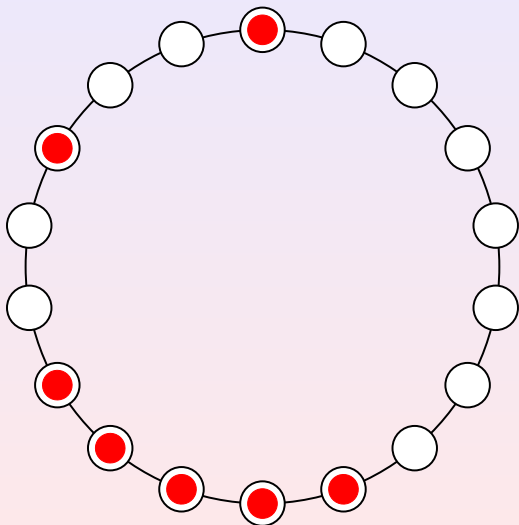
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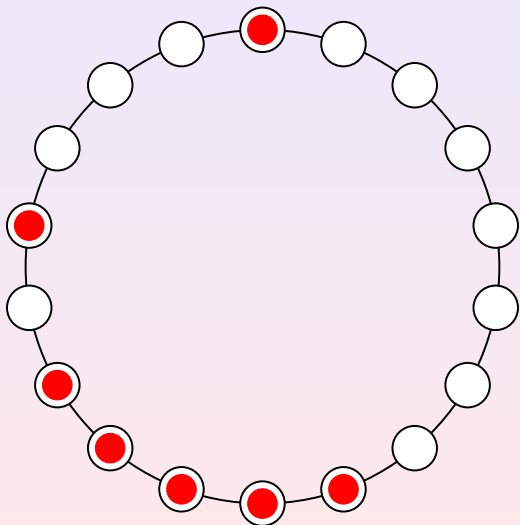
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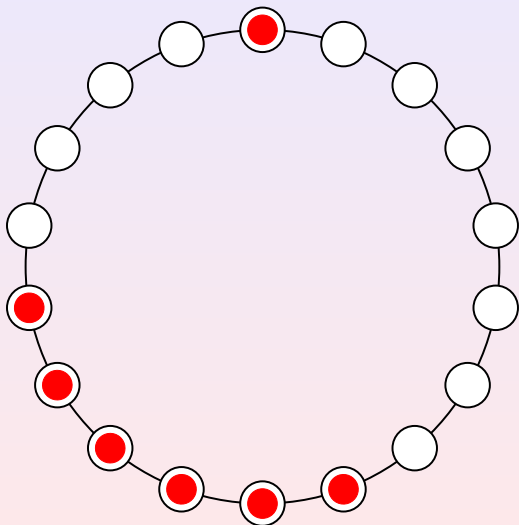
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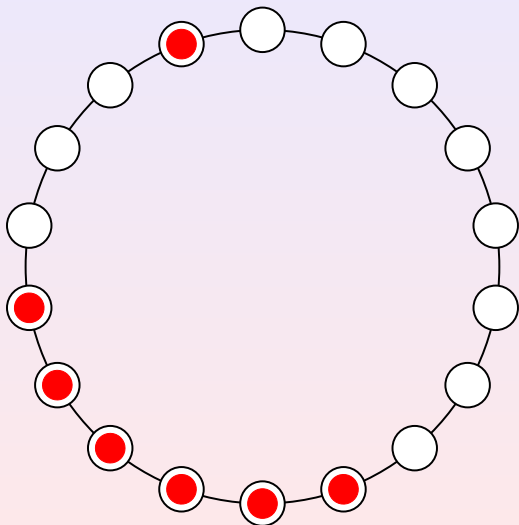
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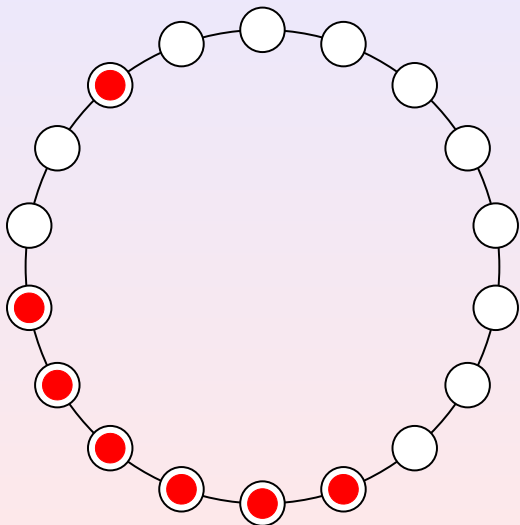
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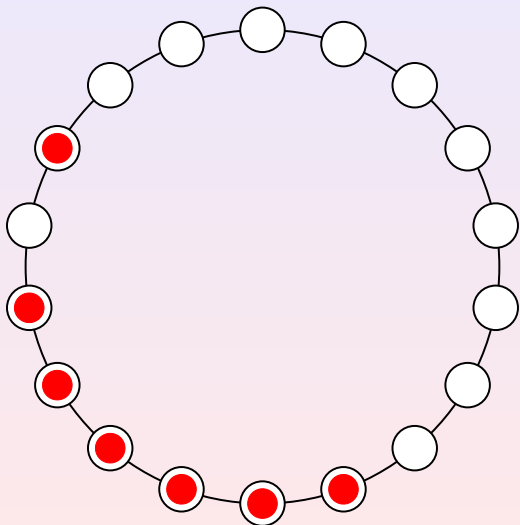
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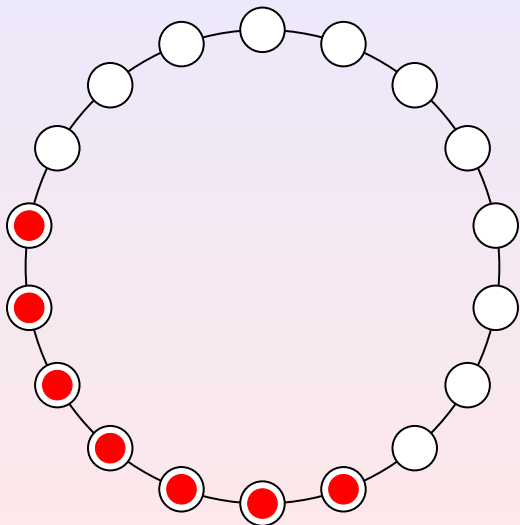
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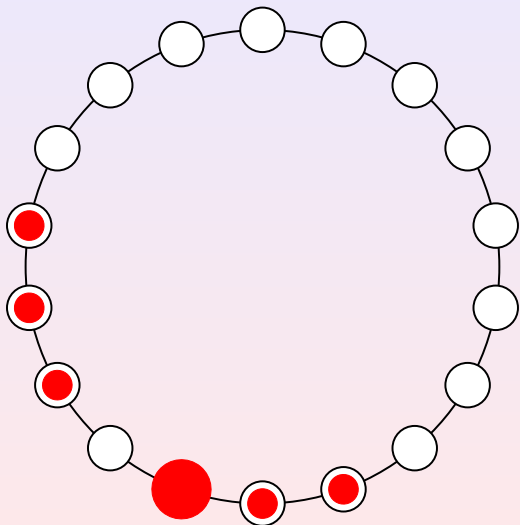
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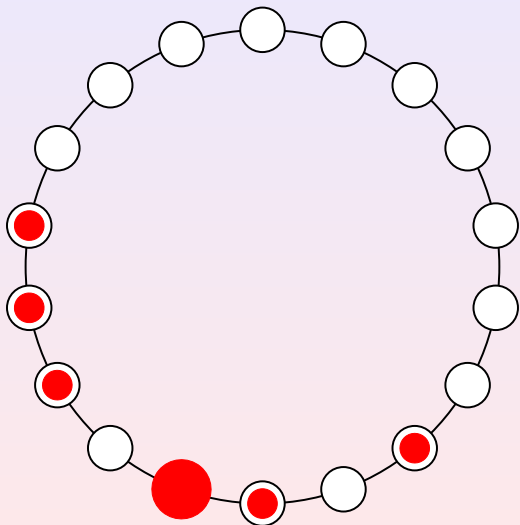
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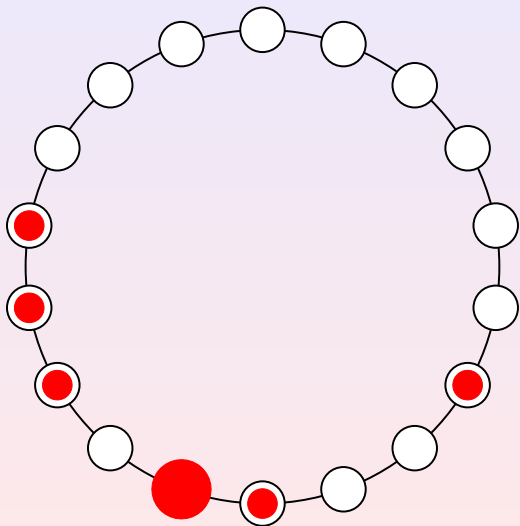
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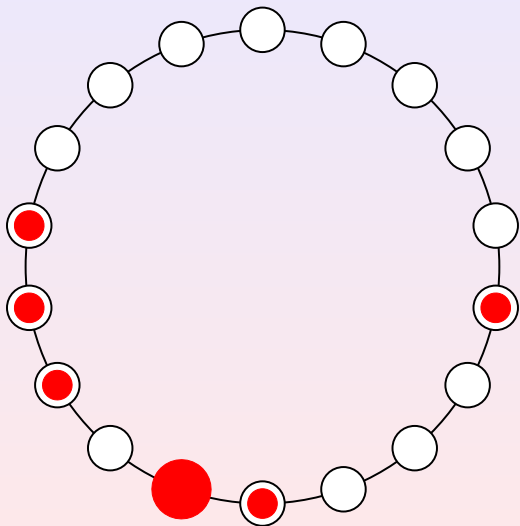
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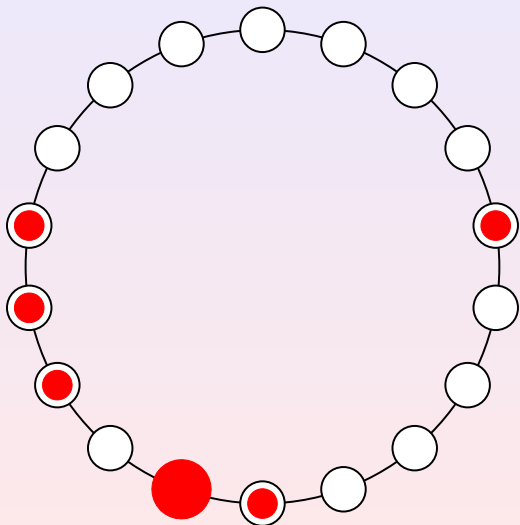
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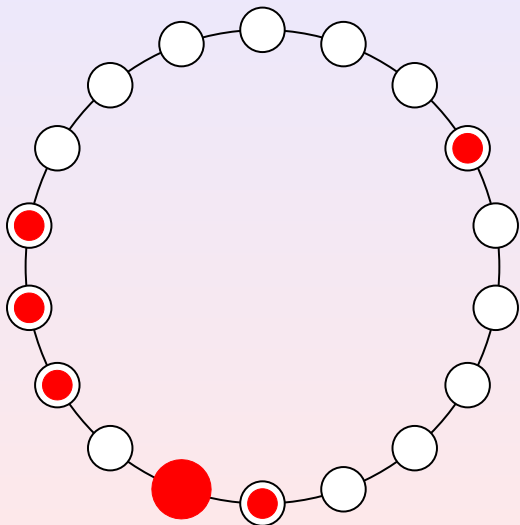
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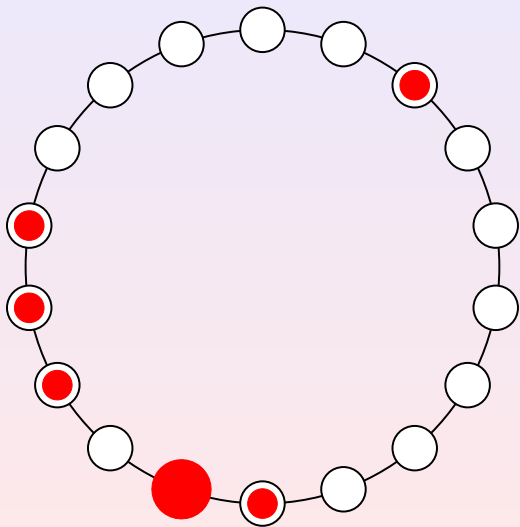
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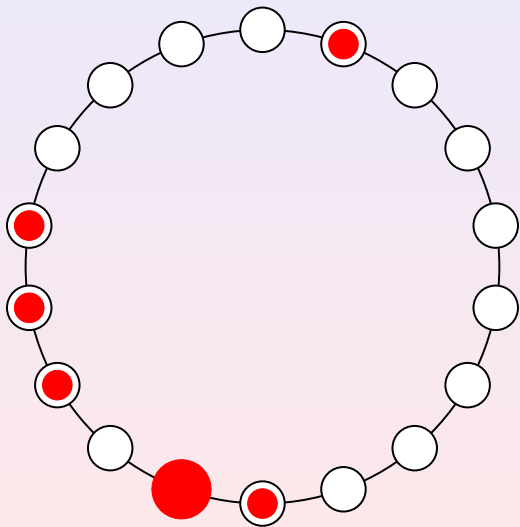
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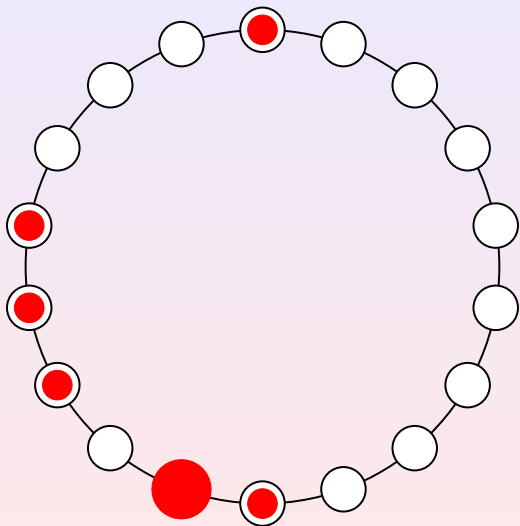
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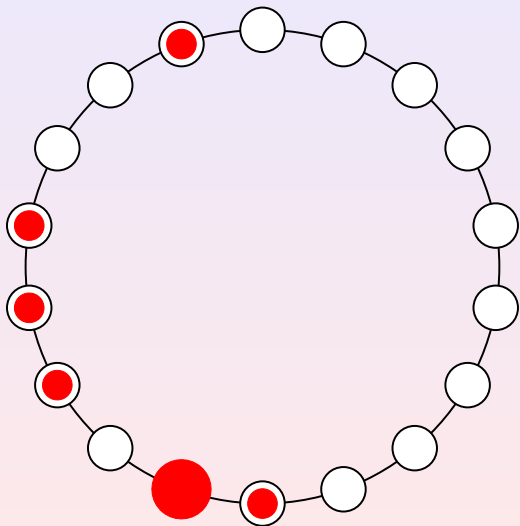
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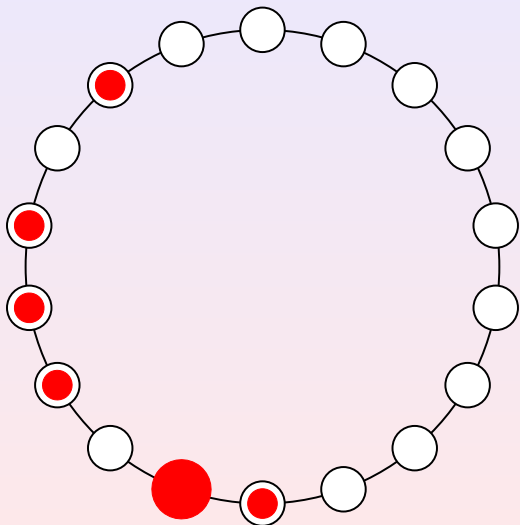
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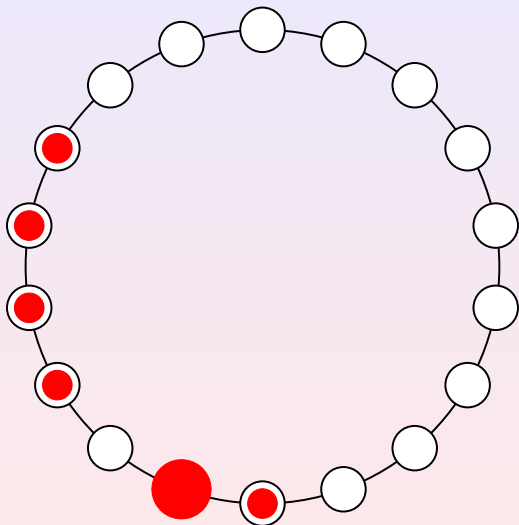
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Algorithmic techniques (exploration with stop)

Phase 1

Form a **special configuration without multiplicities**.

- Uses (co-)primality of n and k .

Phase 2

Create a multiplicity to mark beginning of exploration.

Phase 3

Perform exploration by few (1–3) agents.

- Uses multiplicity and other agents to
 - break symmetry
 - keep track of the progress

But symmetry and asynchrony

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Models of synchrony

FSYNC (fully synchronous)

All agents execute their Look-Compute-Move **simultaneously**.

SSYNC (semi-synchronous)

Look-Compute-Move is **instantaneous**.

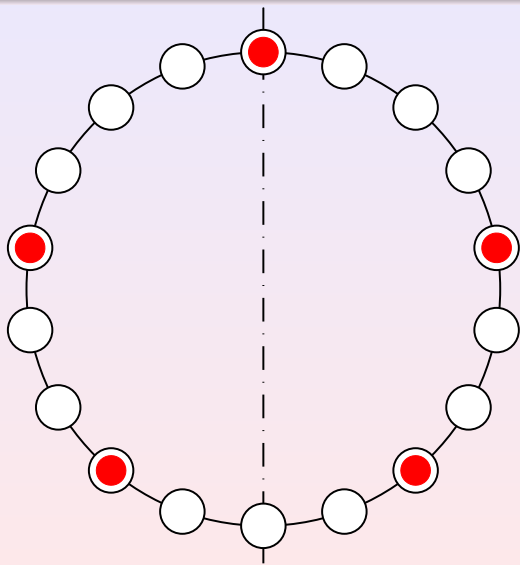
⇒ Difficult to move multiplicities

ASYNC (asynchronous)

The time between Look, Compute, and Move operations is **finite but unbounded**.

⇒ Pending moves (moves based on outdated snapshots)

Symmetries and asynchrony



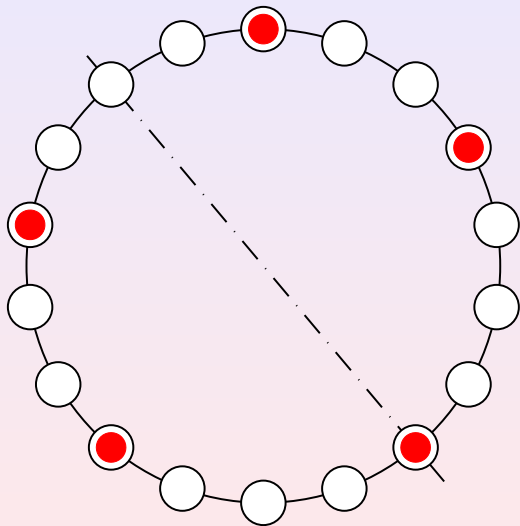
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Move only one agent at a time if possible.

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Symmetries and asynchrony



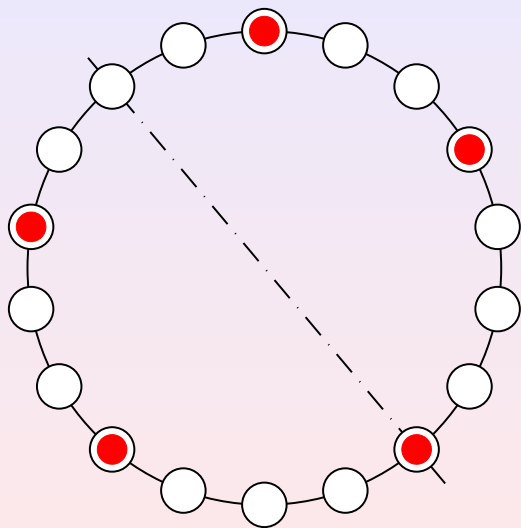
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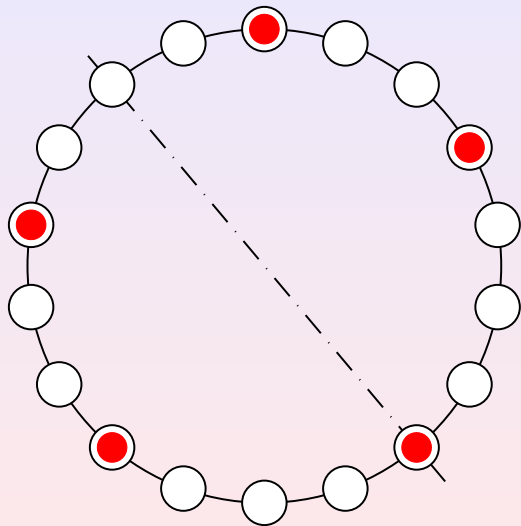
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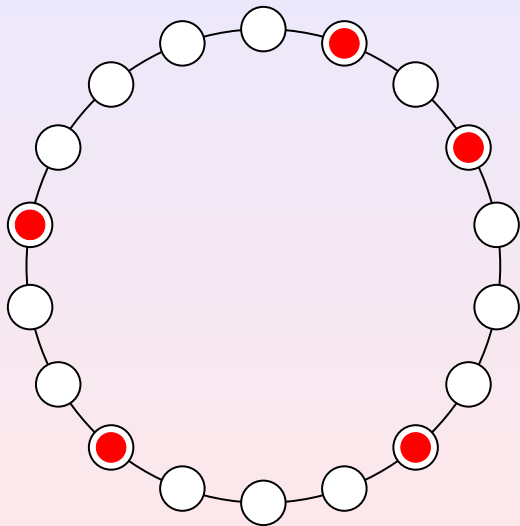
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Known results (explo. with stop) 1/3

In the rings

- [1] ASYNC det.: $\rho^-(n)$ is in $O(\log n)$
- [1] SSYNC det.: $\rho^-(n)$ is in $\Omega(\log n)$ for inf. many n
- [2] ASYNC det.: $\rho^-(n) \leq 5$ when n is not a multiple of 5
- [2] SSYNC det.: $\rho^-(n) = 4$ when n is odd.
- [2] SSYNC det.: $\rho^-(n) \geq 5$ if n is even
- [3] ASYNC prob.: $\rho^-(n)$ is in $\Omega(\log n)$ for inf. many n
- [3] SSYNC prob.: $\rho^-(n) = 4$ for any $n > 4$

[1] Flocchini, Ilcinkas, Pelc, Santoro. Algorithmica 2013

[2] Lamani, Potop-Butucaru, Tixeuil. SIROCCO 2010

[3] Devismes, Petit, Tixeuil. TCS 2013

Known results (explo. with stop) 2/3

Limited visibility: snapshot up to distance φ only
[Datta, Lamani, Larmore, Petit]

In the rings, with $\varphi = 1$ (ICDCS 2013)

- SSYNC det.: impossible
- FSYNC det.: $\rho^-(n) \geq 5$
- FSYNC det.: algo. $k = 5$ when initial config. is special

In the rings, with $\varphi = 2$ and 3 (IPDPS workshop 2015)

For $\varphi = 2$:

- ASYNC det.: algo. $k = 7$ when initial config. is special

For $\varphi = 3$:

- ASYNC det.: $\rho^-(n) \leq 7$ for any $n > 21$

Known results (explo. with stop) 3/3

[Flocchini, Ilcinkas, Pelc, Santoro. TCS 2010]

- **Trees max. deg. 3:** $\rho^-(n) \in \Theta(\log n / \log \log n)$
- **Complete ternary trees:** $\rho^-(n) \in \Theta(n)$

[Flocchini, Ilcinkas, Pelc, Santoro. IPL 2011]

- **Line:** iff $k = 3$, $k \geq 5$, or $k = 4$ and n odd

[Devismes, Lamani, Petit, Raymond, Tixeuil. SSS 2012]

- **Grid:** $\rho^-(n) = 3$ (SSYNC/ASYNC)

[Devismes, Lamani, Petit, Tixeuil. NETYS 2015]

- **Torus:** SSYNC $\rho^-(n) = 4$ prob. (≥ 5 det.)

[Chalopin, Flocchini, Mans, Santoro. WG 2010]

- **Arbitrary graphs** with port numbers: rigid initial config.

Focus on Exclusive Perpetual Exploration

Exclusive perpetual exploration (reminder)

- **Each** node must be visited by **every** agent **infinitely often**.
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First studied family: Partial grids with NSEW

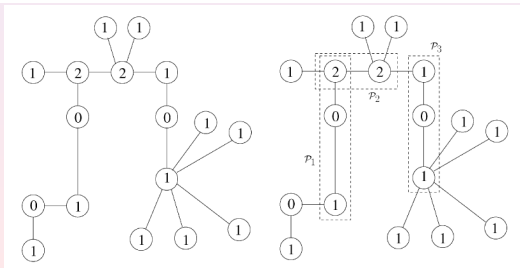
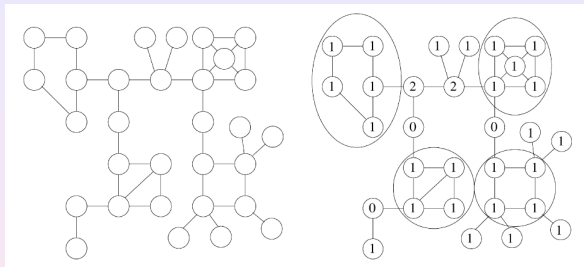
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First studied family: Partial grids with NSEW

Mobility tree and parameter q



Known results (exclusive perpetual explo.) 1/2

In the partial grids (FSYNC)

[Baldoni, Bonnet, Milani, Raynal. IPL 2008]

- Definition of the parameter q
- Solvable iff $k \leq n - q$

[Baldoni, Bonnet, Milani, Raynal. OPODIS 2008]

- Limited visibility $\varphi = 1$
- Solvable iff $k \leq n - q$, except for $q = 0 \implies n - 1$

In the (complete) grids

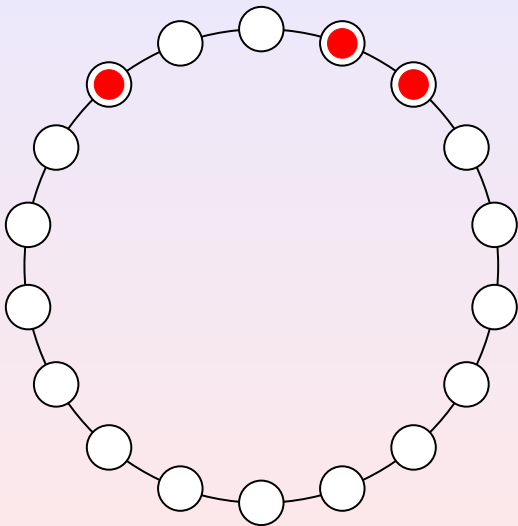
[Bonnet, Milani, Potop-Butucaru, Tixeuil. OPODIS 2011]

- ASYNC, no sense of direction
- Deterministically: $\rho^-(n) = 3$

Algorithmic techniques (exclusive perpetual explo.)

Two phases

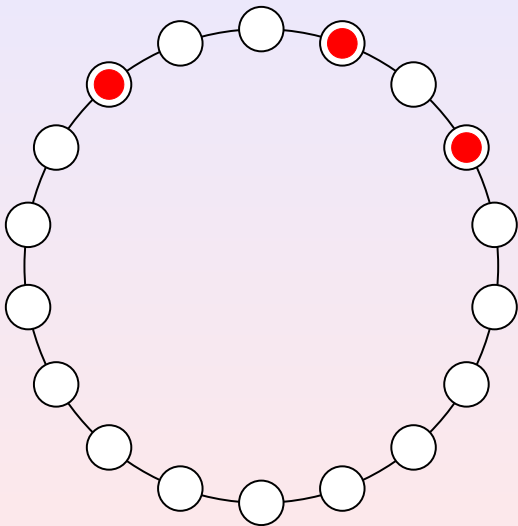
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- Move the pattern through the graph



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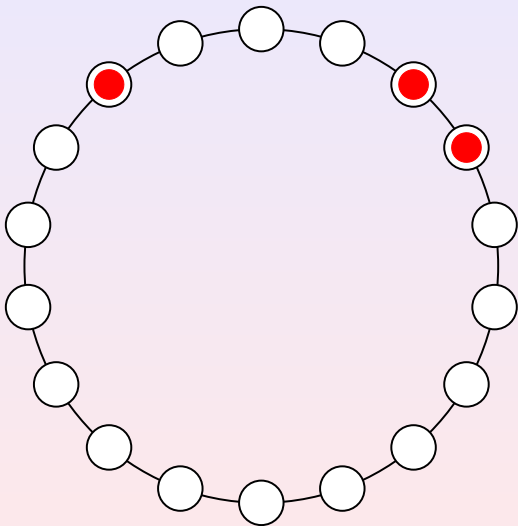
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Algorithmic techniques (exclusive perpetual explo.)

Two phases

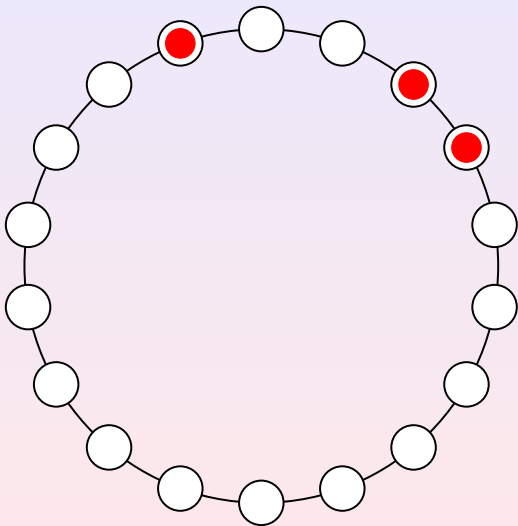
- Form a **special asymmetric pattern**
- Move the pattern through the graph



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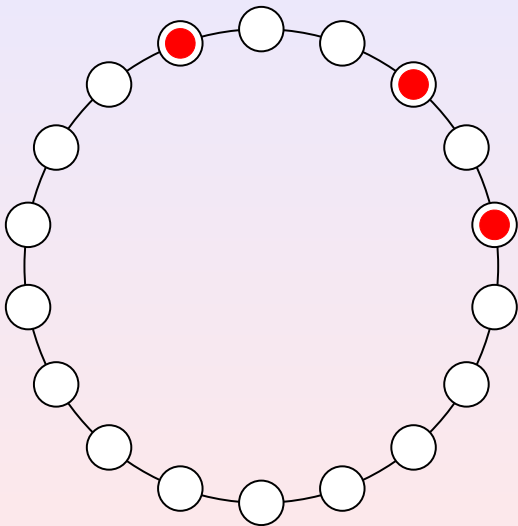
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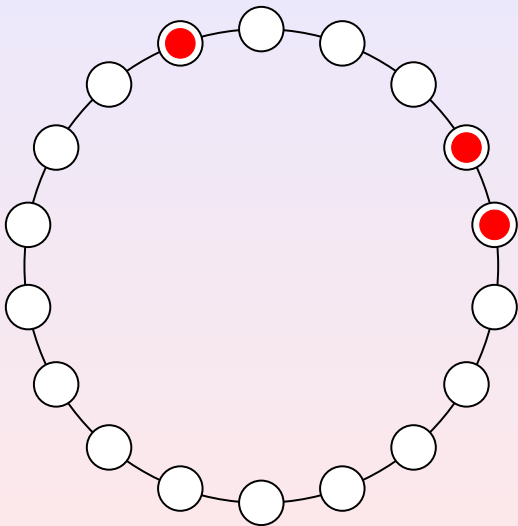
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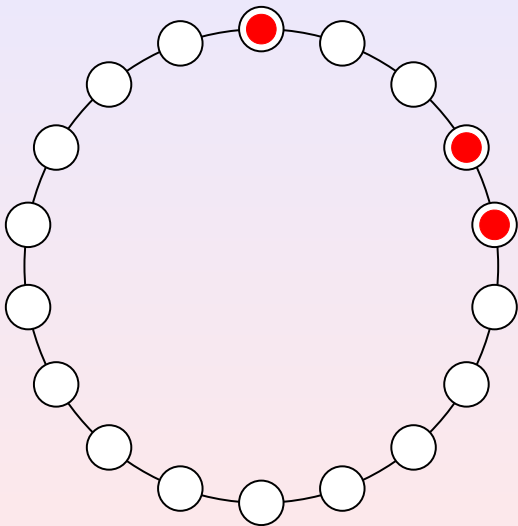
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Known results (exclusive perpetual explo.) 2/2

Case of the rings

[Blin, Milani, Potop-Butucaru, Tixeuil. DISC 2010]

- FSYNC det.: Impossible when k is even
- ASYNC det.: $\rho^-(n) = 3$ for $n \geq 10$ and n and k coprime
- ASYNC det.: $\rho^+(n) = n - 5$ for k odd & n and k coprime

[Bonnet, Défago, Petit, Potop-Butucaru, Tixeuil. SRDS Workshop 2014]

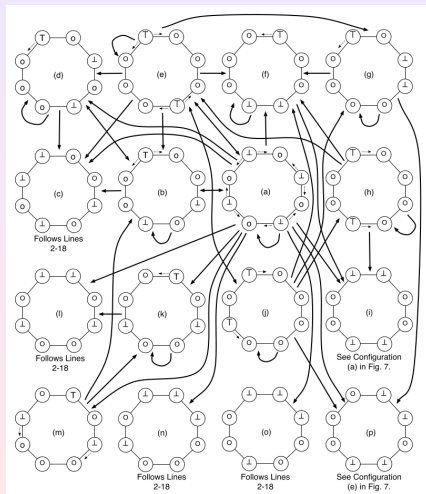
- Generic method to list all protocols (SSYNC)
- Application: impossible for $k = 5$ and $n = 10$

[D'Angelo, Di Stefano, Navarra, Nisse, Suchan. Algorithmica 2015]

- ASYNC but only for rigid initial configurations
- Det. algo for any k, n such that $n \geq 10$ and $5 \leq k \leq n - 3$, except the case $k = 5$ and $n = 10$

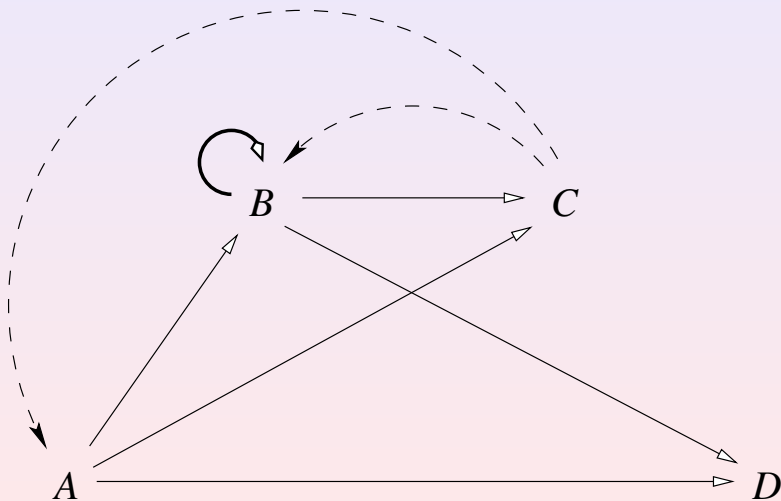
Proofs of correctness (1/2)

Case-by-case analysis



Proofs of correctness (2/2)

DAG with potential functions



Formal verification via model checking

[Bérard, Lafourcade, Millet, Potop-Butucaru, Thierry-Mieg, Tixeuil. Distributed Computing 2016]

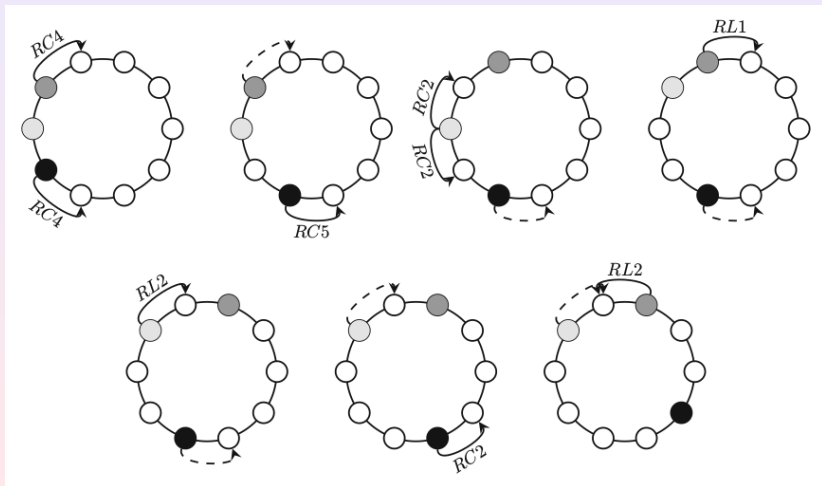
Study of two algorithms

- **Exploration with stop** [FIPS Algorithmica 2013]
 - Correct for $17 \leq k \leq 21$ and $k < n \leq 22$
 - Also correct for smaller values of k (5, 7 and 10)
- **Exclusive perpetual exploration** [BMPT DISC 2010]
 - Counter-example for $k = 3$ and $n = 10$

[Doan, Bonnet, Ogata. SOFG+MSVL 2016]

- Also study of [BMPT DISC 2010]
- **Same counter-example**, with a different technique

The counter-example



Conclusion and perspectives

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- Computability depends on the amount of symmetries
- **Tricky** to (correctly) handle **symmetries and asynchrony**

Interesting perspectives

- Limited visibility
- Formal verification
 - (Parametrized) Model checking
 - Proof assistant

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Thank you
for your attention