Graph Exploration by Automata

David Ilcinkas

LRI, Université Paris-Sud, France

Oct. 4, 2004

Graph exploration

Goal

A mobile entity has to traverse every edge of an unknown anonymous graph.

Motivation

- exploration of environments unreachable by humans
- network maintenance
- map drawing

Graph exploration

Goal

A mobile entity has to traverse every edge of an unknown anonymous graph.

Motivation

- exploration of environments unreachable by humans
- network maintenance
- map drawing

Unknown, anonymous

Unknown

- Unknown topology
- Unknown size (no upper bound)

Anonymous

- No node labeling
- Local edge labeling

Unknown, anonymous

Unknown

- Unknown topology
- Unknown size (no upper bound)

Anonymous

- No node labeling
- Local edge labeling

Unknown, anonymous

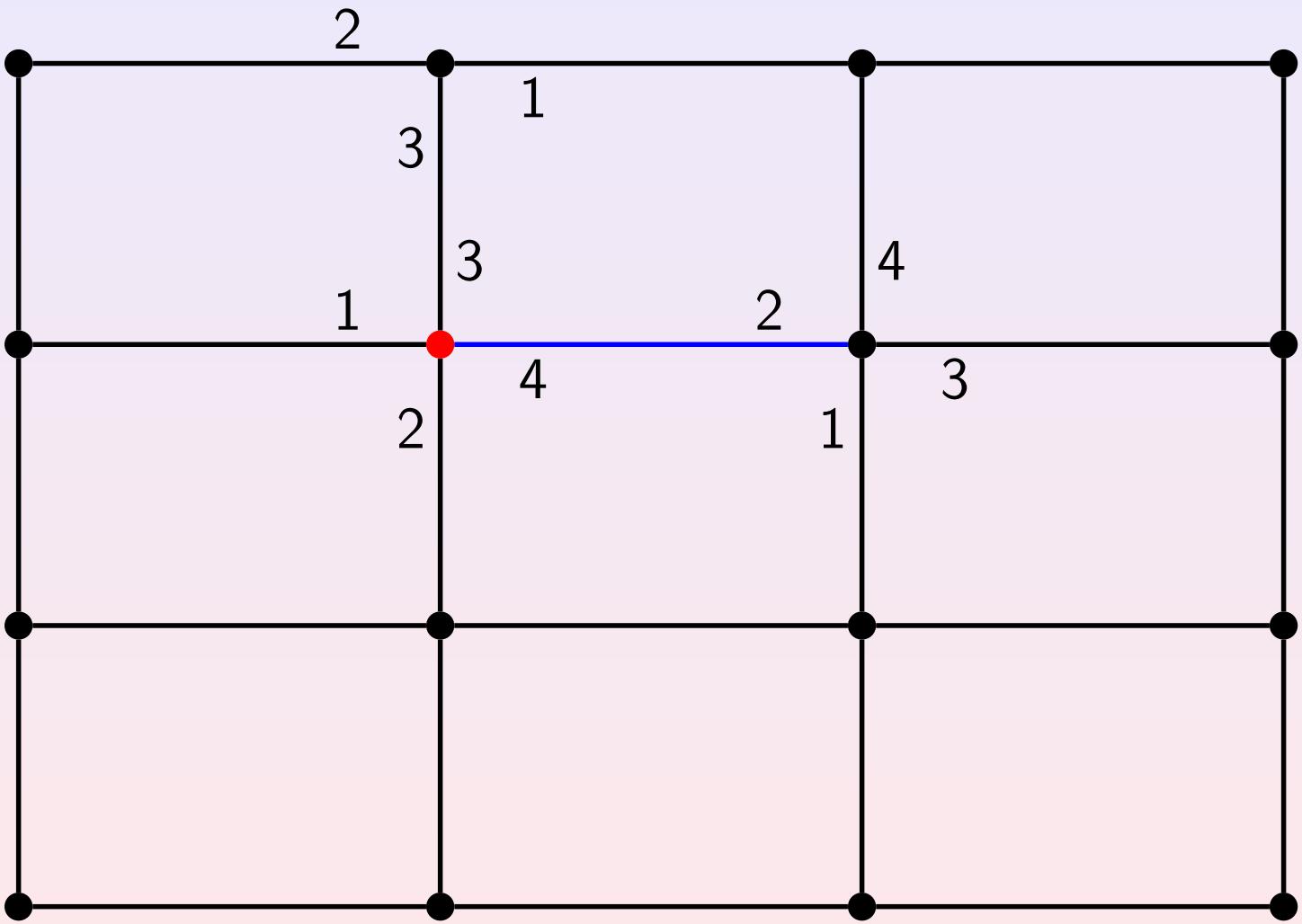
Unknown

- Unknown topology
- Unknown size (no upper bound)

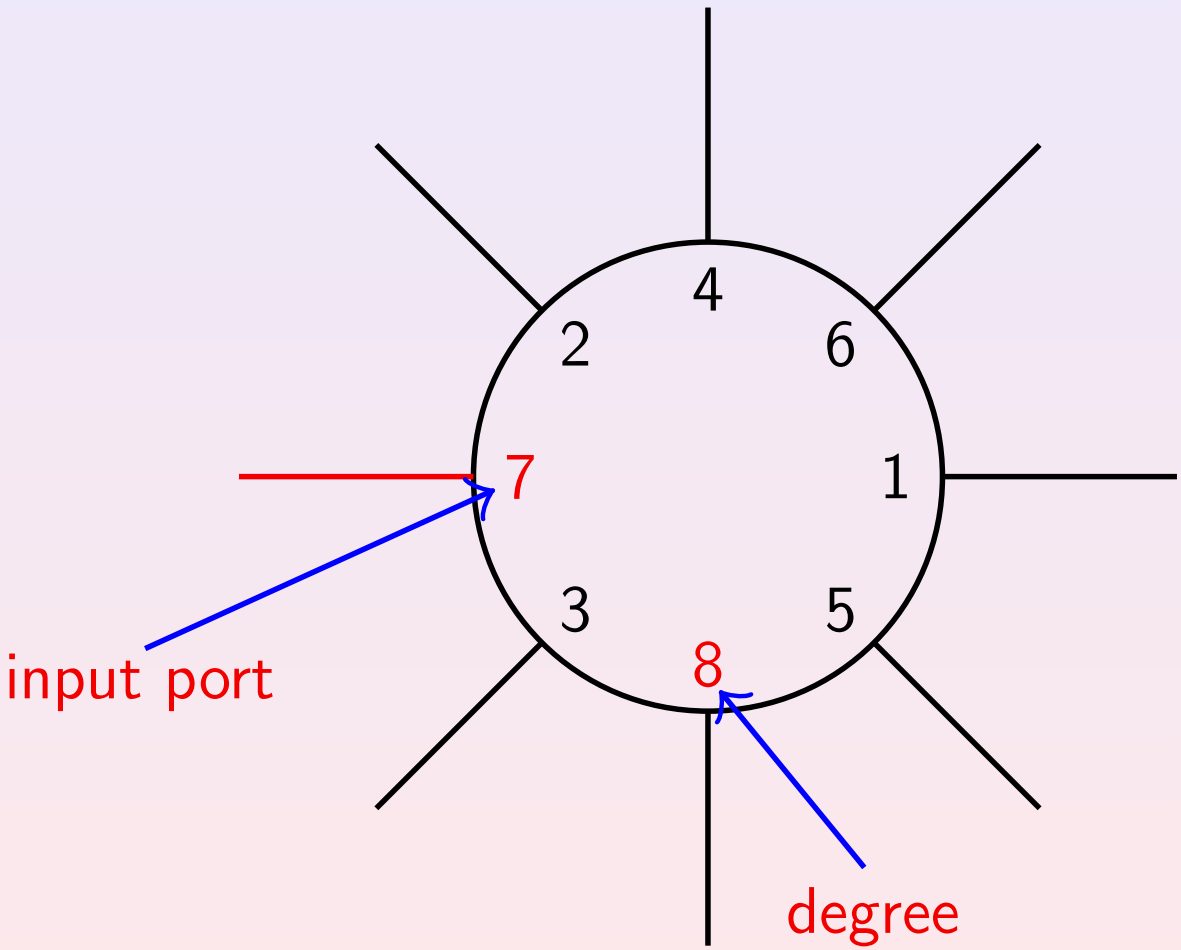
Anonymous

- No node labeling
- Local edge labeling

Example of an anonymous graph



Input of the automaton

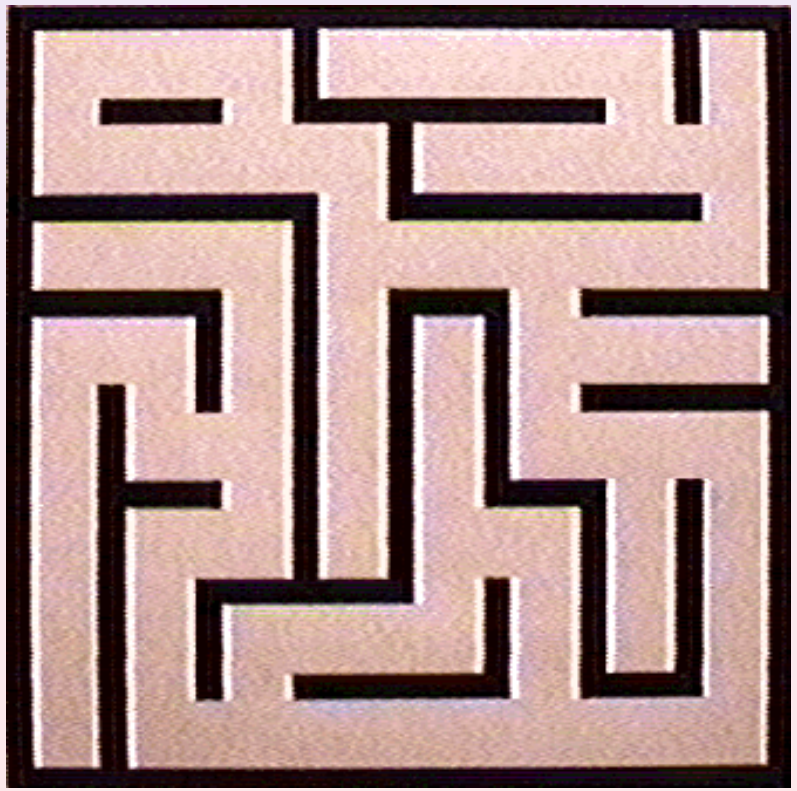


Outline

Particular case: Labyrinths

Definition

- two-dimensional, obstructed chess-board
- directions are known: North, South, East, West



Results on labyrinths

Budach, Math. Nachrichten 1978
Automata and Labyrinths

$\nexists$ universal finite automaton

Blum, Kozen, FOCS 1978
On the power of the compass

$\exists$ a universal two-pebble automaton
$\exists$ two cooperative automata that can explore all labyrinths
$\nexists$ two finite automata that can explore all graphs

Hoffmann, FCT 1981
One pebble does not suffice to search plane labyrinths

$\nexists$ universal one-pebble automaton for labyrinths

Results on labyrinths

Budach, Math. Nachrichten 1978
Automata and Labyrinths

$\nexists$ universal finite automaton

Blum, Kozen, FOCS 1978
On the power of the compass

$\exists$ a universal two-pebble automaton

$\exists$ two cooperative automata that can explore all labyrinths

$\nexists$ two finite automata that can explore all graphs

Hoffmann, FCT 1981
One pebble does not suffice to search plane labyrinths

$\nexists$ universal one-pebble automaton for labyrinths

Results on labyrinths

Budach, Math. Nachrichten 1978
Automata and Labyrinths

$\nexists$ universal finite automaton

Blum, Kozen, FOCS 1978
On the power of the compass

$\exists$ a universal two-pebble automaton
$\exists$ two cooperative automata that can explore all labyrinths
$\nexists$ two finite automata that can explore all graphs

Hoffmann, FCT 1981
One pebble does not suffice to search plane labyrinths

$\nexists$ universal one-pebble automaton for labyrinths

Results on labyrinths

Budach, Math. Nachrichten 1978
Automata and Labyrinths

$\nexists$ universal finite automaton

Blum, Kozen, FOCS 1978
On the power of the compass

$\exists$ a universal two-pebble automaton
$\exists$ two cooperative automata that can explore all labyrinths
$\nexists$ two finite automata that can explore all graphs

Hoffmann, FCT 1981
One pebble does not suffice to search plane labyrinths

$\nexists$ universal one-pebble automaton for labyrinths

Results on labyrinths

Budach, Math. Nachrichten 1978
Automata and Labyrinths

$\nexists$ universal finite automaton

Blum, Kozen, FOCS 1978
On the power of the compass

$\exists$ a universal two-pebble automaton
$\exists$ two cooperative automata that can explore all labyrinths
$\nexists$ two finite automata that can explore all graphs

Hoffmann, FCT 1981
One pebble does not suffice to search plane labyrinths

$\nexists$ universal one-pebble automaton for labyrinths

Arbitrary graphs

Budach, Math. Nachrichten, 1978
Automata and Labyrinths

No finite automaton can explore all labyrinths $\Longrightarrow$ graphs.

Rollik, Acta Informatica, 1980
Automaten in planaren Graphen

No finite team of finite cooperative automata can explore all graphs.

Arbitrary graphs

Budach, Math. Nachrichten, 1978
Automata and Labyrinths

No finite automaton can explore all labyrinths $\Longrightarrow$ graphs.

Rollik, Acta Informatica, 1980
Automaten in planaren Graphen

No finite team of finite cooperative automata can explore all graphs.

How to trap an automaton

Trap for a specific automaton

- finite automaton $\Longrightarrow$ behaviour becomes periodic
- trap the automaton in a cycle

Trap for k non-cooperative automata

- defined recursively
- place a trap for $k-1$ in every edge

Trap for k cooperative automata

- again defined recursively
- place a trap for $k-1$ in every edge
- much more complicated meta-structure

How to trap an automaton

Trap for a specific automaton

- finite automaton $\Longrightarrow$ behaviour becomes periodic
- trap the automaton in a cycle

Trap for k non-cooperative automata

- defined recursively
- place a trap for $k-1$ in every edge

Trap for k cooperative automata

- again defined recursively
- place a trap for $k-1$ in every edge
- much more complicated meta-structure

How to trap an automaton

Trap for a specific automaton

- finite automaton $\Longrightarrow$ behaviour becomes periodic
- trap the automaton in a cycle

Trap for k non-cooperative automata

- defined recursively
- place a trap for $k-1$ in every edge

Trap for k cooperative automata

- again defined recursively
- place a trap for $k-1$ in every edge
- much more complicated meta-structure

Cooperation models

Local cooperation (Rollik)

Robots only communicate if they are at the same node at the same time.

Global cooperation

Robots always communicate.
Especially, robots know when other robots meet.

Cooperation models

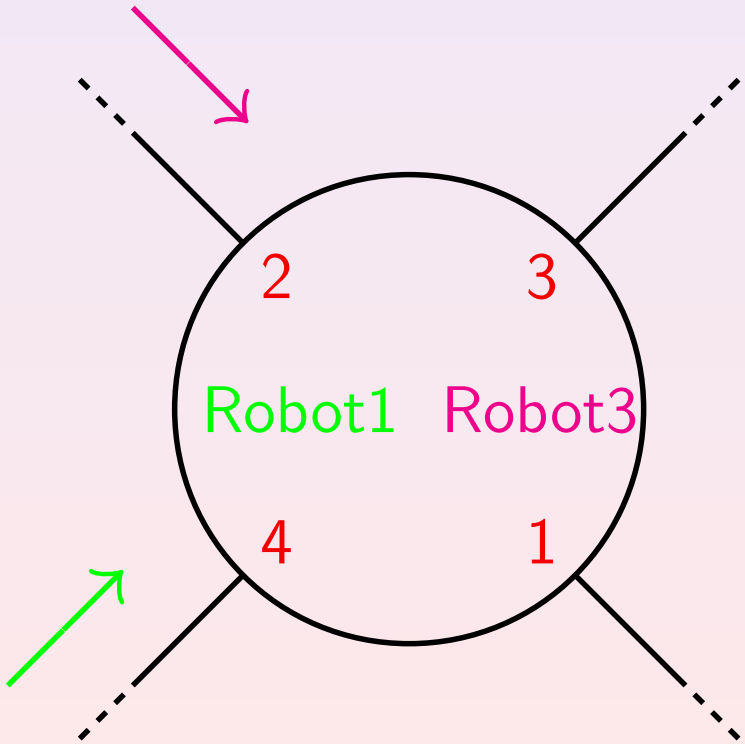
Local cooperation (Rollik)

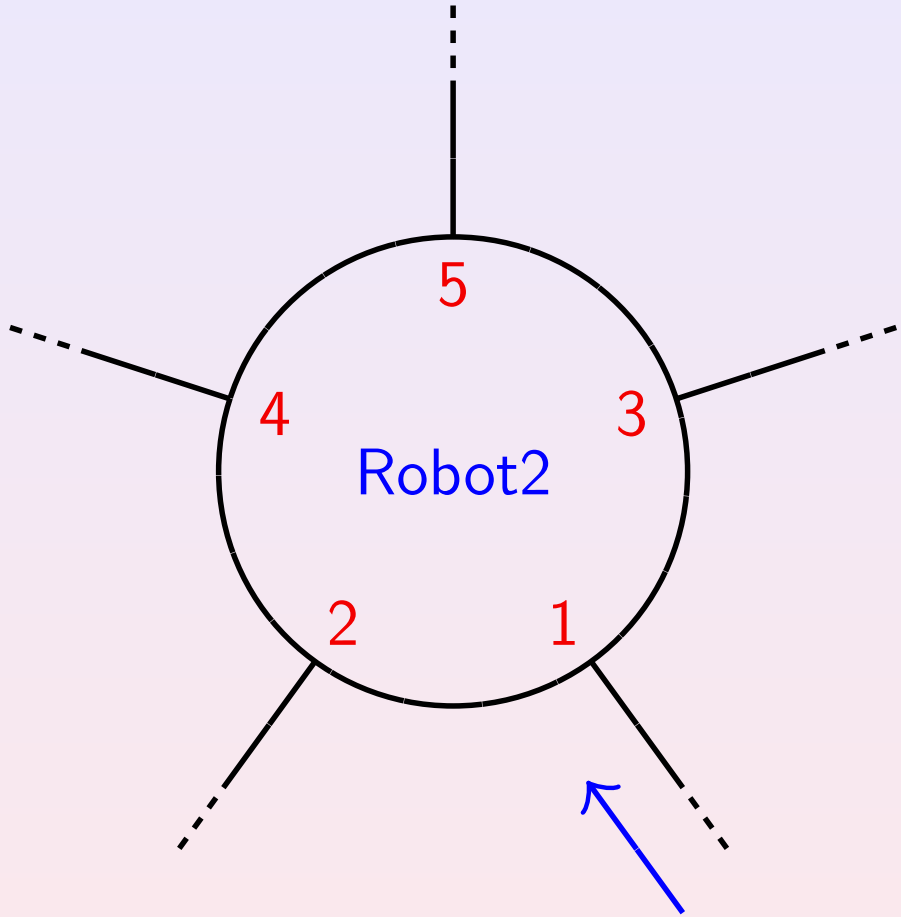
Robots only communicate if they are at the same node at the same time.

Global cooperation

Robots always communicate.
Especially, robots know when other robots meet.

Example





Hydra

Full knowledge $\iff$ Single state

Definition: Hydra

Multi-headed automaton

k globally-cooperative automata $\iff$ k-head hydra

Hydra

Full knowledge $\iff$ Single state

Definition: Hydra

Multi-headed automaton

k globally-cooperative automata $\iff$ k-head hydra

Hydra

Full knowledge $\iff$ Single state

Definition: Hydra

Multi-headed automaton

k globally-cooperative automata $\iff$ k-head hydra

Power of the hydra

Fraigniaud, Ilcinkas, Markou, Pelc

Power of communication in cooperative exploration of graphs

- a 2-head hydra $\not\preceq$ a finite team of automata
- a k-head hydra $\succ$ a team of k locally-cooperative automata
- a 2-head hydra is not universal
- a 3-head hydra $\succ$ a 2-head hydra
- a 3-head hydra is not universal

Conjecture : No hydra is universal

Power of the hydra

Fraigniaud, Ilcinkas, Markou, Pelc

Power of communication in cooperative exploration of graphs

- a 2-head hydra $\not\succ$ a finite team of automata
- a k-head hydra $\succ$ a team of k locally-cooperative automata
- a 2-head hydra is not universal
- a 3-head hydra $\succ$ a 2-head hydra
- a 3-head hydra is not universal

Conjecture : No hydra is universal

Power of the hydra

Fraigniaud, Ilcinkas, Markou, Pelc

Power of communication in cooperative exploration of graphs

- a 2-head hydra $\not\succ$ a finite team of automata
- a k-head hydra $\succ$ a team of k locally-cooperative automata
- a 2-head hydra is not universal
- a 3-head hydra $\succ$ a 2-head hydra
- a 3-head hydra is not universal

Conjecture : No hydra is universal

Power of the hydra

Fraigniaud, Ilcinkas, Markou, Pelc

Power of communication in cooperative exploration of graphs

- a 2-head hydra $\not\succ$ a finite team of automata
- a k-head hydra $\succ$ a team of k locally-cooperative automata
- a 2-head hydra is not universal
- a 3-head hydra $\succ$ a 2-head hydra
- a 3-head hydra is not universal

Conjecture : No hydra is universal

Power of the hydra

Fraigniaud, Ilcinkas, Markou, Pelc

Power of communication in cooperative exploration of graphs

- a 2-head hydra $\not\preceq$ a finite team of automata
- a k-head hydra $\succ$ a team of k locally-cooperative automata
- a 2-head hydra is not universal
- a 3-head hydra $\succ$ a 2-head hydra
- a 3-head hydra is not universal

Conjecture : No hydra is universal

Power of the hydra

Fraigniaud, Ilcinkas, Markou, Pelc

Power of communication in cooperative exploration of graphs

- a 2-head hydra $\not\preceq$ a finite team of automata
- a k-head hydra $\succ$ a team of k locally-cooperative automata
- a 2-head hydra is not universal
- a 3-head hydra $\succ$ a 2-head hydra
- a 3-head hydra is not universal

Conjecture : No hydra is universal

Power of the hydra

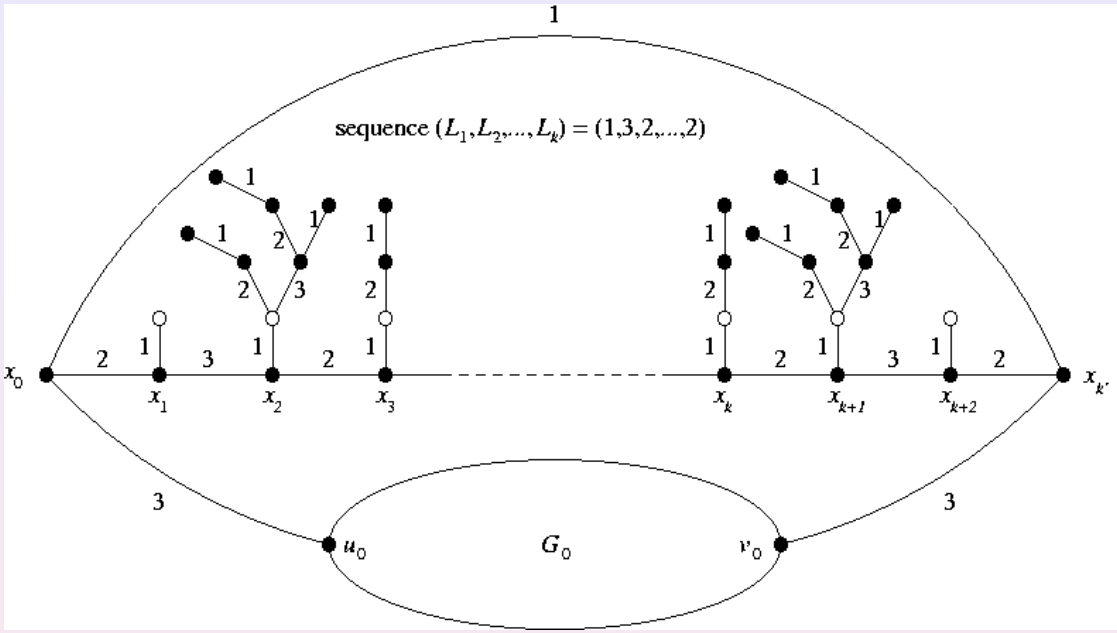
Fraigniaud, Ilcinkas, Markou, Pelc

Power of communication in cooperative exploration of graphs

- a 2-head hydra $\not\succ$ a finite team of automata
- a k-head hydra $\succ$ a team of k locally-cooperative automata
- a 2-head hydra is not universal
- a 3-head hydra $\succ$ a 2-head hydra
- a 3-head hydra is not universal

Conjecture : No hydra is universal

Hydra vs team



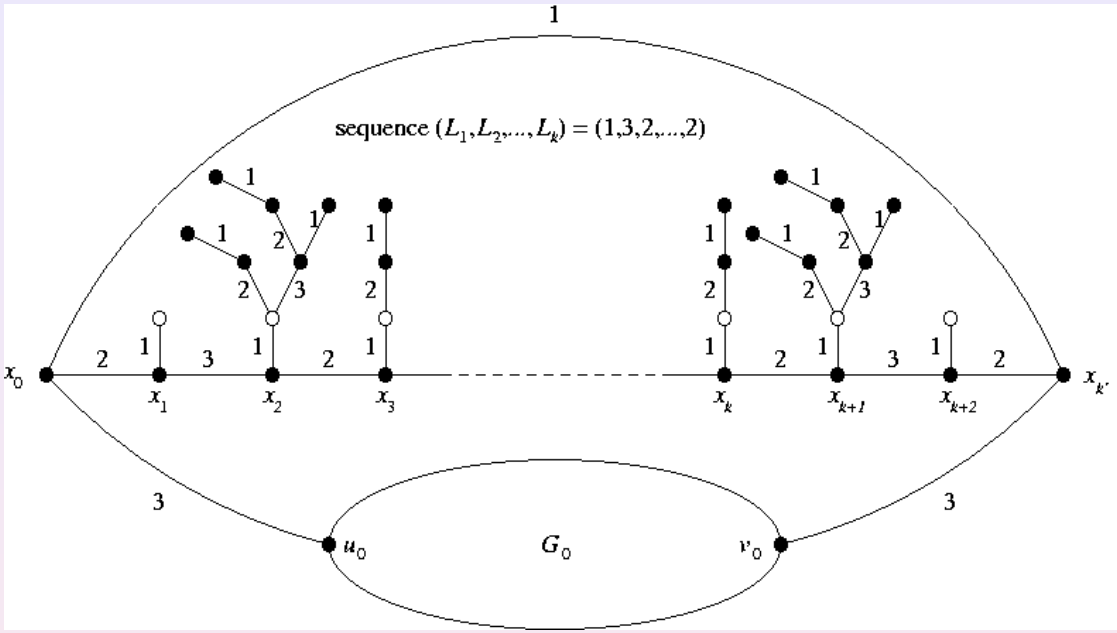
Conclusion

a 2-head hydra $\not\succeq$ a finite team of automata

Corollary

a k-head hydra $\succ$ a team of k locally-cooperative automata

Hydra vs team



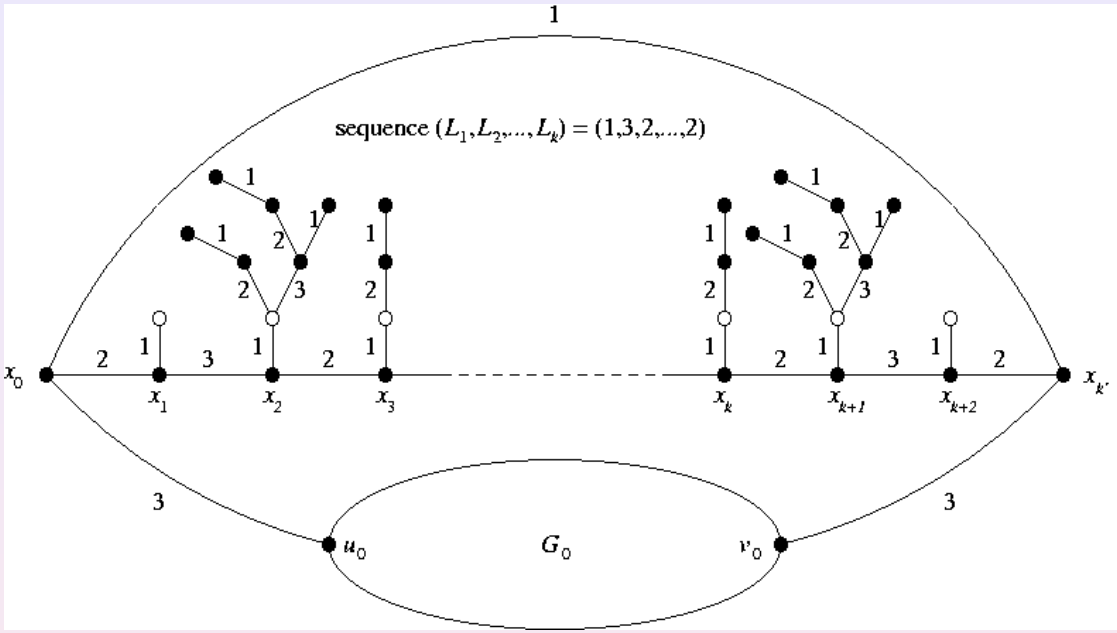
Conclusion

a 2-head hydra $\not\preceq$ a finite team of automata

Corollary

a k-head hydra $\succ$ a team of k locally-cooperative automata

Hydra vs team



Conclusion

a 2-head hydra $\not\preceq$ a finite team of automata

Corollary

a k-head hydra $\succ$ a team of k locally-cooperative automata

Trap for a 2-head hydra

d-homogeneous graph

- d-regular
- edge-colored (same label at both extremities)

In d-homogeneous graphs:
2-head hydra $\equiv$ team of two automata

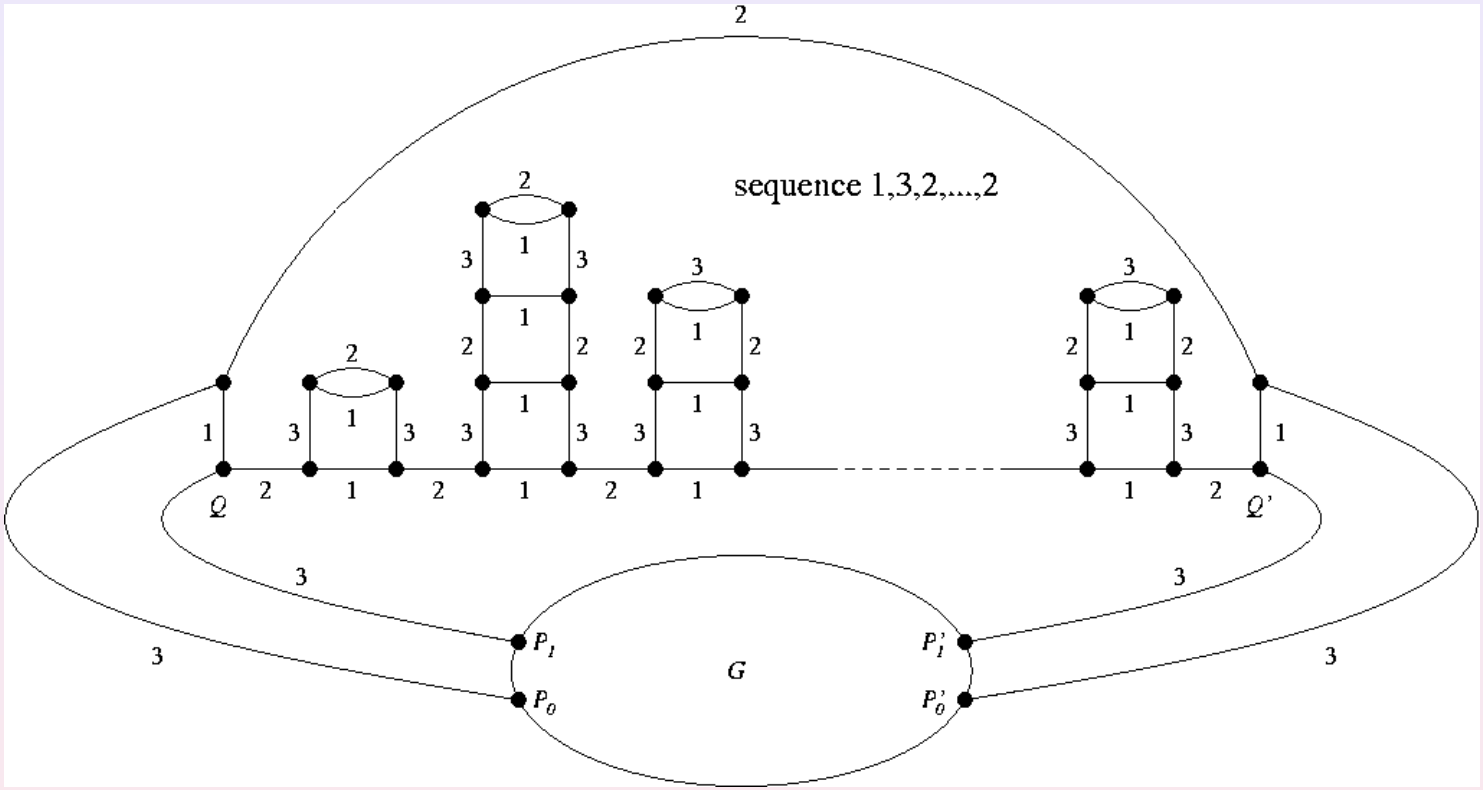
Trap for a 2-head hydra

d-homogeneous graph

- d-regular
- edge-colored (same label at both extremities)

In d-homogeneous graphs:
2-head hydra $\equiv$ team of two automata

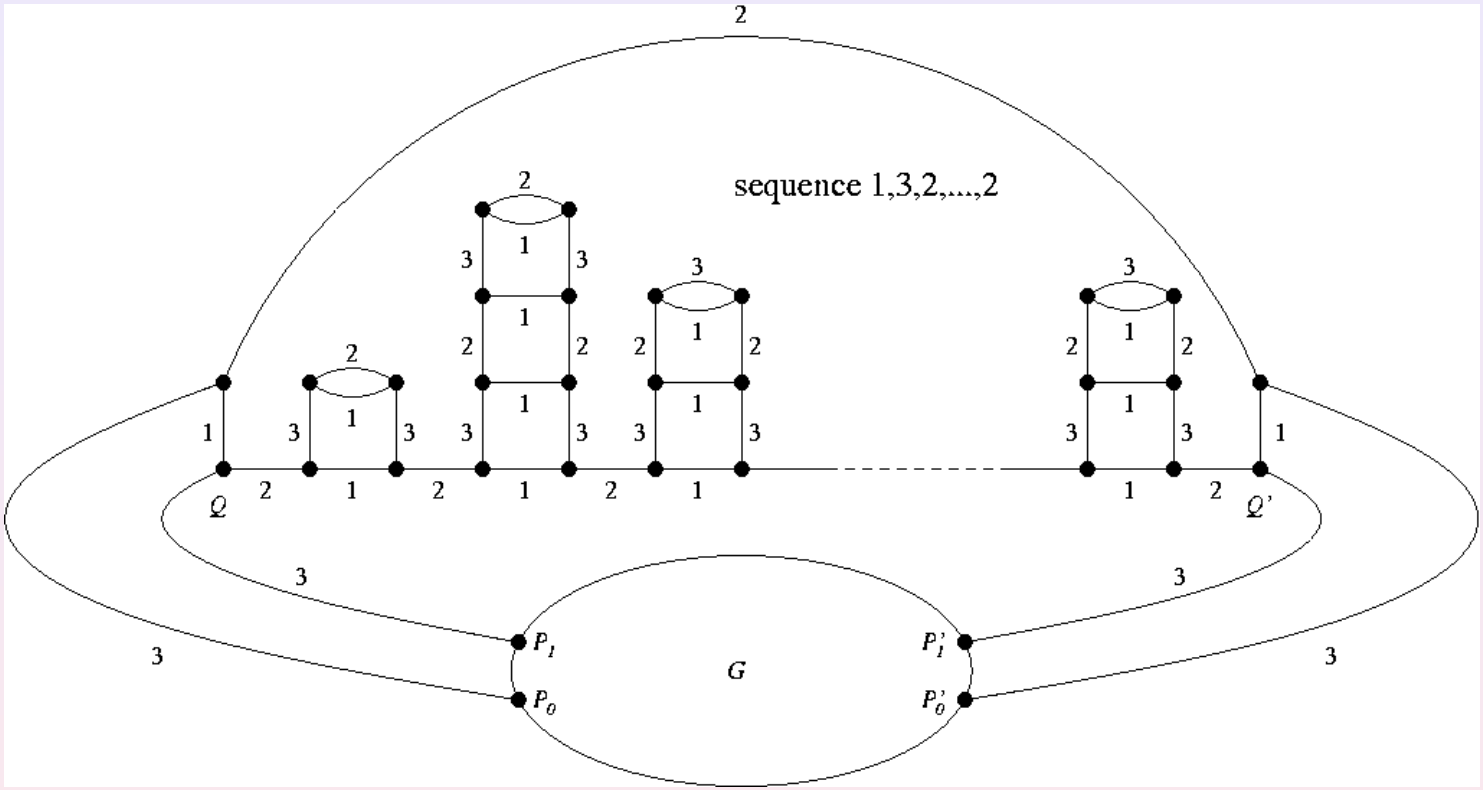
2-head hydra vs 3-head hydra



Conclusion

3-head hydra $\succ$ 2-head hydra

2-head hydra vs 3-head hydra



Conclusion

3-head hydra $\succ$ 2-head hydra

Trap for a 3-head hydra

- If heads are separated
  - Two heads (together or not)
    - The third head does not help
    - Trap for a 2-head hydra
  - One head alone
    - Receive periodic information from the other heads
    - Simulated by a bigger automaton
    - Head trapped in a meta-structure
- If heads are not separated (bounded distance between each other)
  - Simulated by a bigger automaton
  - Heads trapped in a meta-meta-structure

Trap for a 3-head hydra

- If heads are separated
  - Two heads (together or not)
    - The third head does not help
    - Trap for a 2-head hydra
  - One head alone
    - Receive periodic information from the other heads
    - Simulated by a bigger automaton
    - Head trapped in a meta-structure
- If heads are not separated (bounded distance between each other)
  - Simulated by a bigger automaton
  - Heads trapped in a meta-meta-structure

Trap for a 3-head hydra

- If heads are separated
  - Two heads (together or not)
    - The third head does not help
    - Trap for a 2-head hydra
  - One head alone
    - Receive periodic information from the other heads
    - Simulated by a bigger automaton
    - Head trapped in a meta-structure
- If heads are not separated (bounded distance between each other)
  - Simulated by a bigger automaton
  - Heads trapped in a meta-meta-structure

Trap for a 3-head hydra

- If heads are separated
  - Two heads (together or not)
    - The third head does not help
    - Trap for a 2-head hydra
  - One head alone
    - Receive periodic information from the other heads
    - Simulated by a bigger automaton
    - Head trapped in a meta-structure
- If heads are not separated (bounded distance between each other)
  - Simulated by a bigger automaton
  - Heads trapped in a meta-meta-structure

Outline

Different tasks

- Perpetual exploration
- Exploration with stop
  the robot is required to stop after completing exploration
- Exploration with return
  the robot has to stop at its starting node
- Mapping
  the robot has to output an edge-labeled isomorphic copy of the graph

Trees

Diks, Fraigniaud, Kranakis, Pelc, SODA 2002
Tree exploration with little memory

- Perpetual: $\Theta(\log \Delta)$ bits
- With stop: $\Omega(\log \log \log n)$ bits
- With return: $\Omega(\log n)$, $O(\log^2 n)$ bits

Arbitrary graphs

Fraigniaud, Ilcinkas, Peer, Pelc, Peleg, MFCS 2004

Graph exploration by a finite automaton

Perpetual exploration:

- For any K-state automaton, there exists a trap of at most $K + 1$ nodes.
- DFS is space optimal: $\Theta(D \log \Delta)$ bits

DFS is optimal

Algorithm

- Depth-first search (DFS) of increasing depth
- Memory: a stack of port numbers leading to the root
  $\longrightarrow O(D \log \Delta) bits$

Lower bound

Attach a tree to the trap in order to reduce the diameter.

DFS is optimal

Algorithm

- Depth-first search (DFS) of increasing depth
- Memory: a stack of port numbers leading to the root
  $\longrightarrow O(D \log \Delta) bits$

Lower bound

Attach a tree to the trap in order to reduce the diameter.

Outline

Minimizing time

Time = number of edge traversals

Dudek, Jenkin, Milios, Wilkes, IEEE TRA 1991
Robotic exploration as graph construction

- Mapping with pebbles in $O(mn)$

Deng, Mirzaian, IEEE 1996
Competitive robot mapping with homogeneous markers

- Competitive ratio: mapping / map verification
- With a single pebble, DJMW's algorithm is optimal (relaxed depth-one strategies)

Minimizing time

Time = number of edge traversals

Dudek, Jenkin, Milios, Wilkes, IEEE TRA 1991
Robotic exploration as graph construction

- Mapping with pebbles in $O(mn)$

Deng, Mirzaian, IEEE 1996
Competitive robot mapping with homogeneous markers

- Competitive ratio: mapping / map verification
- With a single pebble, DJMW's algorithm is optimal (relaxed depth-one strategies)

A lot of frameworks

- Piecemeal exploration
- Digraphs
- Geometric exploration
- Rooms with obstacles
- UTS, UXS

A lot of frameworks

- Piecemeal exploration
- Digraphs
- Geometric exploration
- Rooms with obstacles
- UTS, UXS

A lot of frameworks

- Piecemeal exploration
- Digraphs
- Geometric exploration
- Rooms with obstacles
- UTS, UXS

A lot of frameworks

- Piecemeal exploration
- Digraphs
- Geometric exploration
- Rooms with obstacles
- UTS, UXS

A lot of frameworks

- Piecemeal exploration
- Digraphs
- Geometric exploration
- Rooms with obstacles
- UTS, UXS

Outline

Coloring nodes

An oracle colors (labels) the nodes to help the automaton.

Basic idea

spanning tree: the label tells which edges are in the tree

Enhanced labeling

only edge leading to the parent $\longrightarrow \Delta$ colors

Coloring nodes

An oracle colors (labels) the nodes to help the automaton.

Basic idea

spanning tree: the label tells which edges are in the tree

Enhanced labeling

only edge leading to the parent $\longrightarrow \Delta$ colors

Coloring nodes

An oracle colors (labels) the nodes to help the automaton.

Basic idea

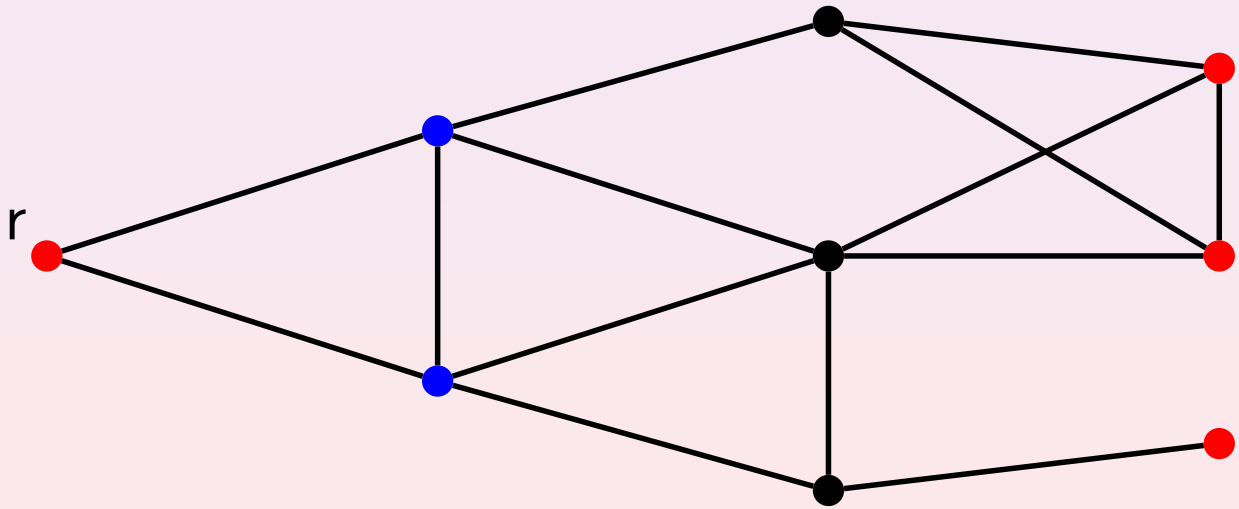
spanning tree: the label tells which edges are in the tree

Enhanced labeling

only edge leading to the parent $\longrightarrow$ Δ colors

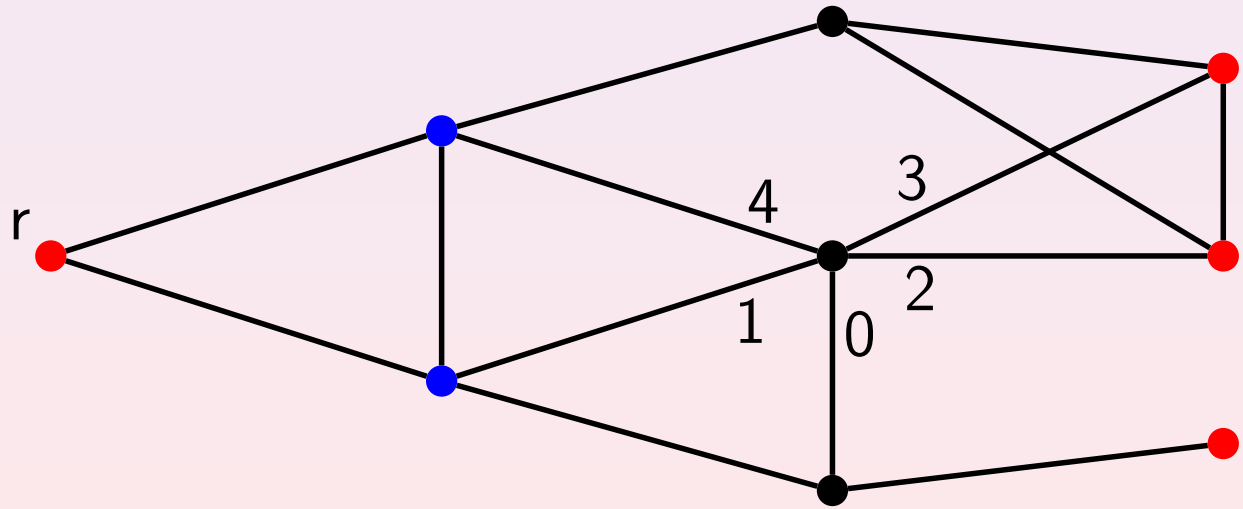
Three colors are enough

- choose arbitrarily a node as the root
- colors all nodes according to their distance d to the root
  - distance $d \cong 0[n]$ red
  - distance $d \cong 1[n]$ blue
  - distance $d \cong 2[n]$ black



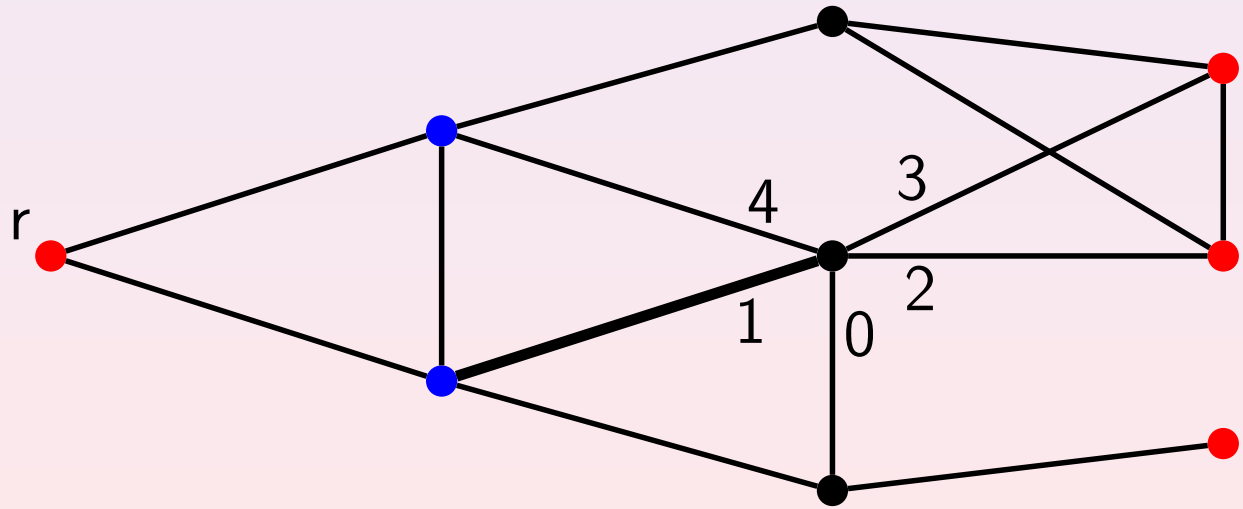
Three colors are enough

- choose arbitrarily a node as the root
- colors all nodes according to their distance d to the root
  - distance $d \cong 0[n]$ red
  - distance $d \cong 1[n]$ blue
  - distance $d \cong 2[n]$ black



Three colors are enough

- choose arbitrarily a node as the root
- colors all nodes according to their distance d to the root
  - distance $d \cong 0[n]$ red
  - distance $d \cong 1[n]$ blue
  - distance $d \cong 2[n]$ black



Only two colors ?

- Layer 1: red
- Layer 2: blue
- Layer 3: red
- Layer 4: red
- Layer 5: red
- Layer 6: blue
- Layer 7: blue
- Layer 8: blue

Outline

Open problems

Hydra

- $\forall k$ $(k+1)$-head hydra $\succ$ k-head hydra ?
- Is there a k such that the k-head hydra is universal ?

USTCON

- Is USTCON in log-space ?
- Better understanding of UTS, UXS, automata.

Open problems

Hydra

- $\forall k$ $(k+1)$-head hydra $\succ$ k-head hydra ?
- Is there a k such that the k-head hydra is universal ?

USTCON

- Is USTCON in log-space ?
- Better understanding of UTS, UXS, automata.