Computing Without Communicating: Ring Exploration by Asynchronous Oblivious Robots

Paola Flocchini¹  David Ilcinkas²
Andrzej Pelc³  Nicola Santoro⁴

¹University of Ottawa, Canada
²CNRS, Université Bordeaux I, France
³Université du Québec en Outaouais, Canada
⁴Carleton University, Canada

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Problem

Model/context
- Team of robots
  - sensing the environment by taking a snapshot of it
  - that do not communicate
  - that are anonymous and oblivious
- Anonymous unoriented rings.

Goal: exploration with stop
- Each node must be visited by at least one robot.
- All robots must stop after finite time.
The Look-Compute-Move cycle

**Look**
The robot takes an **instantaneous snapshot** of the network and its robots, with multiplicity detection ("zero", "one", or "more than one" robots).

**Compute**
Based on this observation, it **decides to stay idle or to move to some neighbouring node**.

**Move**
In the latter case it **instantaneously moves** towards its destination.
Identical oblivious asynchronous robots

**Identical**
Robots have **no IDs**. They execute the **same program**.

**Oblivious**
The robots have **no memory** of observations, computations and moves made in previous cycles.

**Asynchronous**
The time between Look, Compute, and Move operations is **finite but unbounded**.
Some remarks
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Related work

In the plane

Suzuki, Yamashita, SIAMJC’99, semi-synchronous model
Flocchini et al., ISAAC’99, asynchronous model
Cieliebak et al., ICALP’03, gathering assuming multiplicity detection
Prencipe, SIROCCO’05, gathering unsolvable without multiplicity detection
Flocchini et al., TCS’05 - limited visibility
Agmon, Peleg, SODA’04, fault tolerant gathering
Czyzowicz et al., OPODIS’06, gathering of few fat robots

In graphs

Klasing et al., ISAAC’06, gathering in rings

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Ring Exploration by Asynchronous Oblivious Robots
Smallest exploring team

The minimum number of robots that can explore a $n$-node ring is denoted by $\rho(n)$.

Definition

We say that exploration of a $n$-node ring is possible with $k$ robots, if there exists an algorithm enabling the robots to perform exploration with stop starting from any initial configuration of the $k$ robots without multiplicity (at most one robot per node).
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We say that exploration of a $n$-node ring is possible with $k$ robots, if there exists an algorithm enabling the robots to perform exploration with stop starting from any initial configuration of the $k$ robots **without multiplicity** (at most one robot per node).
Our results

**Lemma**

Exploration of a $n$-node ring by $k$ robots is

- impossible if $k|n$ but $k \neq n$;
- possible if $\gcd(n, k) = 1$, for $k \geq 17$.

**Main result**

- $\rho(n) \in \Theta(\log n)$
- There exists a constant $c$ such that, for infinitely many $n$, we have $\rho(n) \geq c \log n$.
- $\rho(n) \in O(\log n)$
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Impossible to stop (and sometimes to explore) when $k|n$. 
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Introducing... Lower bound. Upper bound. Conclusion.
Theorem

There exists a constant $c$ such that, for infinitely many $n$, we have $\rho(n) \geq c \log n$.

Proof

- Let $n$ be the least common multiple of integers $1, 2, \ldots, q$.
- From the previous slide, we have $\rho(n) \geq q + 1$.
- The Prime Number Theorem implies $\frac{\ln n}{q} \to 1$.
- This implies the existence of a constant $c$ such that, for infinitely many $n$, $\rho(n) \geq c \log n$. 

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Ring Exploration by Asynchronous Oblivious Robots
Lower bound (2/2)

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Let $n$ be the least common multiple of integers $1, 2, \ldots, q$. From the previous slide, we have $\rho(n) \geq q + 1$. 

The Prime Number Theorem implies $\ln n \sim q \to 1$. This implies the existence of a constant $c$ such that, for infinitely many $n$, $\rho(n) \geq c \log n$. 

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**Upper bound**

**Lemma**

Exploration is possible if $\gcd(n, k) = 1$, for $k \geq 17$.

**Theorem**

The size $\rho(n)$ of the smallest exploring team is in $O(\log n)$.

**Proof**

Let $p_j$ be the $j$-th prime, and $p_j\# = \prod_{i=1}^{j} p_i$ the $p_j$-primorial.

- Take $j$ such that $\frac{p_j\#}{13\#} \leq n < \frac{p_{j+1}\#}{13\#}$. We have $\rho(n) \leq p_{j+1}$.
  
  (all primes in $\{17, \ldots, p_{j+1}\}$ divide $n \implies n \geq \frac{p_{j+1}\#}{13\#}$)

- From [Ruiz, Math. Gaz. '97], we have $\frac{\ln(p_j\#)}{p_j} \to 1$.

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Some definitions

**Interdistance**
Minimum distance taken over all pairs of distinct robots.

Here interdistance=2.

**Block**
Maximal set of robots, of size at least 2, forming a line with a robot every \( d \) nodes. \( (d = \text{interdistance}) \)
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Our algorithm

**Set-Up Phase**

**Goal:** to transform the (arbitrary) initial configuration into a configuration of interdistance 1 where there is a single block or two blocks of the same size.

**Method:** decrease the number of blocks whenever possible. Otherwise, decrease the interdistance.

**Tower-Creation Phase**

**Goal:** to create one or two multiplicities inside each block; furthermore a number of robots become uniquely identified as explorers.

**Exploration Phase**

**Goal:** to perform exploration thanks to the explorers until reaching an identified final configuration.
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Conclusion and perspectives

Open problem

- Cycles: $\Theta(\log n)$
- Trees of maximum degree 3: $\Theta(\log n / \log \log n)$
- Arbitrary trees: $\Theta(n)$

What about arbitrary graphs (of maximum degree 3)?

Perspectives

- Limited visibility
- Fault tolerant protocols
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Thank You
for your attention