

# Optimal Exploration of Terrains with Obstacles

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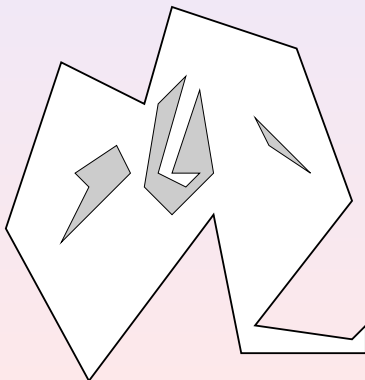
<sup>2</sup>CNRS & Univ. de Bordeaux, France

Séminaire "Algorithmique distribuée", LaBRI  
May 3, 2010

# The problem

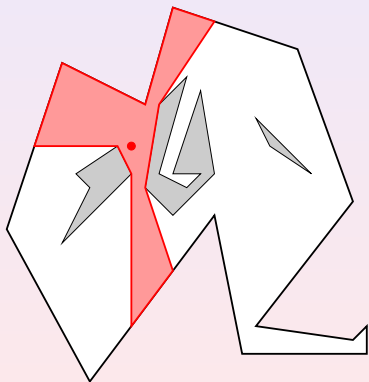
## Exploration of terrains with obstacles

A mobile robot has to **explore/see all points** of an **unknown** bounded terrain, possibly with obstacles.



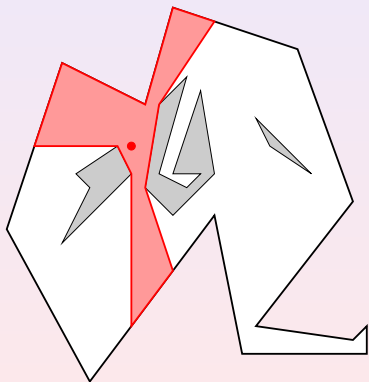
# Vision

## Unlimited vision

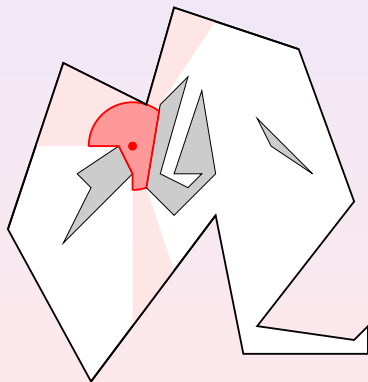


# Vision

Unlimited vision



Limited vision



# Motivations

## Motivations / Applications

### Exploration / searching / mapping of

- unsafe environments
- unreachable environments

### Related (off-line) problems

- with unlimited vision
  - Gallery tour problem (NP-hard)
  - Watchman's route (polynomial)
- with limited vision
  - Lawn mowing problem (NP-hard)
  - Pocket milling problem (NP-hard)

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# Performance measures

Algorithm  $\mathcal{A}$  having **no a priori knowledge** on the terrain.

Worst-case complexity

Worst-case length of  $\mathcal{A}$ 's trajectory in the family of terrains of bounded parameters

- $P$ : total perimeter
- $A$ : area of the terrain
- $k$ : number of obstacles

Competitive ratio

$\max_T \frac{\text{length of } \mathcal{A}'\text{s trajectory}}{\text{length of an optimal algorithm knowing the terrain } T}$



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# Related work (unlimited vision)

## Rectilinear polygon without obstacles

- [Deng, Kameda, Papadimitriou, 1998]: competitive ratio 2
- [Hammar, Nilsson, Schuierer, 2002]: competitive ratio  $\frac{5}{3}$
- [Hammar, Nilsson, Persson, 2006]: competitive ratio  $\frac{3}{2}$
- [Kleinberg, 1994]: (randomized) competitive ratio  $\frac{5}{4}$   
no deterministic algorithm with competitive ratio  $< \frac{5}{4}$

## Polygonal terrain with obstacles

- [Kalyanasundaram, Pruhs, 1993]: competitive ratio  $O(k)$   
( $O(\sqrt{k})$  in some special cases)
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- Y. Gabriely, E. Rimon, Int. Conf. of Robotics and Automaton (ICRA), 2001.
- Y. Gabriely, E. Rimon, Computational Geometry: Theory and Applications, 2003.
- S.K. Ghosh, J.W. Burdick, A. Bhattacharya, S. Sarkar, Robotics & Automation Magazine, 2008.
- C. Icking, T. Kamphans, R. Klein, E. Langetepe, European Workshop on Computational Geometry, 2000.
- A. Kolenderska, A. Kosowski, M. Malafiejski, P. Zylinski, SIROCCO, 2009.
- S. Ntafos, Comput. Geom. Theory Appl., 1992.

# Our results

We consider the **worst-case trajectory length** measure.

- $P$ : total **perimeter**
- $D$ : **diameter** of the (convex hull of the) terrain
- $A$ : **area** of the terrain
- $k$ : number of **obstacles**

## Unlimited vision

- $\Omega(P + D\sqrt{k})$ , even if the terrain is a priori known
- $O(P + D\sqrt{k})$ , constructive proof (algorithm)

## Limited vision

- $\Omega(P + A + \sqrt{Ak})$ , even if the terrain is a priori known
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# Outline

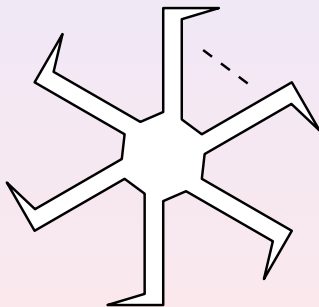
- 1 Introduction
- 2 Unlimited vision**
  - Lower bound
  - Upper bound
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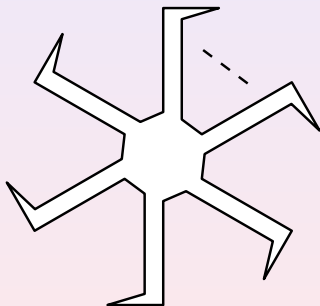
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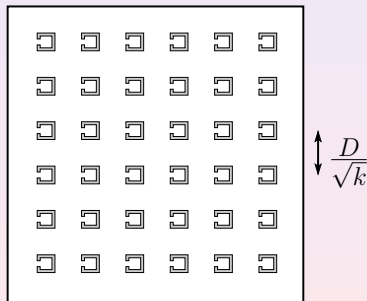
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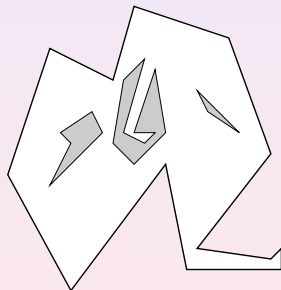
Lower bound  $\Omega(D\sqrt{k})$

# Upper bound

Upper bound  $O(P + D \cdot k)$ , without a priori knowledge.  
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## Naive algorithm

- Reach the boundary
- Follow/explore the boundary
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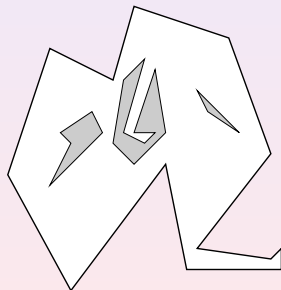


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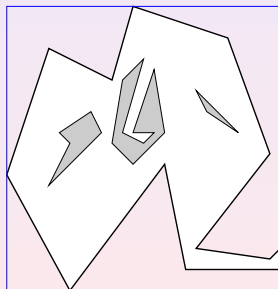


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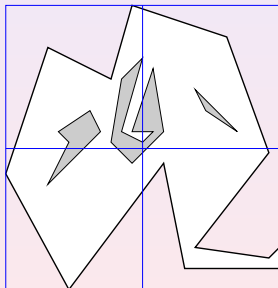


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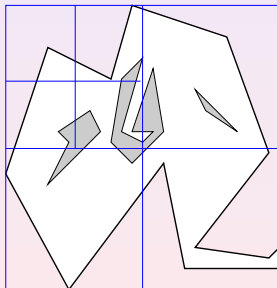


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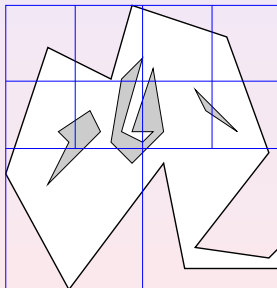


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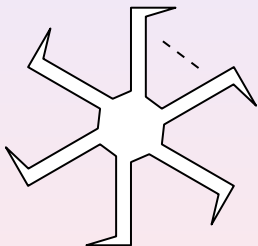


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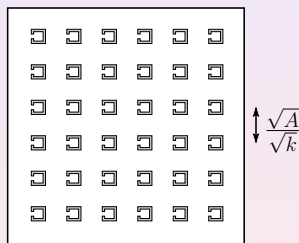
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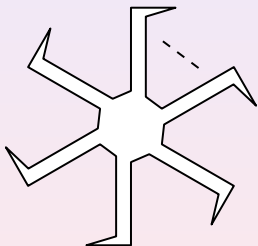
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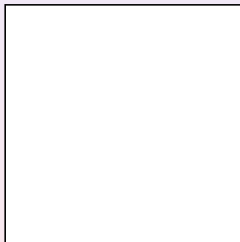
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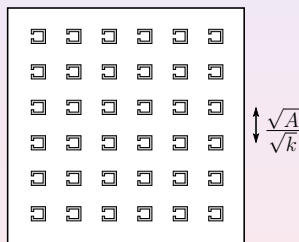
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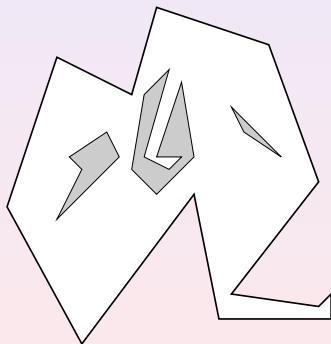
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# Upper bound (1)

Basic idea: use the unlimited vision algorithm

## Algorithm LET( $F$ )

- Reach the boundary
- Follow/explore the boundary
- **Partition** the terrain **in cells**  
(of side  $F \leq 1$ )
- $\forall$  cell in **DFS** order
  - Explore the cell

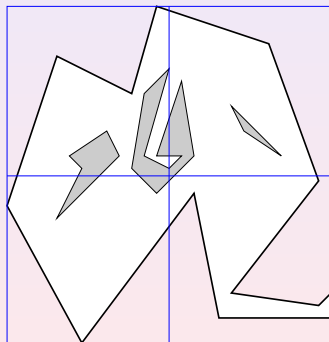


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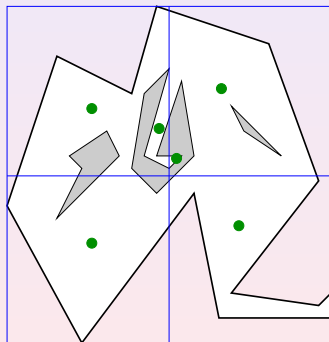


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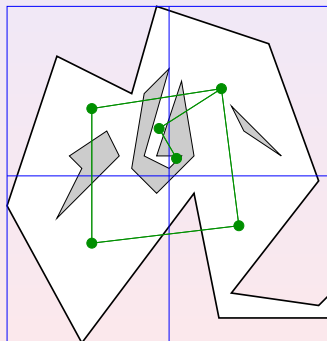


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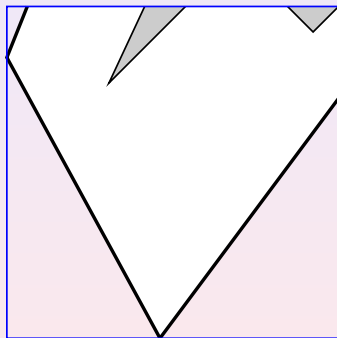


# Cell exploration

Basic idea: use the unlimited vision **naive** algorithm

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- Reach the boundary of **cell c**
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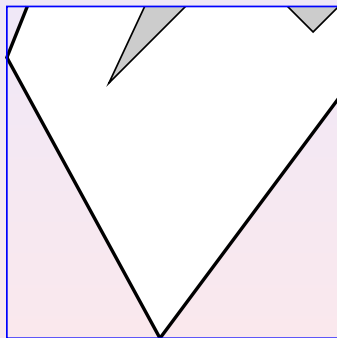
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# Upper bound (2)

Complexity of  $\text{LET}(F)$ :  $O(P + A/F + k \cdot F)$

Goal (lower bound):  $\Omega(P + A + \sqrt{A \cdot k})$

Solutions intuition: choose  $F = \min\{1, \sqrt{\frac{A}{k}}\}$

If only  $A$  is known (case when  $k$  known is similar):

Algorithm  $\text{LET}_A$

- Set  $F = 1$
- Apply  $\text{LET}(F)$  until the error is “too” large
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# Complexity analysis of $LET_A$ ( $A$ known)

$P^*, k^*$ : current discovered values of  $P$  and  $k$

$P_j, k_j$ : values of  $P^*, k^*$  at the end of Phase  $j$

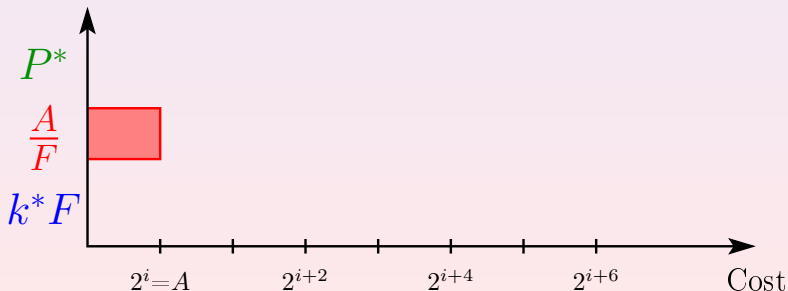
- $F_1 = 1$
- Stopping condition:  $\{k^*F_j \geq 2\frac{A}{F_j}$  and  $k^*F_j \geq P^*\}$
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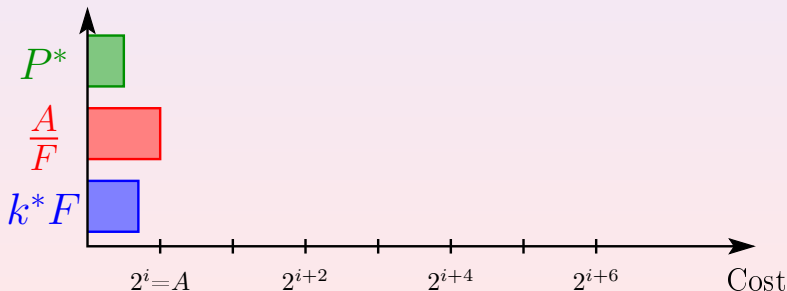


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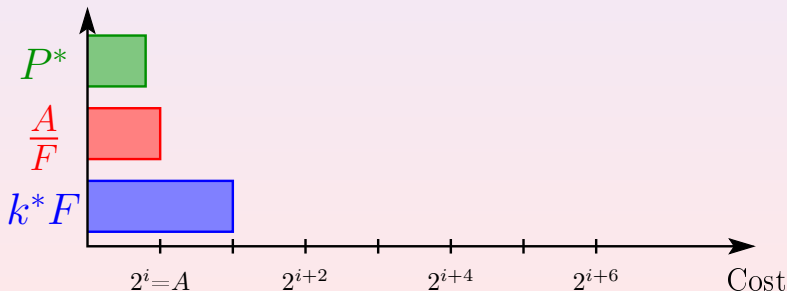


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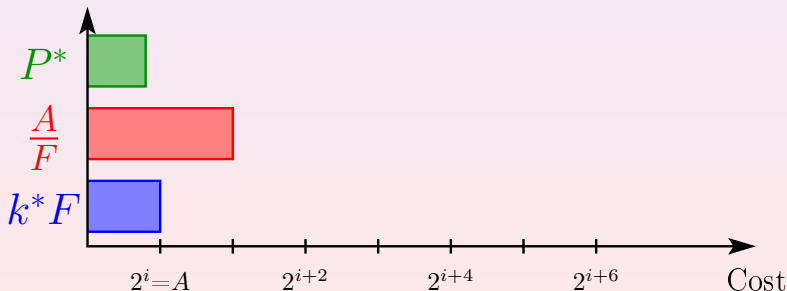


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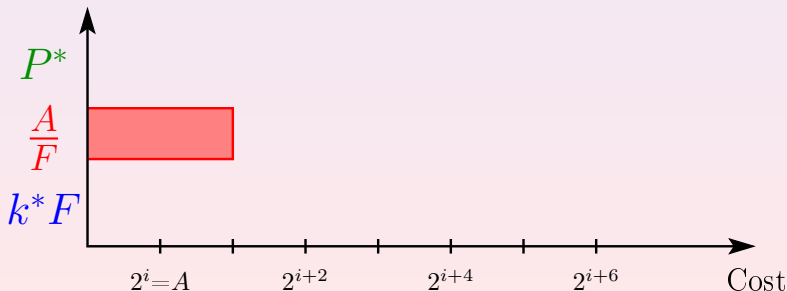


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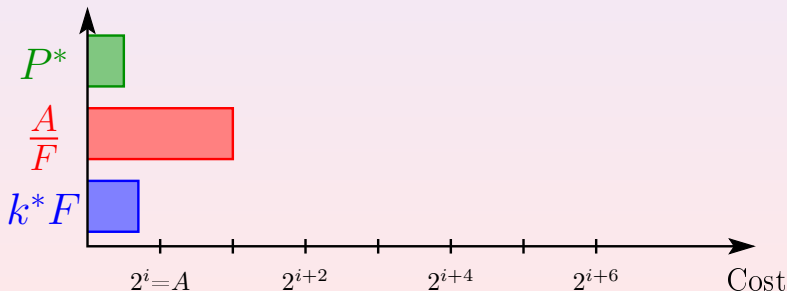


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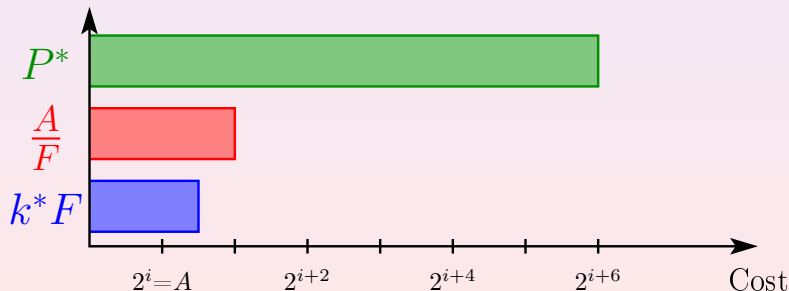


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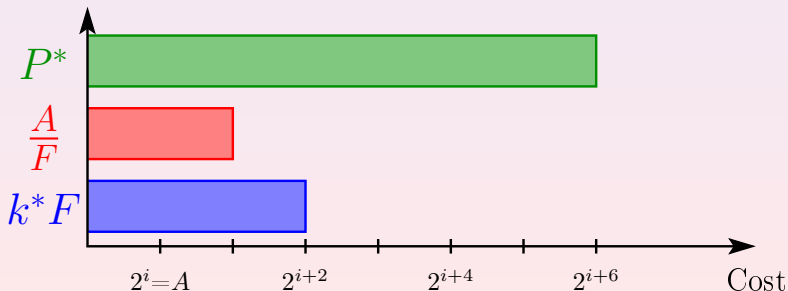


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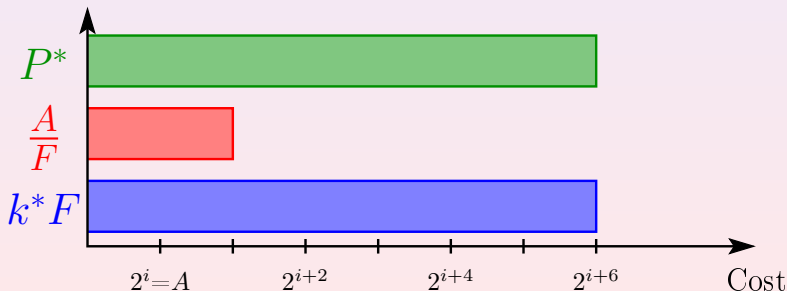


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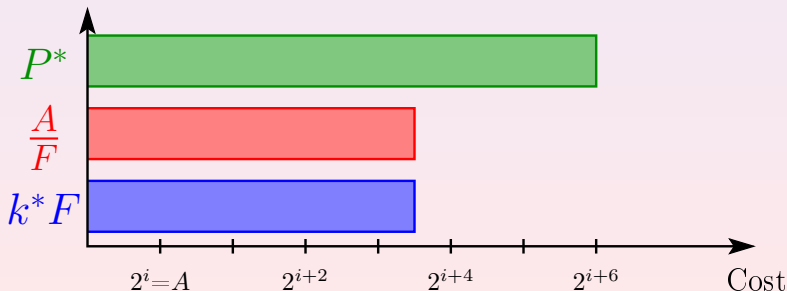


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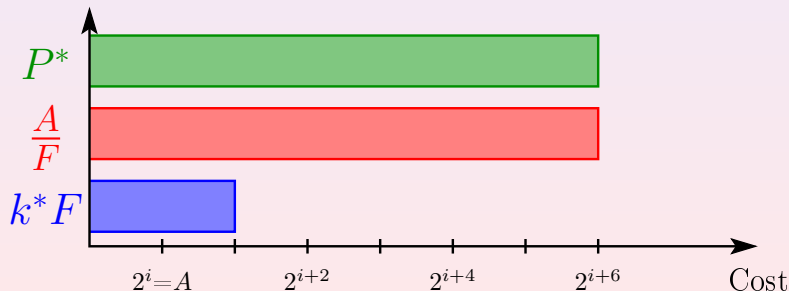


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# Outline

- 1 Introduction
- 2 Unlimited vision
- 3 Limited vision
- 4 Conclusion**

# Conclusion and perspectives

(Reminder:  $P$  = perimeter;  $A$  = area;  $k$  = # obstacles)

## Limited vision

- $\Omega(P + A + \sqrt{Ak})$ , even if the terrain is a priori known
- $O(P + A + \sqrt{Ak})$ , if  $k$  or  $A$  is known (algorithm)
- $O(\min\{\log A, \log k\}(P + A + \sqrt{Ak}))$ , nothing known

## Open problem

- *Worst-case complexity without knowledge?*
- *Competitive ratio in any polygonal terrain with polygonal obstacles (limited and unlimited)?*

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Thank You  
for your attention