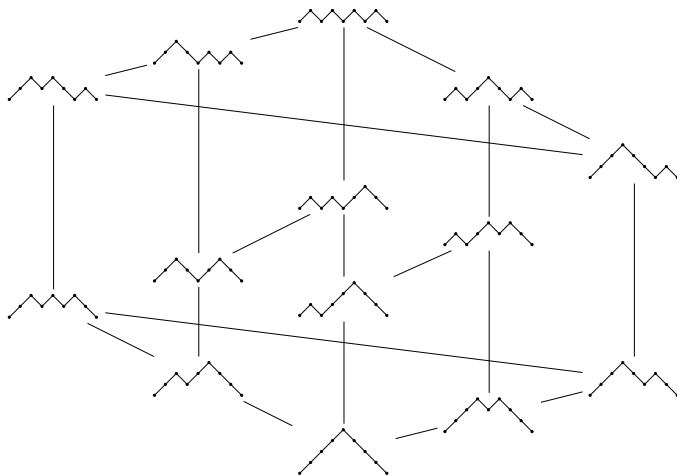


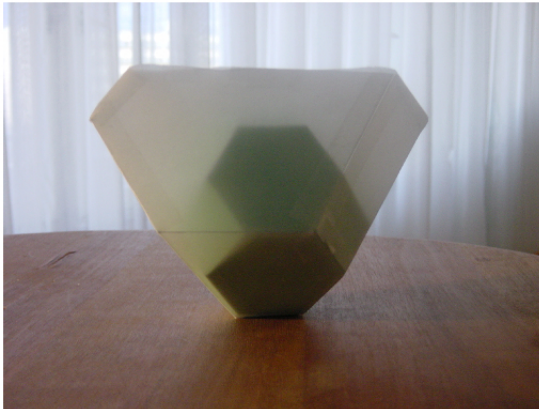
Intervalles du treillis de Tamari

Viviane Pons

Universität Wien

JCB 2014





(image due à Jean-Louis Loday)

Treillis de Tamari

Treillis de Tamari

- ▶ 1962, Tamari : ordre partiel sur les parenthésages formels

Treillis de Tamari

- ▶ 1962, Tamari : ordre partiel sur les parenthésages formels
- ▶ 1972, Huang, Tamari : structure de treillis

Treillis de Tamari

- ▶ 1962, Tamari : ordre partiel sur les parenthésages formels
- ▶ 1972, Huang, Tamari : structure de treillis
- ▶ 2007, Chapoton : nombre d'intervalles

$$\frac{2}{n(n+1)} \binom{4n+1}{n-1}$$

Treillis de m -Tamari

Treillis de m -Tamari

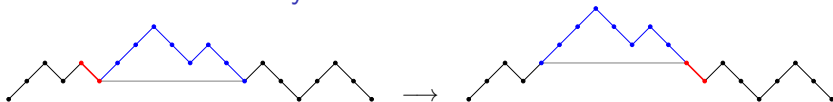
- ▶ Bergeron, Préville-Ratelle : Posets de m -Tamari

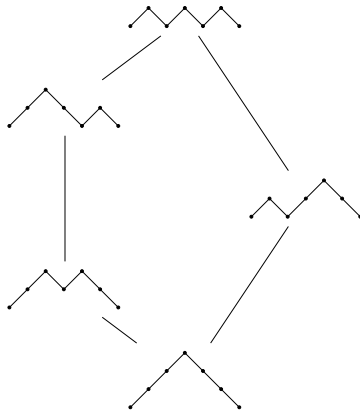
Treillis de m -Tamari

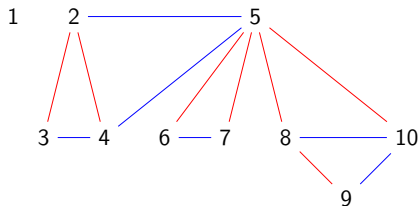
- ▶ Bergeron, Préville-Ratelle : Posets de m -Tamari
- ▶ Bousquet-Mélou, Fusy, Préville-Ratelle : structure de treillis et comptage des intervalles

$$\frac{m+1}{n(mn+1)} \binom{(m+1)^2 n + m}{n-1}$$

Rotation chemins de Dyck





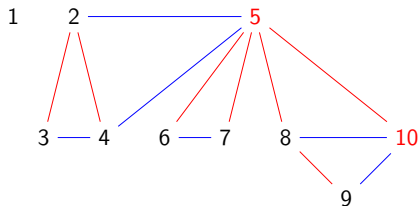


Définition

Un intervalle-poset est un poset de taille n étiqueté par $1, \dots, n$ tel que

- ▶ Si $a < c$ et $a \triangleleft c$ alors $b \triangleleft c$ pour tout $a < b < c$.
- ▶ Si $a < c$ et $c \triangleleft a$ alors $b \triangleleft a$ pour tout $a < b < c$.

On note $a \triangleleft b$ pour a inférieure à b dans le poset.

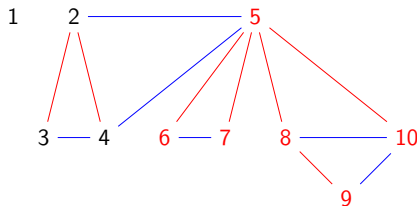


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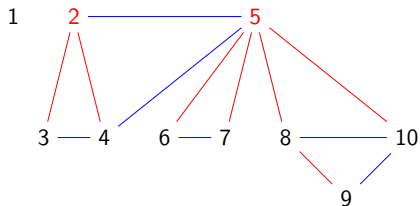


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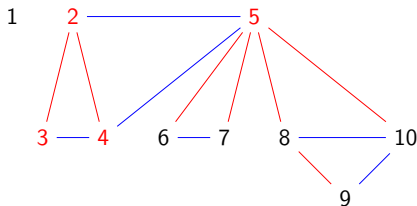


Définition

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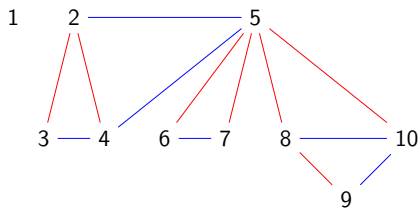


Définition

Un intervalle-poset est un poset de taille n étiqueté par $1, \dots, n$ tel que

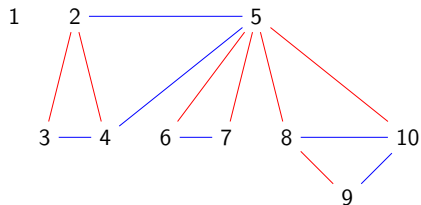
- ▶ Si $a < c$ et $a \triangleleft c$ alors $b \triangleleft c$ pour tout $a < b < c$.
- ▶ Si $a < c$ et $c \triangleleft a$ alors $b \triangleleft a$ pour tout $a < b < c$.

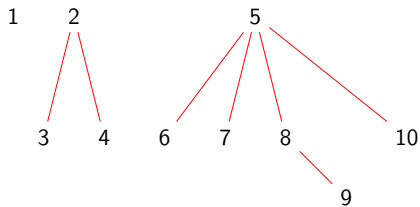
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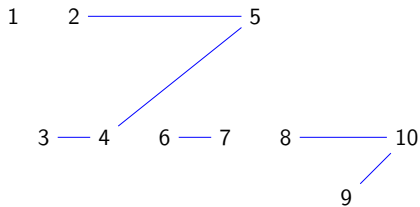


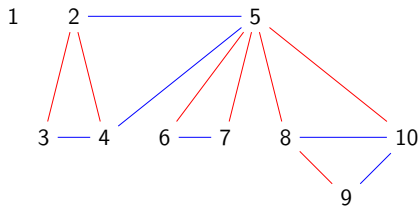
Théorème (Châtel, P.)

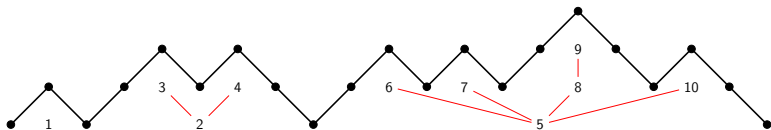
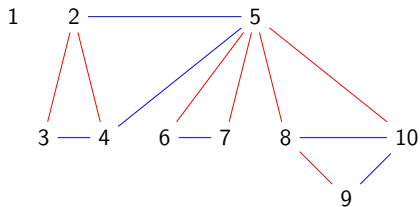
Les intervalles-posets sont en bijection avec les intervalles du treillis de Tamari.

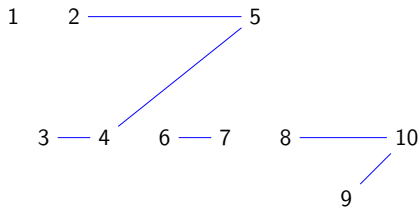


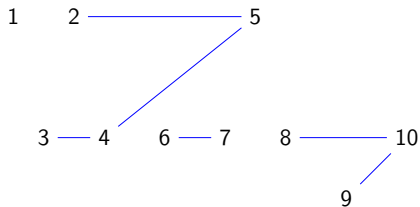


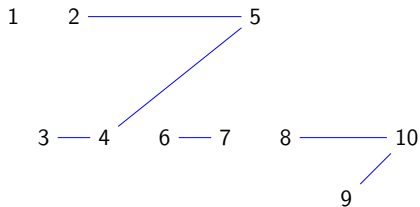


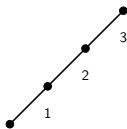
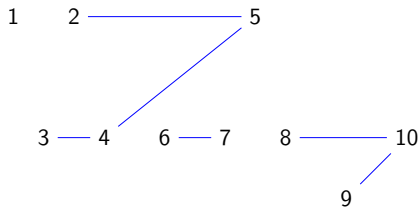


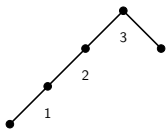
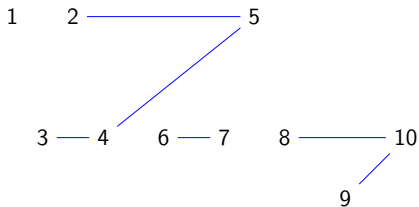


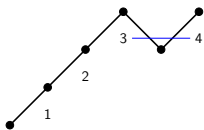
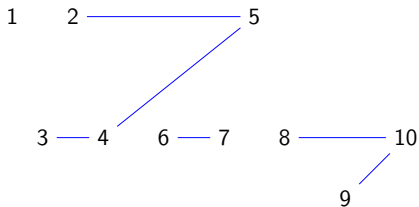


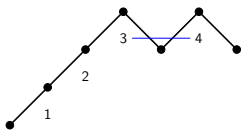
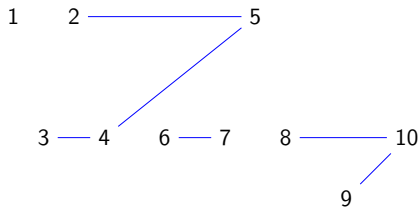


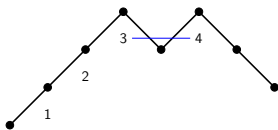
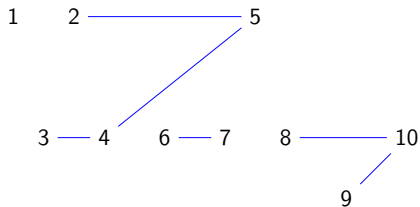


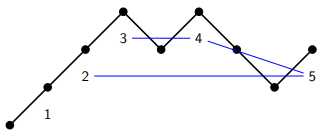
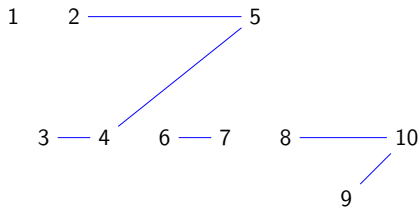


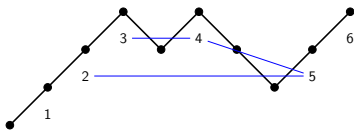
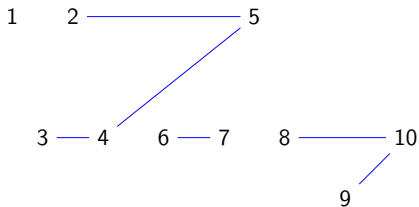


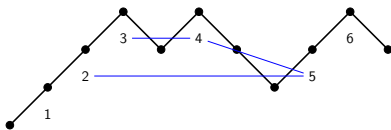
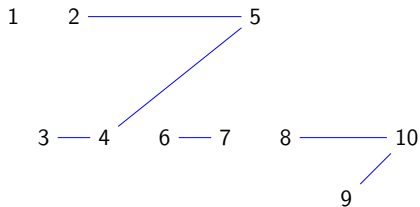


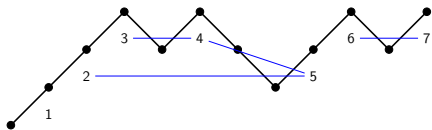
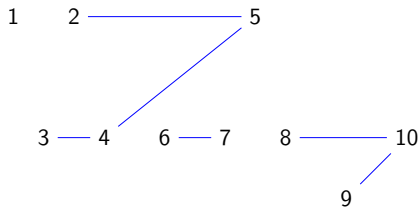


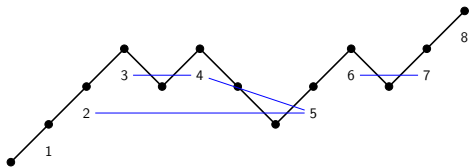
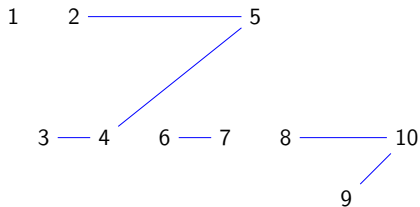


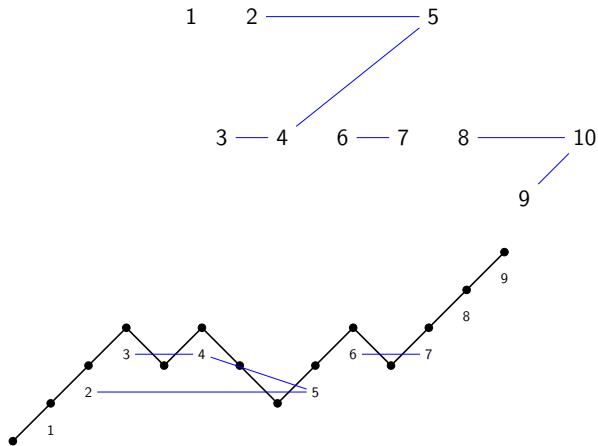


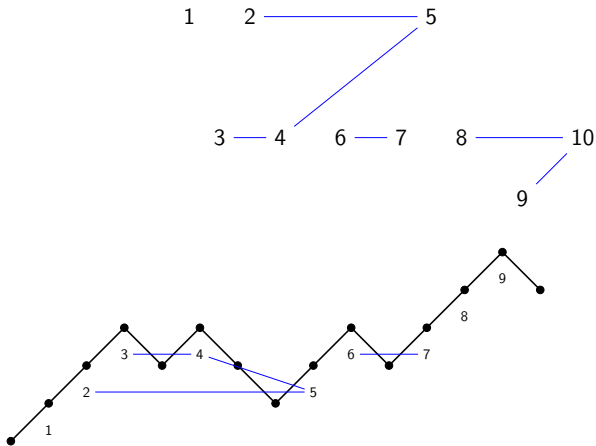


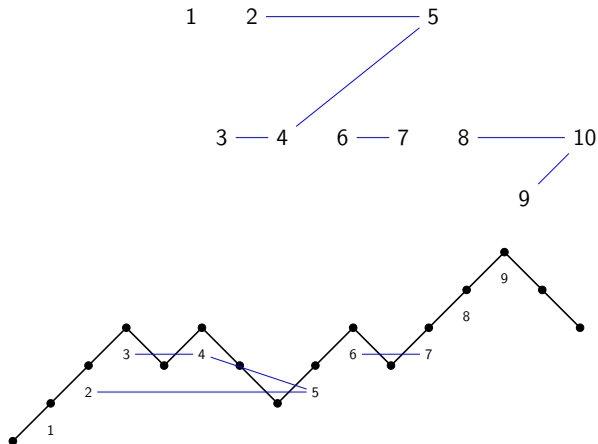


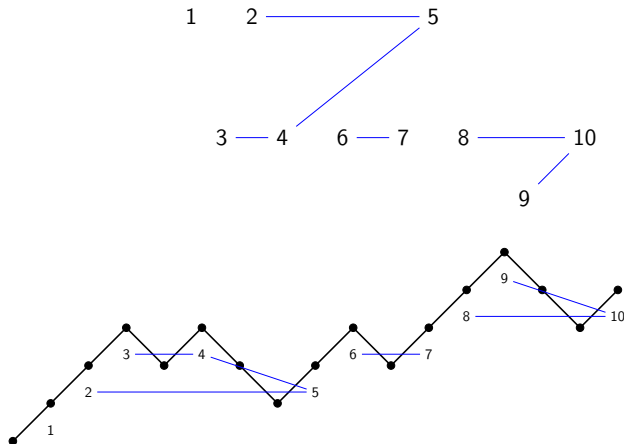


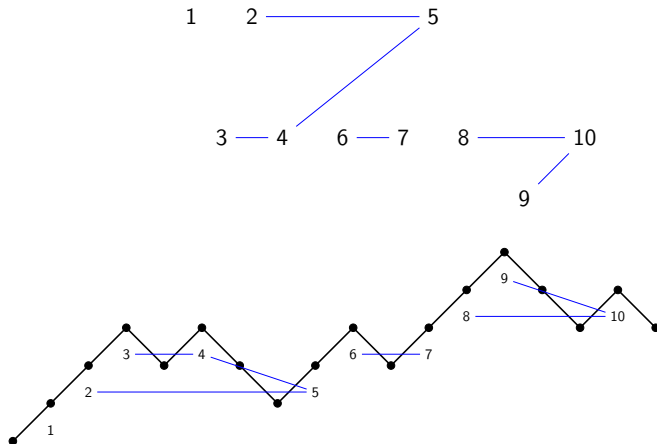


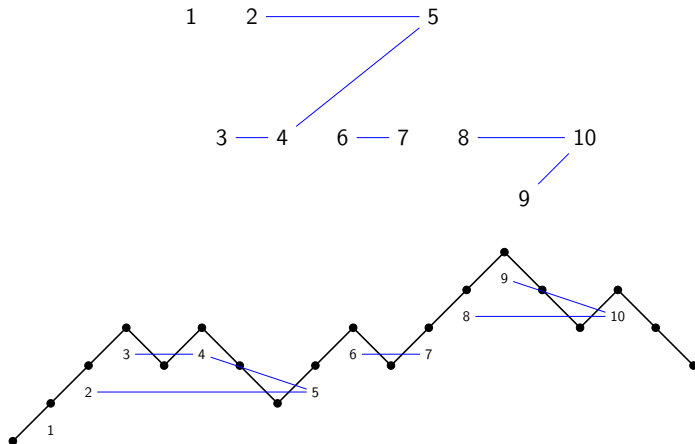


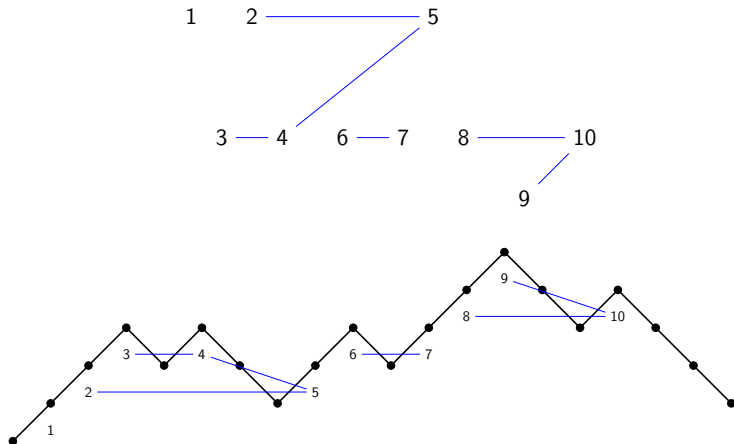


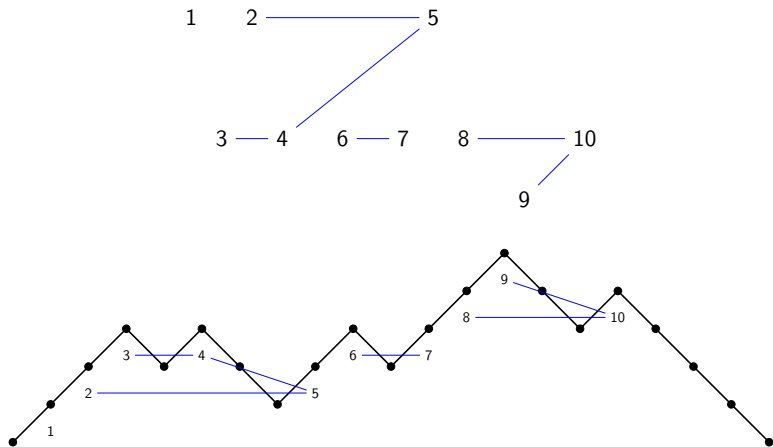


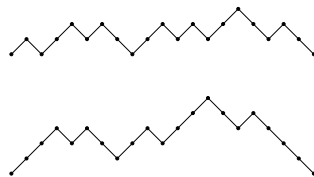
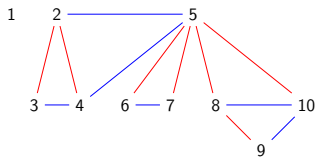




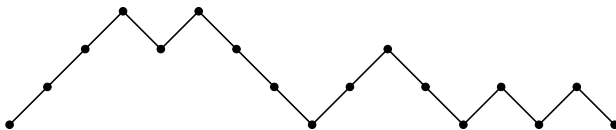




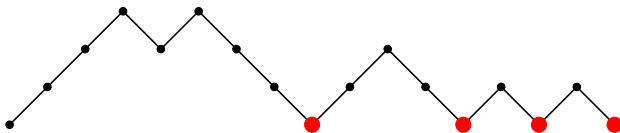




Montée initiale et nombre de contacts

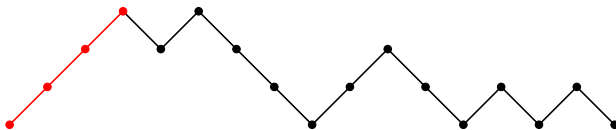


Montée initiale et nombre de contacts



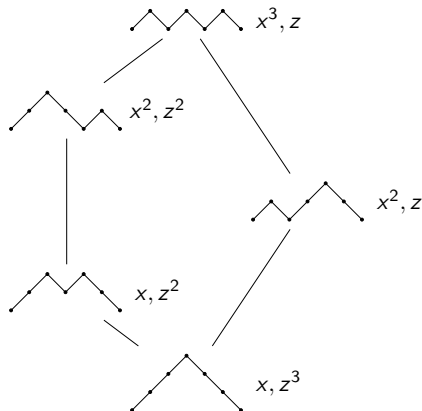
x^4

Montée initiale et nombre de contacts



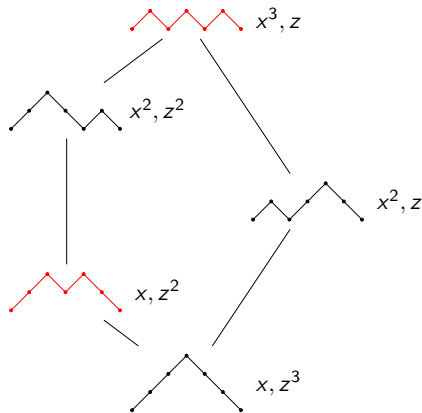
$$x^4, z^3$$

Distribution symétrique



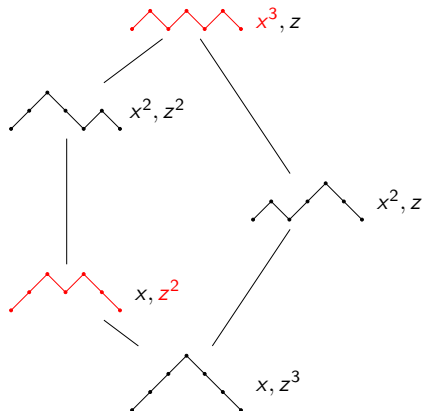
(Bousquet-Mélou, Fusy, Préville-Ratelle)

Distribution symétrique



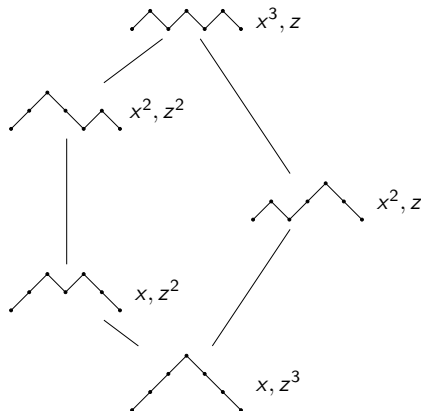
(Bousquet-Mélou, Fusy, Préville-Ratelle)

Distribution symétrique



$$x^3 z^2$$

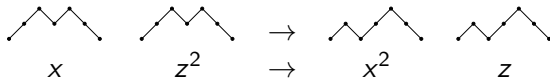
(Bousquet-Mélou, Fusy, Préville-Ratelle)

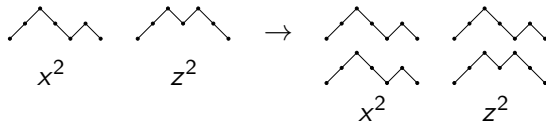
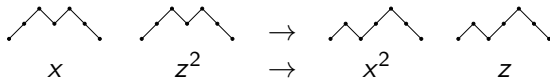


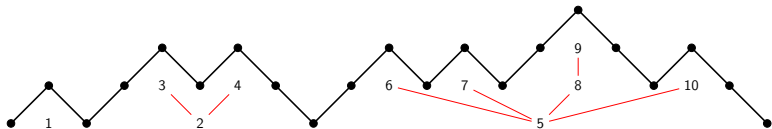
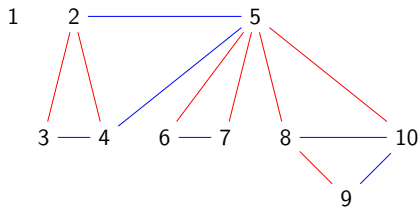
Distribution symétrique

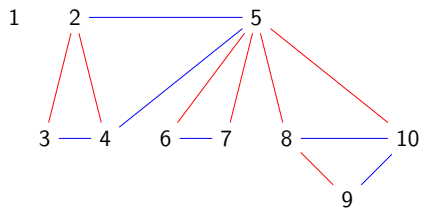
$$\begin{aligned}
 &2xz^3 + 2x^2z^3 + x^3z^3 \\
 &+ xz^2 + 2x^2z^2 + 2x^3z^2 \\
 &+ x^2z + 2x^3z
 \end{aligned}$$

(Bousquet-Mélou, Fusy, Préville-Ratelle)

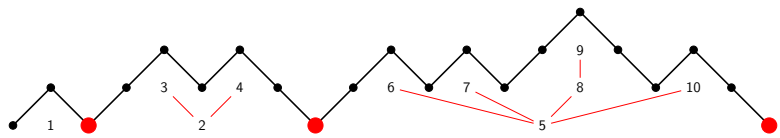


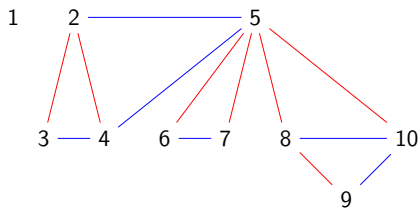




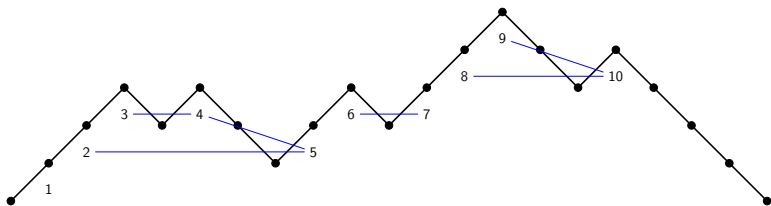


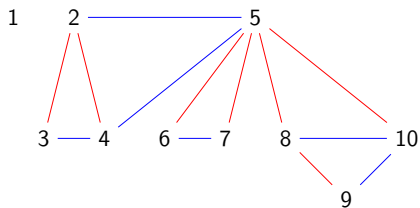
\times^3



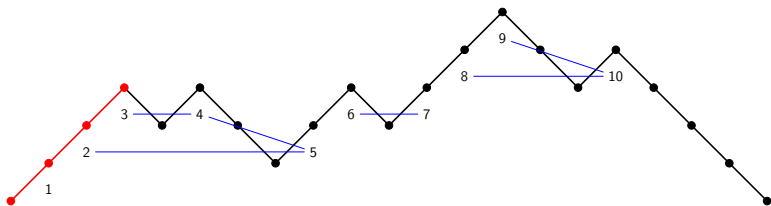


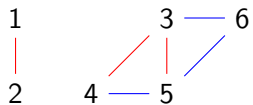
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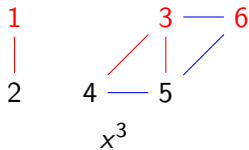


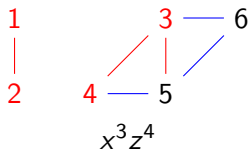


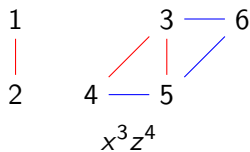
$$x^3 z^3$$



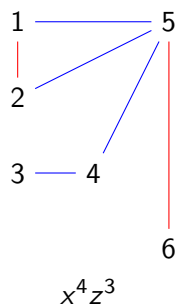


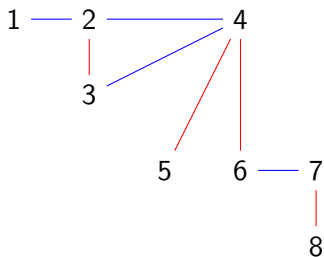


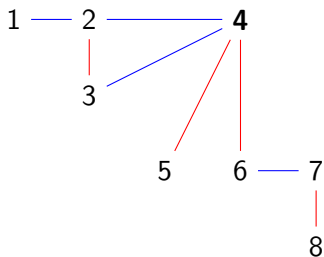


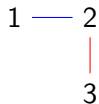
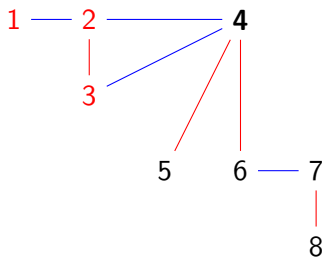


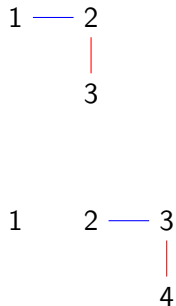
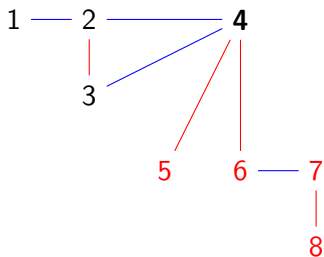
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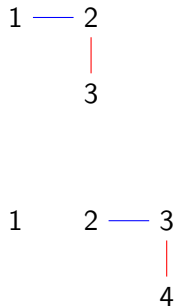
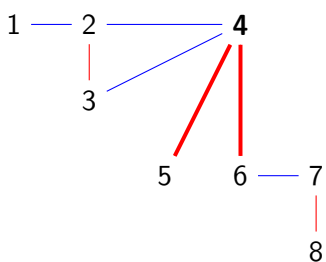




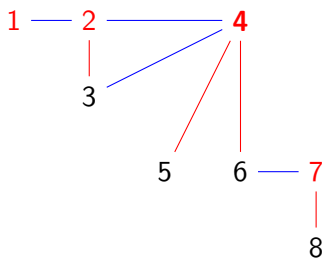




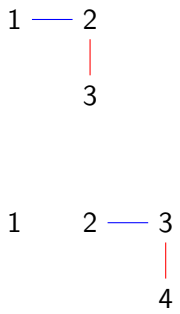




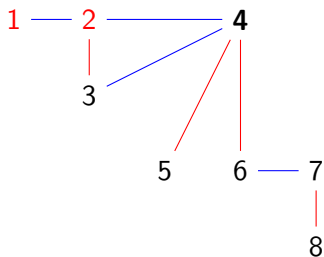
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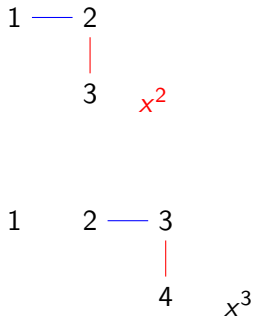
x^4



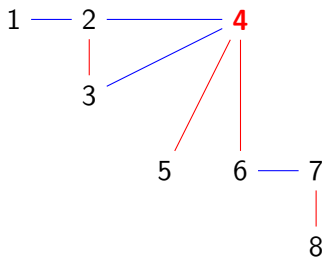
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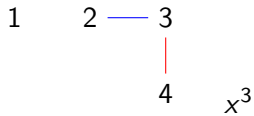
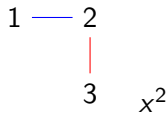
$$x^4 = x^2 \cdot x \cdot \frac{x^3}{x^2}$$



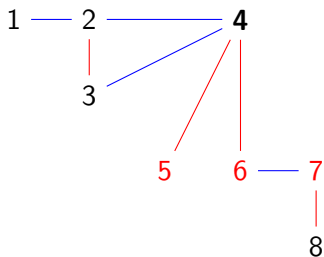
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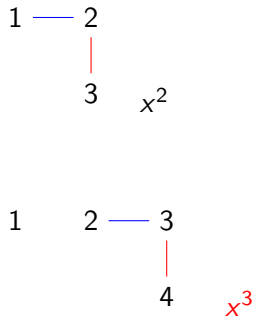
$$x^4 = x^2 \cdot x \cdot \frac{x^3}{x^2}$$



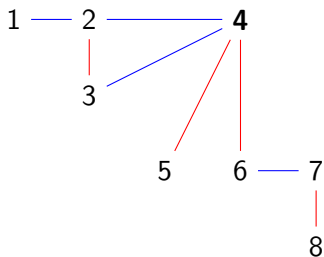
2



$$x^4 = x^2 \cdot x \cdot \frac{x^3}{x^2}$$

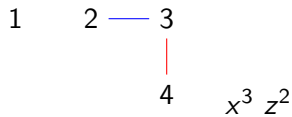
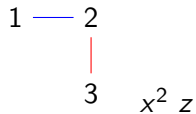


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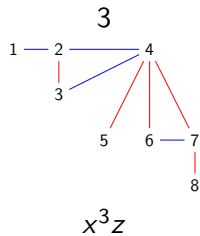
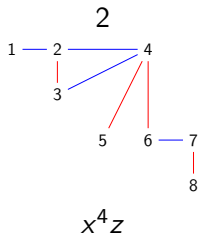
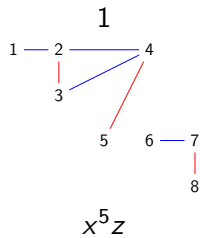
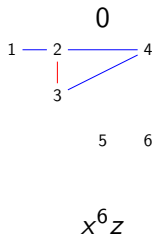


$$x^4 = x^2 \cdot x \cdot \frac{x^3}{x^2}$$

z



2



Une deuxième composition

1 — 2

x^2z

0



1



2

1



2

xz^2

Une deuxième composition

1 — 2

1
|
2

x^2z

xz^2

