

Planar graphs with  $\Delta \geq 8$   
are  $(\Delta + 1)$ -edge-choosable.

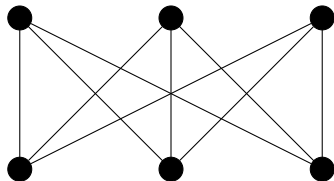
Marthe Bonamy

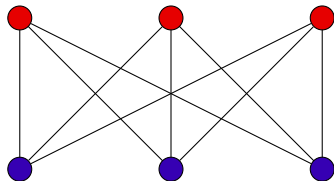
LIRMM, Montpellier, France



*September 11, 2013*

# Vertex coloring

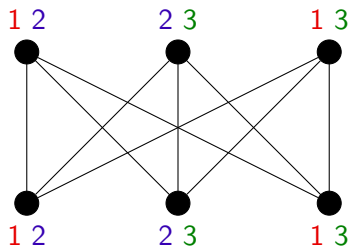




$\chi$ : Minimum number of colors to ensure that

$$\textcircled{a} \text{---} \textcircled{b} \Rightarrow a \neq b.$$

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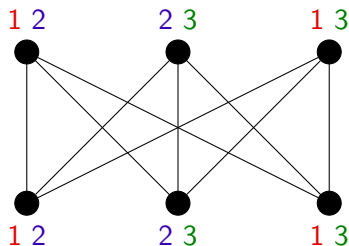
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$\chi_\ell$ : Minimum size of every  $L(v)$  such that

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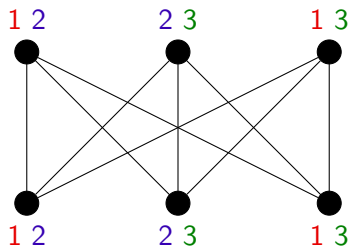
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$\omega$ : Maximum size of a clique.

$$\omega \leq \chi \leq \chi_\ell$$

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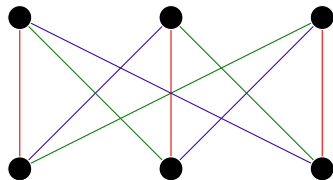
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$\Delta$ : Maximum degree.

$$\omega \leq \chi \leq \chi_\ell \leq \Delta + 1.$$

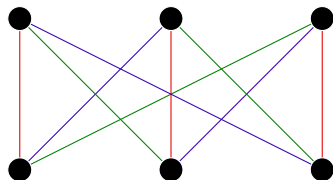
# Edge coloring



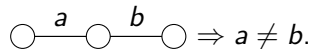
$\chi'$ : Minimum number of colors to ensure that

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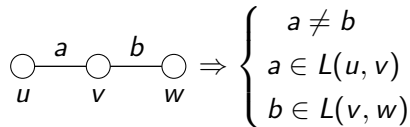
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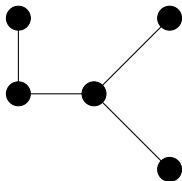
$\chi'_e$ : Minimum size of every  $L(e)$  such that





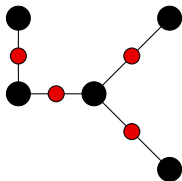
# A subcase of vertex coloring

Vertex coloring of **Line graphs**.



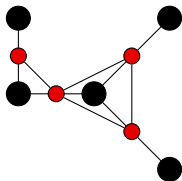
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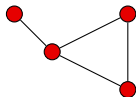
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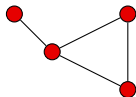
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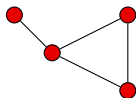
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Theorem (Beineke '70)

Characterized by *nine forbidden induced subgraphs*.

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Conjecture (List Coloring Conjecture '85)

For any graph,  $\chi'_e = \chi'$ .

# Knowing $\chi'$ is useful

On some subclasses:

Theorem (König '16)

For any *bipartite* graph,  $\chi' = \Delta$ .

Theorem (Sanders Zhao '01)

For any *planar* graph with  $\Delta \geq 7$ ,  $\chi' = \Delta$ .

Asymptotically true:

Theorem (Erdős Wilson '77)

For *almost every* graph,  $\chi' = \Delta$ .

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$\chi'_e - \chi' = o(\chi')$  when  $\chi' \rightarrow \infty$ .

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Theorem (Holyer '81)

It is *NP-complete* to compute  $\chi'$ .

# The weak List Coloring Conjecture

Theorem (Vizing '64)

*For any graph,  $\chi' \leq \Delta + 1$ .*

Conjecture (Vizing '76)

*For any graph,  $\chi'_\ell \leq \Delta + 1$ .*

## Theorem (Borodin '90)

For any *planar* graph with  $\Delta \geq 9$ ,  $\chi'_e \leq \Delta + 1$ .

## Theorem (Vizing '76, Juvan Mohar Škrekovski '99)

For any *planar* graph with  $\Delta \leq 4$ ,  $\chi'_e \leq \Delta + 1$ .

+ Many results for planar graphs with restrictions on cycles (no 4-cycle,  $\Delta \geq 6$  and no chordal 5-cycle,  $\Delta \geq 6$  and no chordal 4-cycle...)

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## Theorem (B. '13)

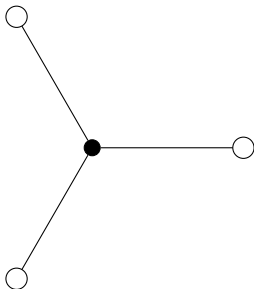
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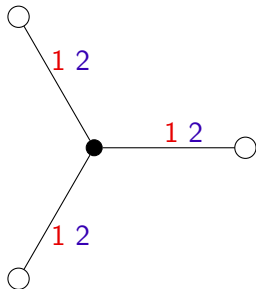
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# Key idea

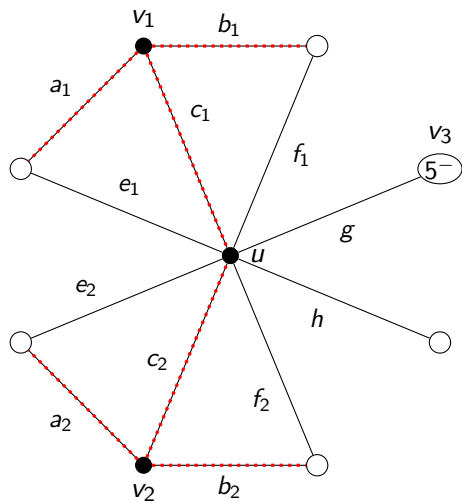


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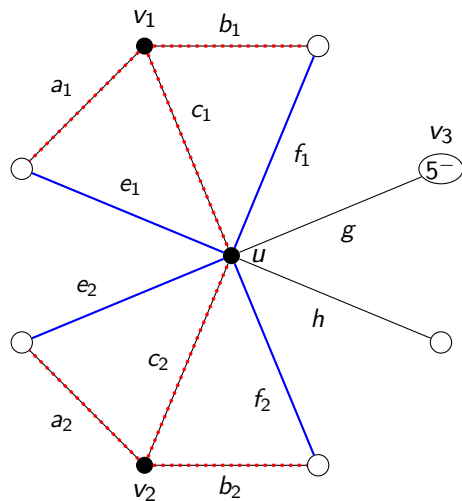




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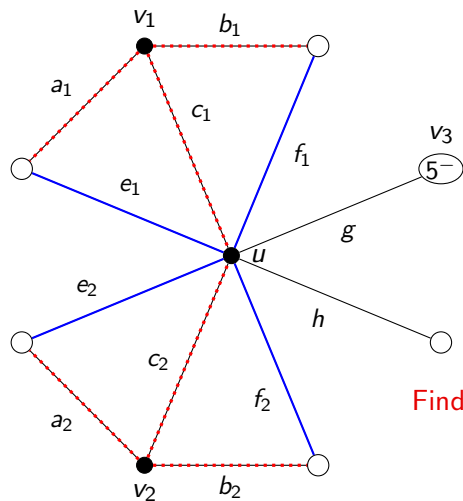
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Goal: recolor any of  $\{e_1, f_1, e_2, f_2\}$ .

Consider the digraph of constraints on  $\{e_1, f_1, e_2, f_2, g, h\}$ .

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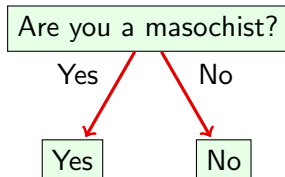
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Consider the digraph of constraints on  $\{e_1, f_1, e_2, f_2, g, h\}$ .

Find a good cycle or maximal path.

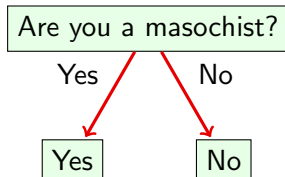
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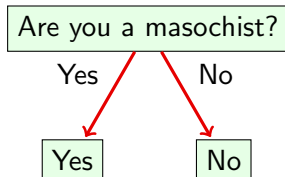
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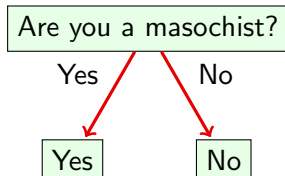
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# Thanks!