

On the diameter of reconfiguration graphs for vertex colourings

Marthe Bonamy

Matthew Johnson

Ioannis Lignos

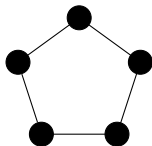
Viresh Patel

Daniël Paulusma

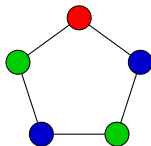
2 September 2011



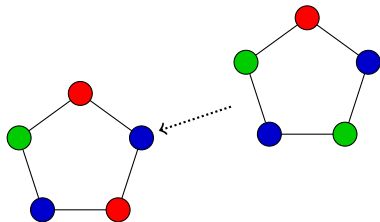
Graph recolouring



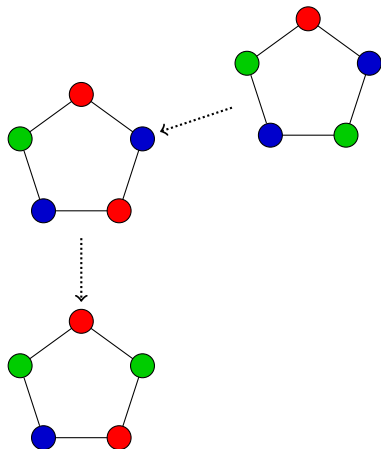
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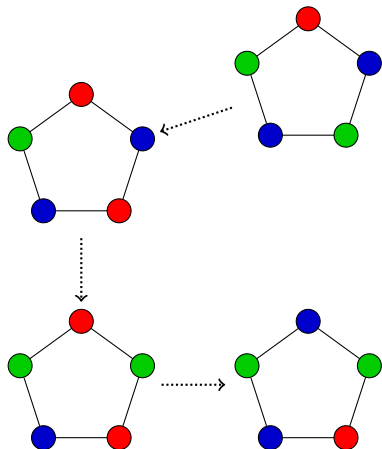
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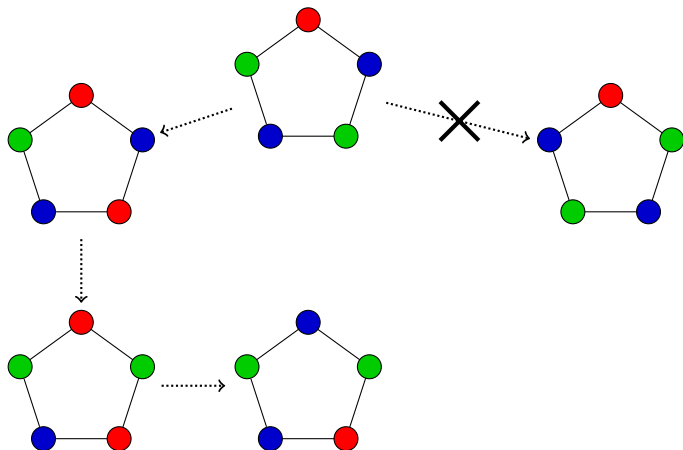
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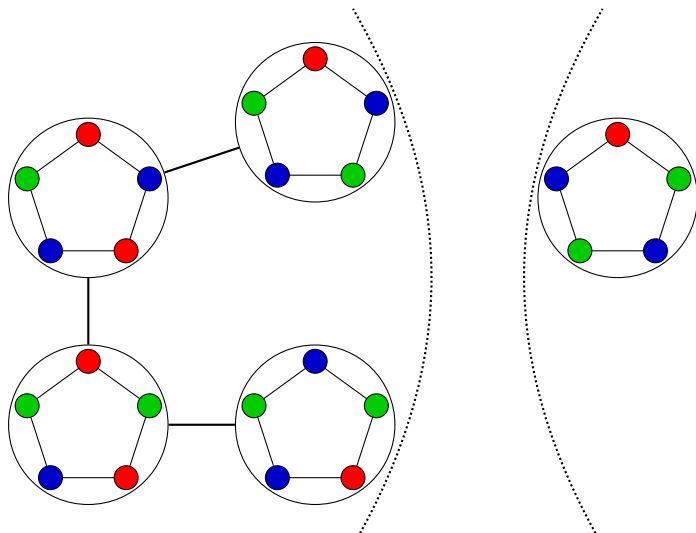


Graph recolouring



Graph recolouring \Rightarrow Reconfiguration graph

Solutions // Nodes. Most similar solutions // Neighbours.



- Two solutions:
 - In the same connected component?
 - What distance between them?

Reconfiguration graphs

- Two solutions:
 - In the same connected component?
 - What distance between them?
- Reconfiguration graph:
 - Connected?
 - Maximal diameter of a component?

Definition

k -mixing: Reconfiguration graph of the k -colourings is connected

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Theorem (Cereda, van den Heuvel, Johnson '07)

Bipartite graph: 3-mixing? \Rightarrow *coNP-hard*

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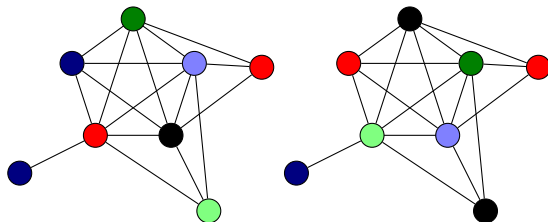
- $k = \chi(G) = \omega(G) \Rightarrow G$ not *k*-mixing
- $k = \chi(G) + 1$?

Cliques and split graphs

- Cliques are k -mixing in $\mathcal{O}(n)$.

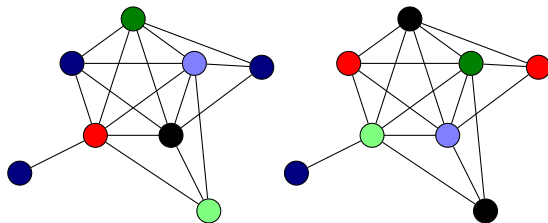
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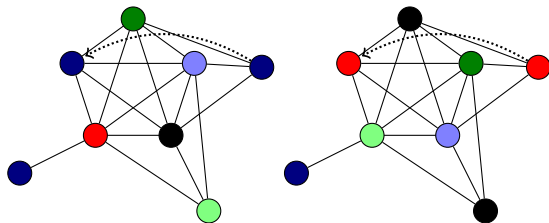
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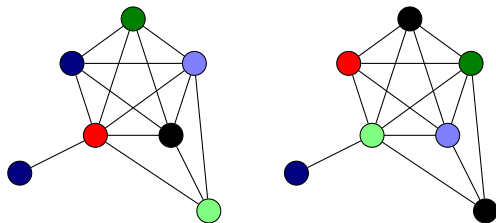
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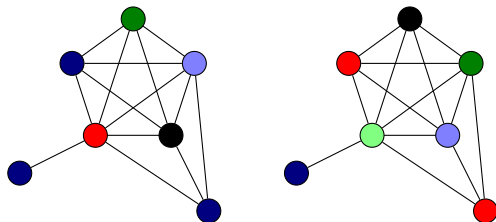
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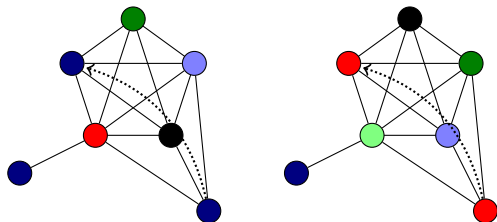
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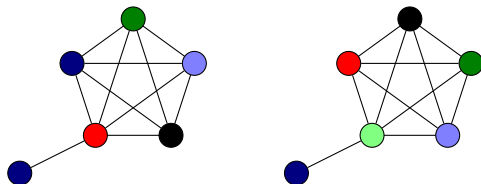
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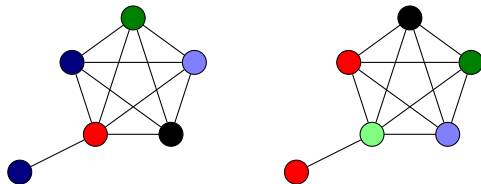
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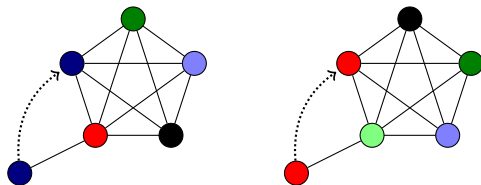
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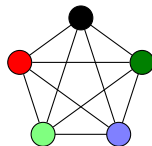
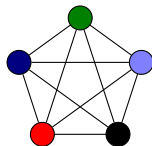
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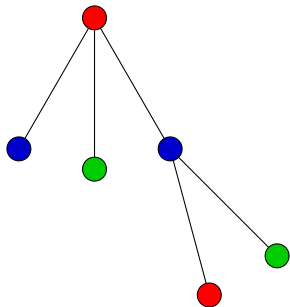
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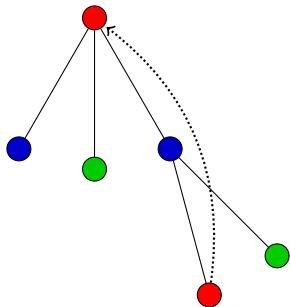
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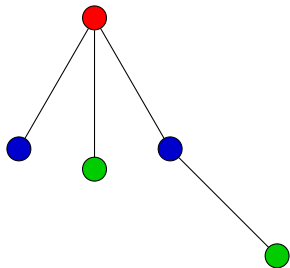
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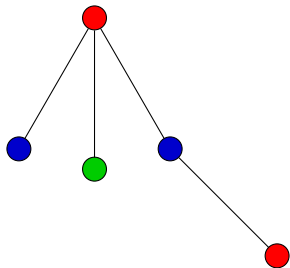
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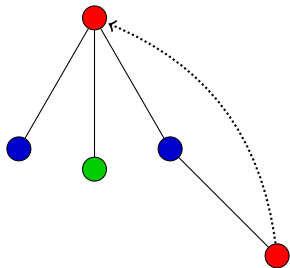
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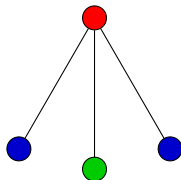
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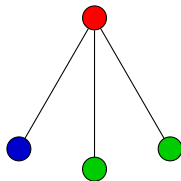
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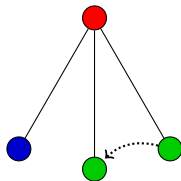
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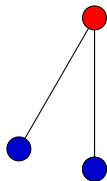
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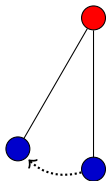
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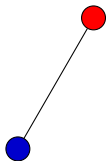
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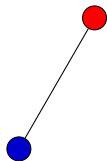
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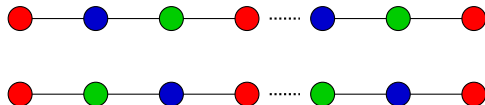


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And this is optimal.



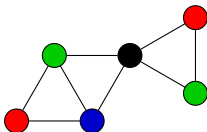
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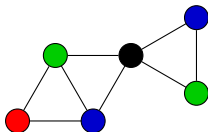


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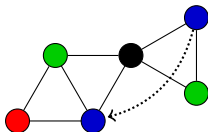


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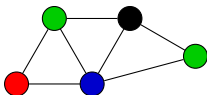


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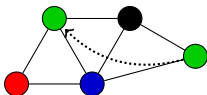


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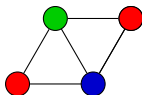


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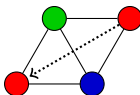


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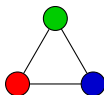


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Thanks for your attention!