

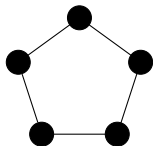
Reconfiguring Independent Sets in Cographs

Marthe Bonamy
Nicolas Bousquet

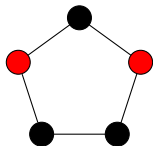
July 3, 2014



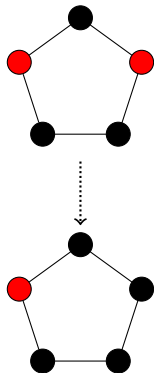
Independent Set Reconfiguration



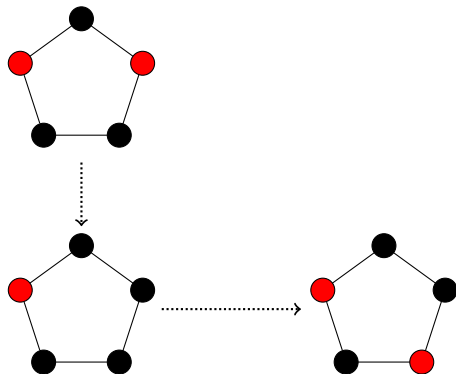
Independent Set Reconfiguration



Independent Set Reconfiguration

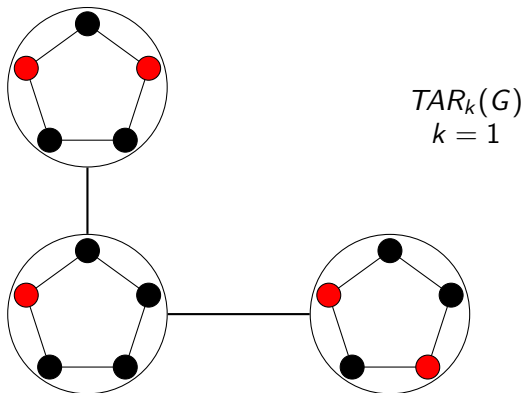


Independent Set Reconfiguration



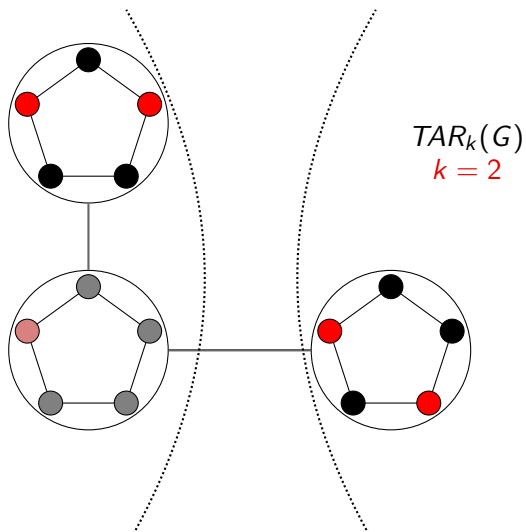
Independent Set Reconfiguration \Rightarrow Reconfiguration Graph

Solutions // Vertices. Closest solutions // Neighbors.



Independent Set Reconfiguration \Rightarrow Reconfiguration Graph

Solutions // Vertices. Closest solutions // Neighbors.



- Two solutions:
 - In the same connected component?
 - What distance between them?

- Two solutions:
 - In the same connected component?
 - What distance between them?
- Reconfiguration graph:
 - Connected?
 - Maximal diameter of a connected component?

- Two solutions:
 - In the same connected component?
 - What distance between them?
- Reconfiguration graph:
 - Connected?
 - Maximal diameter of a connected component?

Colorings, Dominating sets, Vertex covers...

- Two solutions:
 - In the same connected component?
 - What distance between them?
- Reconfiguration graph:
 - Connected?
 - Maximal diameter of a connected component?

Colorings, Dominating sets, Vertex covers...

Token Addition & Removal, Token Jumping, Token Sliding...

Theorem (Hearn, Demaine '05, Kamiński, Medvedev, Milanič '12)

G known to be *perfect* or *subcubic planar*:

Are α, β in the *same connected component* of $TAR_k(G)$?

PSPACE-complete.

Theorem (Hearn, Demaine '05, Kamiński, Medvedev, Milanič '12)

G known to be *perfect* or *subcubic planar*:
Are α, β in the *same connected component* of $TAR_k(G)$?
PSPACE-complete.

Efficient algorithms for:

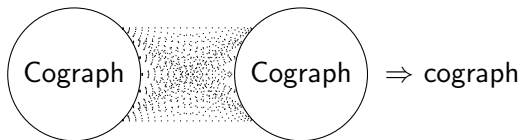
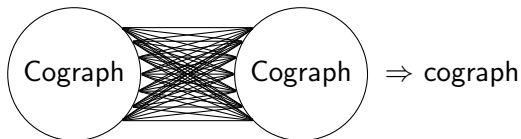
- claw-free graphs,
- line graphs,
- chordal graphs...

Cographs: P_4 -free graphs.

Cographs: P_4 -free graphs.



\Rightarrow cograph



Theorem (Bonsma '14)

G *cograph*, $\alpha, \beta \in \text{TAR}_k(G) \Rightarrow$ Decide in $\mathcal{O}(n^2)$ whether α and β are in the same connected component.

Question (Bonsma '14)

G *cograph* $\stackrel{?}{\Rightarrow}$ Decide in $\text{Poly}(n)$ whether $\text{TAR}_k(G)$ is *connected*.

Theorem (Bonsma '14)

G *cograph*, $\alpha, \beta \in \text{TAR}_k(G) \Rightarrow$ Decide in $\mathcal{O}(n^2)$ whether α and β are in the same connected component.

Question (Bonsma '14)

G *cograph* $\stackrel{?}{\Rightarrow}$ Decide in $\text{Poly}(n)$ whether $\text{TAR}_k(G)$ is *connected*.

Theorem (B., Bousquet '14+)

G *cograph* \Rightarrow Decide in $\mathcal{O}(n^3)$ whether $\text{TAR}_k(G)$ is *connected*.

Our Results

Theorem (Bonsma '14)

G *cograph*, $\alpha, \beta \in TAR_k(G) \Rightarrow$ Decide in $\mathcal{O}(n^2)$ whether α and β in the same connected component.

Question (Bonsma '14)

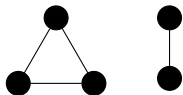
G *cograph* $\stackrel{?}{\Rightarrow}$ Decide in $Poly(n)$ whether $TAR_k(G)$ is *connected*.

Theorem (B., Bousquet '14+)

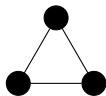
G *cograph*, $\alpha, \beta \in TAR_k(G) \Rightarrow$ Decide in $\mathcal{O}(n)$ whether α and β in the same connected component.

Theorem (B., Bousquet '14+)

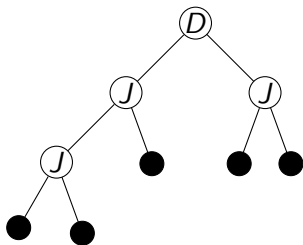
G *cograph* \Rightarrow Decide in $\mathcal{O}(n^3)$ whether $TAR_k(G)$ is *connected*.

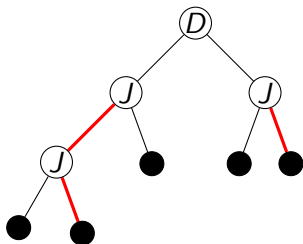
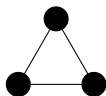


- Take your favorite G and k .

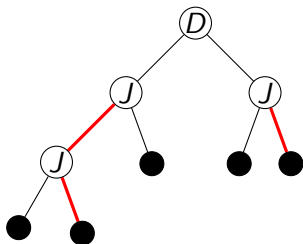
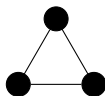


- Take your favorite G and k .
- Build the decomposition tree in $\mathcal{O}(n)$.

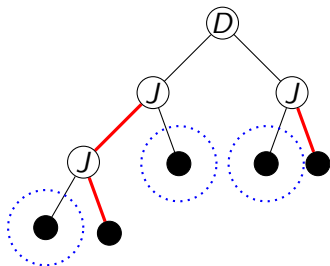
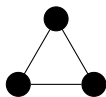




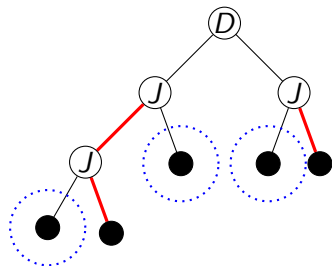
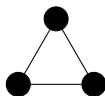
- Take your favorite G and k .
- Build the decomposition tree in $\mathcal{O}(n)$.
- Pick good and bad sides.



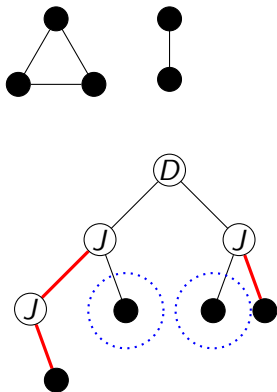
- Take your favorite G and k .
- Build the decomposition tree in $\mathcal{O}(n)$.
- Pick good and bad sides.
- Maximal stable sets \Leftrightarrow "Stable-searches".



- Take your favorite G and k .
- Build the decomposition tree in $\mathcal{O}(n)$.
- Pick good and bad sides.
- Maximal stable sets \Leftrightarrow "Stable-searches".
- Find bad side B with smallest $\alpha(B)$.



- Take your favorite G and k .
- Build the decomposition tree in $\mathcal{O}(n)$.
- Pick good and bad sides.
- Maximal stable sets \Leftrightarrow "Stable-searches".
- Find bad side B with smallest $\alpha(B)$.
- Maximal stable set in $G \setminus (B \cup N(B))$ of size $k - \alpha(B) \leq \cdot \leq k + \alpha(B) - 1$?



- Take your favorite G and k .
- Build the decomposition tree in $\mathcal{O}(n)$.
- Pick good and bad sides.
- Maximal stable sets \Leftrightarrow "Stable-searches".
- Find bad side B with smallest $\alpha(B)$.
- Maximal stable set in $G \setminus (B \cup N(B))$ of size $k - \alpha(B) \leq \cdot \leq k + \alpha(B) - 1$?

Question (Bonsma'14)

G *cograph*, $\alpha, \beta \in \text{TAR}_k(G)$ $\stackrel{?}{\Rightarrow}$ Decide in $\text{Poly}(n)$ whether α and β *at distance at most ℓ* .

Question (Bonsma'14)

G *cograph*, $\alpha, \beta \in TAR_k(G)$ $\stackrel{?}{\Rightarrow}$ Decide in $Poly(n)$ whether α and β *at distance at most ℓ* .

Thanks for your attention!