

Procédures de déchargement global pour des problèmes de coloration de graphes

Marthe Bonamy

Encadrants : Benjamin Lévêque et Alexandre Pinlou

9 février 2015



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ECOLE DOCTORALE



Global discharging methods for coloring problems in graphs

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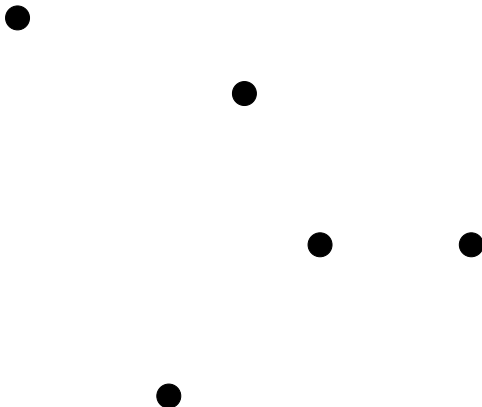
February 9, 2015

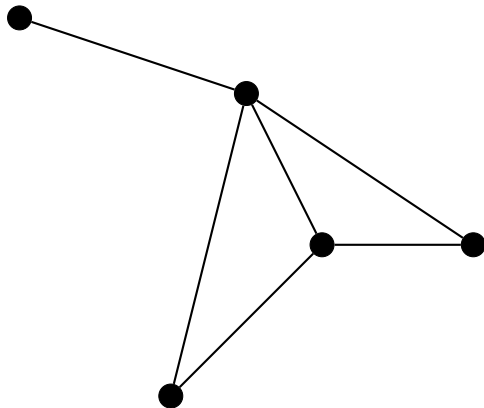


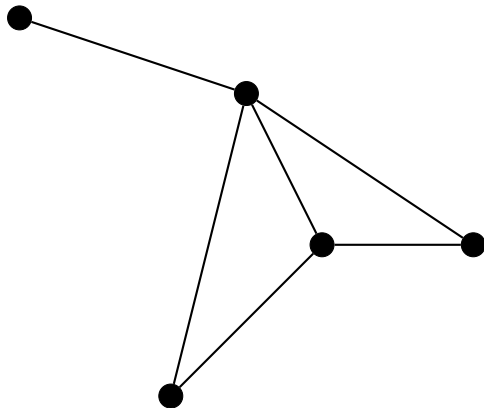
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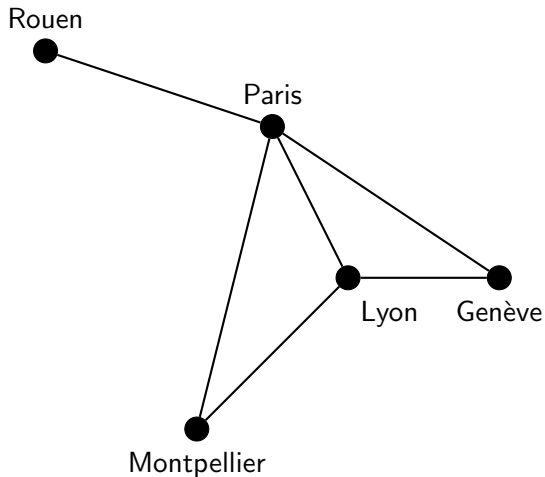
- 1 Introduction
- 2 Square coloring
- 3 Edge coloring
- 4 Conclusion



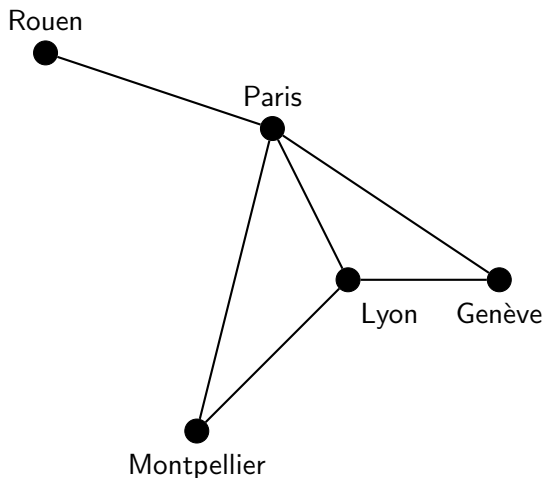




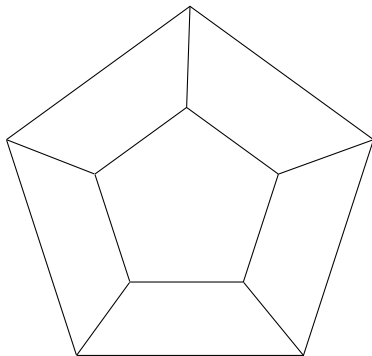
Train network.



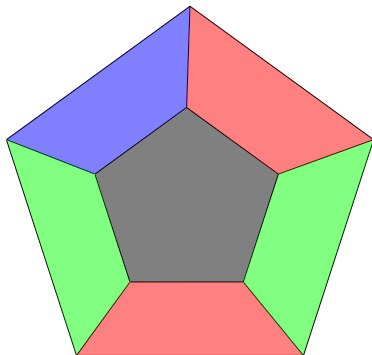
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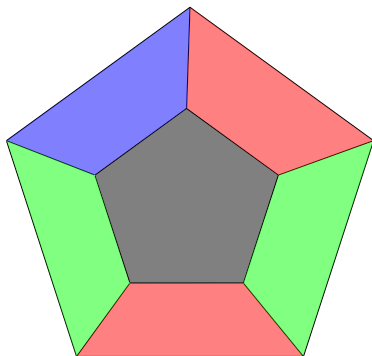


Train network. \rightsquigarrow **Any constraint representation.**



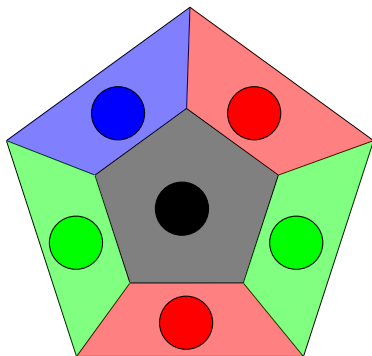
Coloring problems





Question (Guthrie 1852)

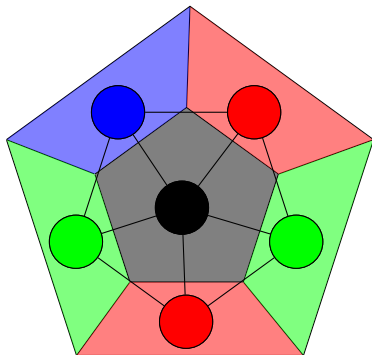
Are all maps 4-colorable?



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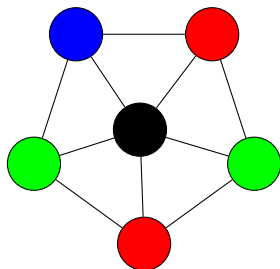
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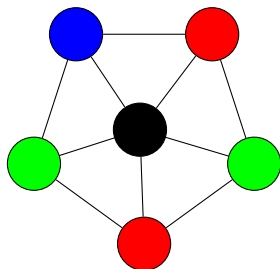
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The Four Color Theorem

A graph is **planar** if it can be drawn in a plane without crossing edges ()

Theorem (Appel, Haken, Koch 1976)

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Two characteristics:

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- Relies upon a **discharging argument**.

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Discharging methods

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The diagram shows the arithmetic expression $1+2+3+4+5+6$. Below the numbers, there are three curved arrows pointing from left to right. The first arrow starts under the '1' and ends under the '3'. The second arrow starts under the '3' and ends under the '5'. The third arrow starts under the '5' and ends under the '6'. This represents a sequence of discharging steps where charge is transferred from vertex 1 to 3, then from 3 to 5, and finally from 5 to 6.

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$$0+0+0+7+7+7$$

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Euler's formula for planar graphs

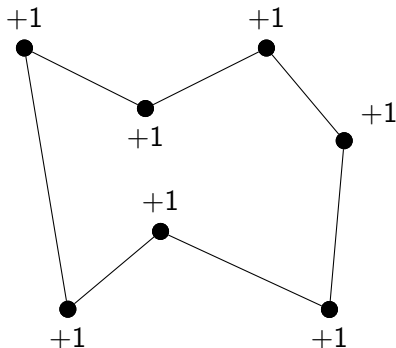
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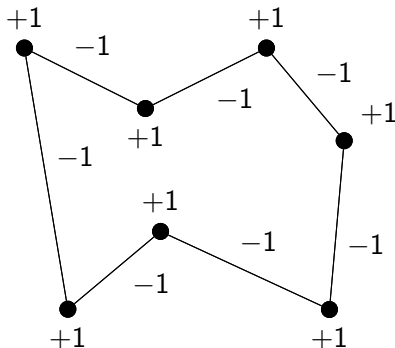
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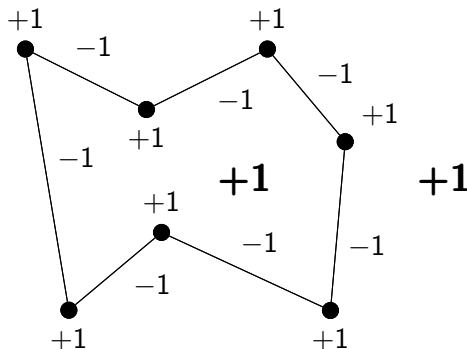


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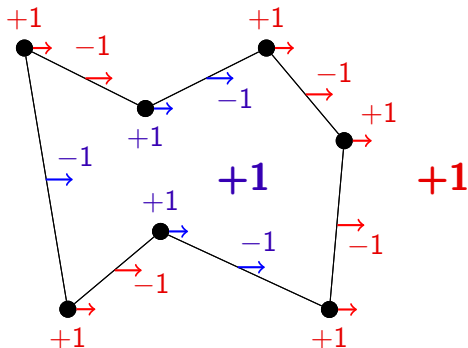


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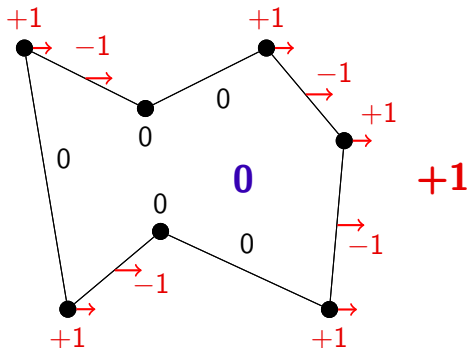


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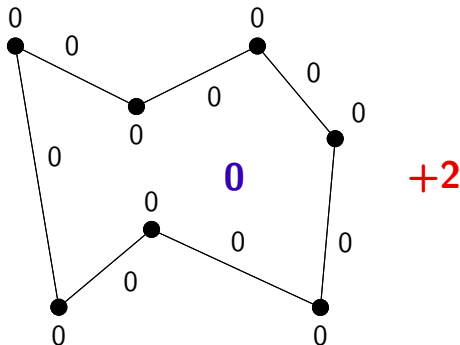


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Discharging methods for coloring problems

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$\Rightarrow \forall v, f, \omega'(v) \geq 0, \omega'(f) \geq 0$ **Contradiction!**

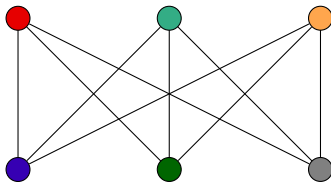
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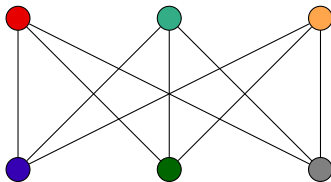
Square coloring



χ^2 : Minimum number of colors to ensure that

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$$\Delta + 1 \leq \chi^2 \leq \Delta^2 + 1$$

Δ : maximum number of neighbors

Historical motivation: Radio frequencies assignment.

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Conjecture (Wegner 1977)

If G is a *planar* graph, then:

- $\chi^2 \leq 7$ if $\Delta = 3$
- $\chi^2 \leq \Delta + 5$ if $4 \leq \Delta \leq 7$
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Theorem (Havet, van den Heuvel, McDiarmid, Reed '07)

Wegner's conjecture is *asymptotically true*.

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- For *planar* graphs, $\exists d$, $g \geq 7$, $\Delta \geq d \Rightarrow \chi^2 = \Delta + 1$.
- $\exists (G_n)_{n \in \mathbb{N}}$, *planar*, $g = 6$, $\Delta \nearrow$, $\chi^2 > \Delta + 1$.

Another measure of sparsity

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$g(G) \geq$	3	4	5	6	7	8	9	10	11	12	13	...	$+\infty$
$\text{mad}(G) <$	6	4	$\frac{10}{3}$	3	$\frac{14}{5}$	$\frac{8}{3}$	$\frac{18}{7}$	$\frac{5}{2}$	$\frac{22}{9}$	$\frac{12}{5}$	$\frac{26}{11}$...	2

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What is the maximum/supremum $m \mid \exists d$,
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Answer: $m = \frac{14}{5}$?

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\Rightarrow What about $\Delta(G) + \mathcal{O}(1)$ colors?

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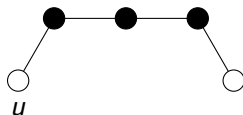
Reducible configurations:

Glimpse of the proof

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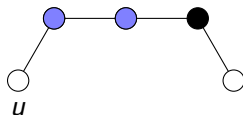
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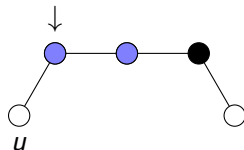
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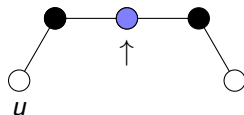
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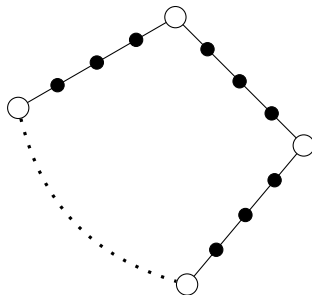
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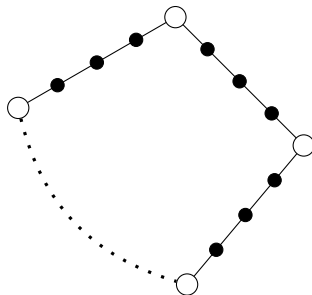


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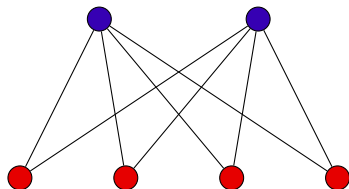
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\Rightarrow **Global** discharging argument.

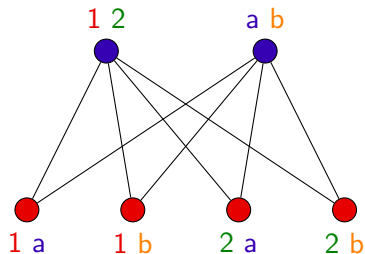
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List Coloring



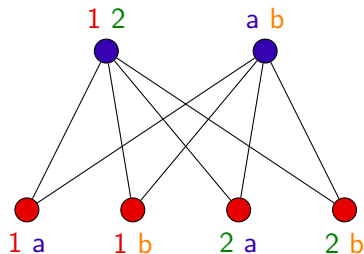
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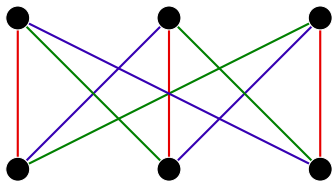
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$\chi_\ell - \chi$ can be arbitrarily large!

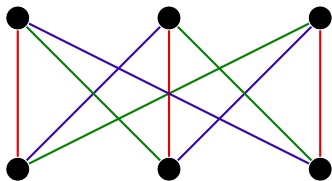
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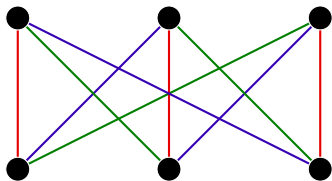
χ' : Minimum number of colors to ensure that

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χ'_e : Minimum size of every $L(e)$ such that

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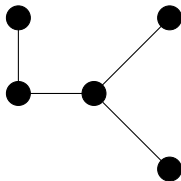
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$$\Delta \leq \chi' \leq \chi'_e \leq 2\Delta - 1.$$

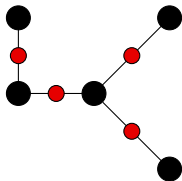
Edge Coloring (2)

Vertex coloring of **Line graphs**.



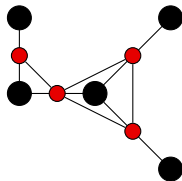
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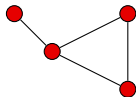
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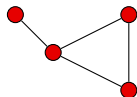
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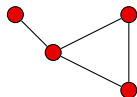


Conjecture (List Coloring Conjecture '85)

For any graph, $\chi'_e = \chi'$.

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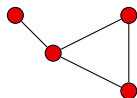
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Theorem (Vizing '64)

For any graph, $\chi' \leq \Delta + 1$.

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Conjecture (List Coloring Conjecture '85)

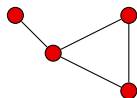
For any graph, $\chi'_l = \chi'$.

Conjecture (weak List Coloring Conjecture)

For any graph, $\chi'_l \leq \Delta + 1$.

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Conjecture (List Coloring Conjecture '85)

For any graph, $\chi'_e = \chi'$.

Conjecture (weak List Coloring Conjecture)

For any graph, $\chi'_e \leq \Delta + 1$.

Theorem (Kahn 1996)

The weak List Coloring Conjecture is *asymptotically true*.

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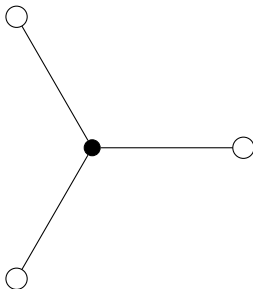
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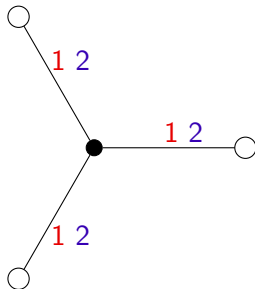
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Conjecture still open for planar graphs with $\Delta(G) \in \{5, 6, 7\}$.

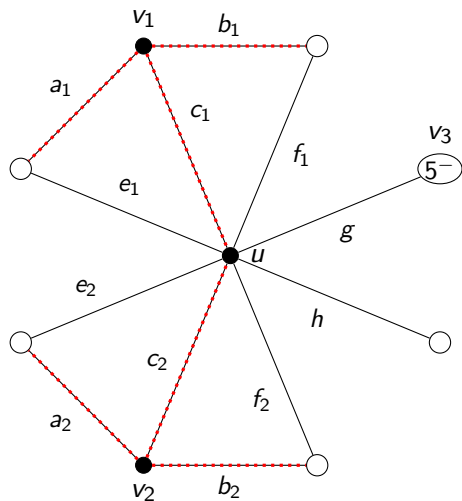
Key idea



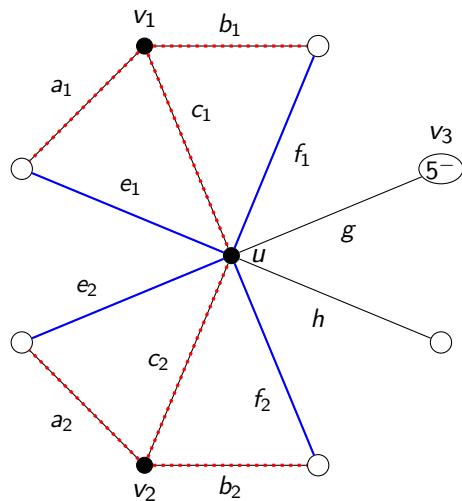
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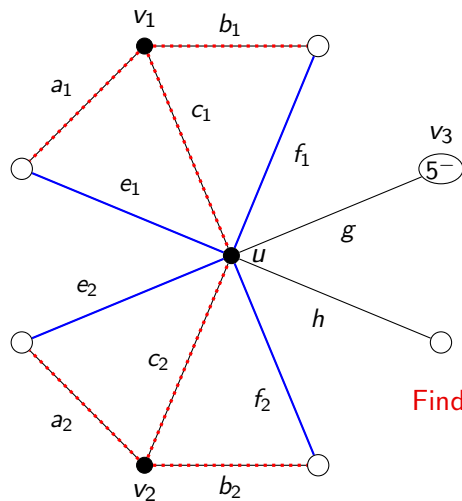
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Goal: recolor any of $\{e_1, f_1, e_2, f_2\}$.

Consider the digraph of constraints on $\{e_1, f_1, e_2, f_2, g, h\}$.

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Find a good cycle or maximal path.

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Two (broad) directions:

- Discharging methods for **algorithmic** purpose.
- Any hope of partial **automatization**?

Not in this talk

Other contributions online:

- **More** on discharging methods for coloring problems (*with Bousquet, Hocquard, Lévêque, Pinlou, Przybyło*).
- Graph **recoloring** (and related problems) (*with Bousquet, Johnson, Lignos, Patel, Paulusma*).
- Graphs with **large chromatic number** induce a **3k-cycle** (*with Charbit, Thomassé*).
- **Erdos-Hajnal conjecture** for graphs with no large induced cycle or anti-cycle (*with Bousquet, Thomassé*).
- Improved **kernel** for **Feedback Vertex Set** in planar graphs (*with Kowalik*).

Thanks!