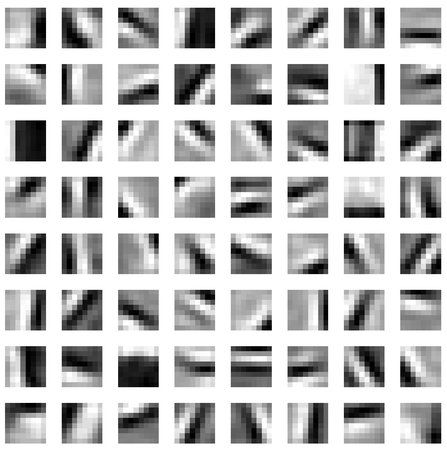
What are the sparse components of 2D shapes?

Michaël Clément James H. Elder

Centre for Vision Research, York University, Toronto, Canada

Context and motivation

- Receptive fields of neurons in early visual cortex resemble the sparse components of natural image patches [Olshausen *et al.* 1996].
- Can neural selectivity in higher visual areas also be understood in terms of sparse coding?
- Here we examine the sparse components of 2D shapes to make predictions about neural tuning in intermediate and higher areas of the object pathway.



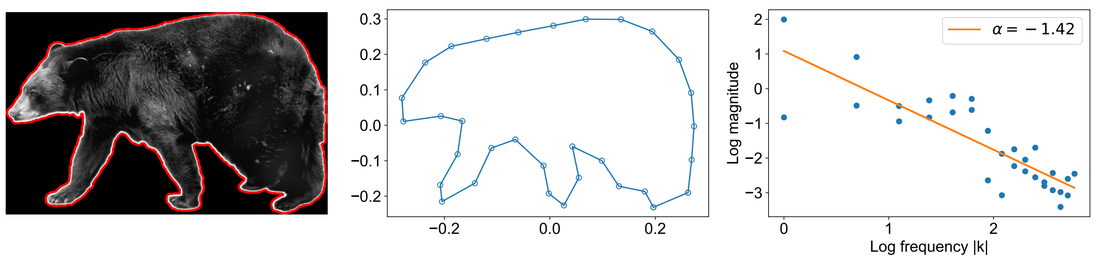
What is sparse coding?

- Sparse coding is a type of linear coding method in which a signal is approximated by a weighted sum of basis functions.
- In principal components analysis (PCA), these functions are selected to minimize the squared error with the training signals.
- In sparse coding, an additional requirement is to minimize the average magnitude of the weights.This results in only a few basis functions being used for any signal.

2D shape stimuli

Hemera dataset: 136 259 shapes of various objects from binary masks:

- Polygons are sampled at $m = 32$ discrete points, then coordinates are concatenated into a real vector $\mathbf{x}_i \in \mathbb{R}^{2m}$.
- 2D shapes can also be represented in the Fourier descriptor domain. Note that natural shapes are mostly lowpass.



Methods: Sparse coding of shapes

- **Components**: We seek to learn a dictionary $\mathbf{\Phi} \in \mathbb{R}^{2m \times k}$ such that any shape $\mathbf{x}_i$ can be encoded by a linear combination of its k columns.
- **Sparsity**: We want to enforce that only few components of this dictionary are used to encode a given shape.

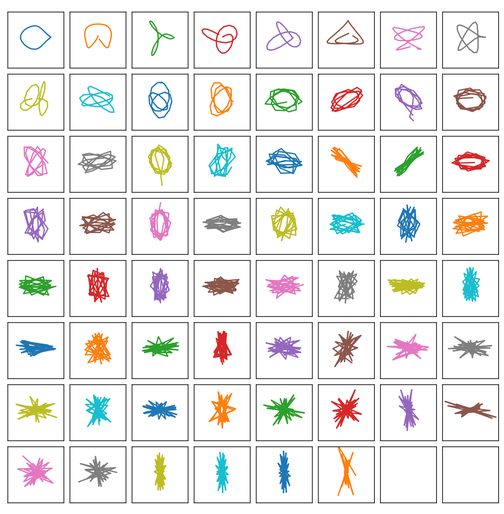
For a set of n shapes, this leads to the following optimization problem:

$$\min_{\mathbf{\Phi},\,\boldsymbol{\alpha}_i} \frac{1}{n} \sum_{i=1}^{n} \Big(\underbrace{\|\mathbf{x}_i - \mathbf{\Phi}\boldsymbol{\alpha}_i\|_2^2}_{\text{reconstruction error}} + \lambda \underbrace{\|\boldsymbol{\alpha}_i\|_1}_{\text{sparsity}} \Big)$$

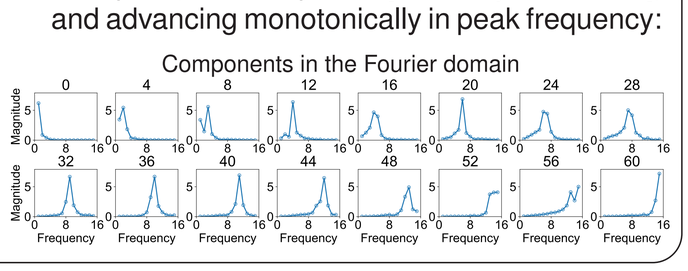
where $\lambda \in \mathbb{R}$ is a regularization parameter and $\boldsymbol{\alpha}_i \in \mathbb{R}^k$ are coefficients applied to the components for each training shape.

- Here we set the number of components to $k = 64$ (complete code).
- We use the optimization strategy proposed by [Mairal *et al.* 2010].

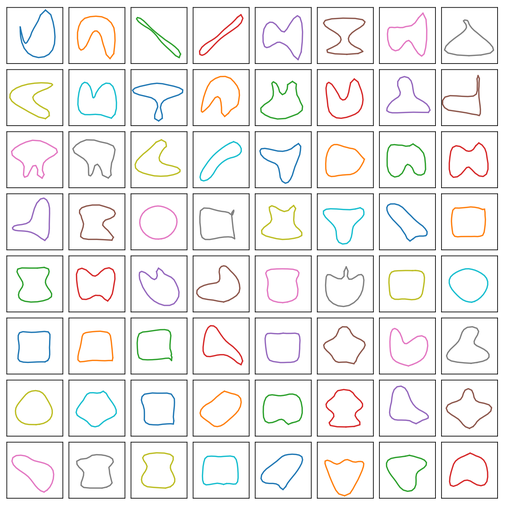
Results: Principal components of 2D shapes



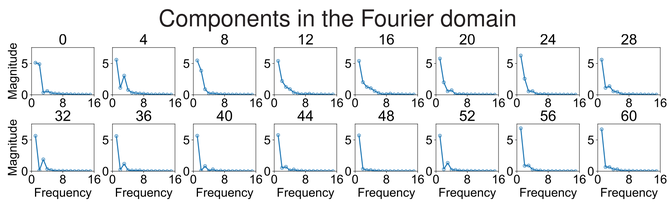
- The principal components are determined by setting the regularization parameter to $\lambda = 0$.
- In the spatial domain, the principal components are not readily interpretable.
- In the Fourier domain, it is clear that they are bandpass, like shapelets [Dubinskiy *et al.* 2003], and advancing monotonically in peak frequency:



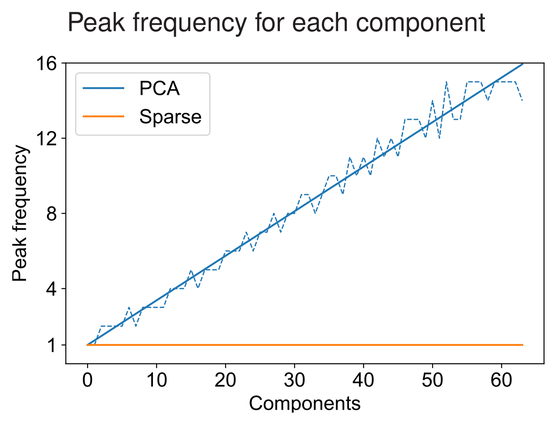
Results: Sparse components of 2D shapes

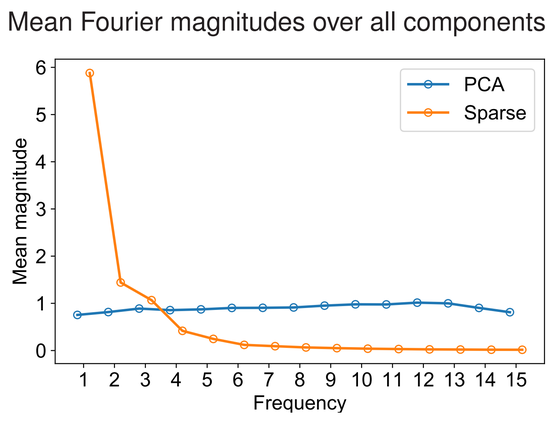


- Regularization parameter fixed to $\lambda = 0.1$ (this could be learned by cross validation).
- In contrast to PCA, the sparse components are readily interpretable in the spatial domain.
- The sparse components are all lowpass, similar to naturalistic shapes:

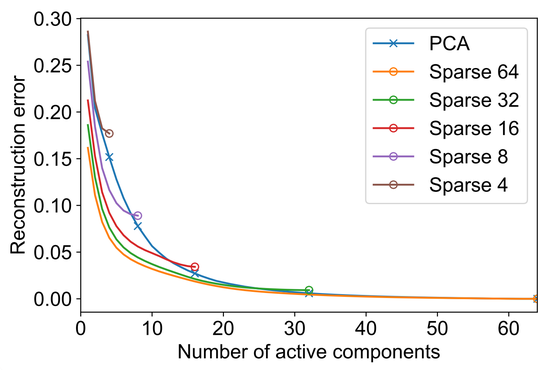


Analysis: Frequency domain



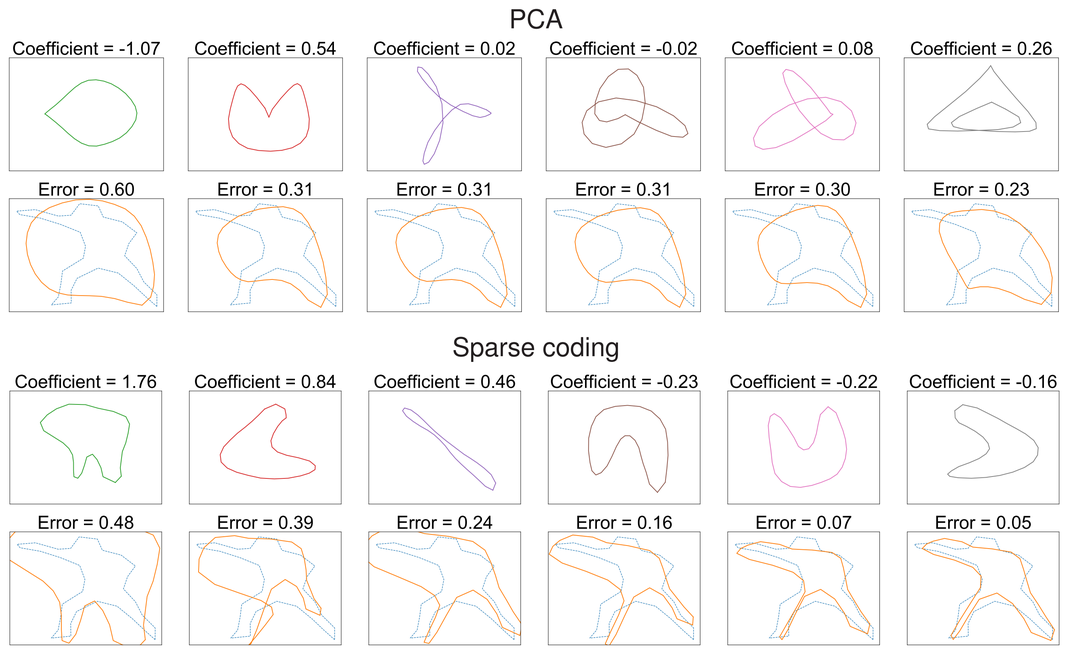


Analysis: Reconstruction error



- PCA is optimal at minimizing the reconstruction error for a fixed number of components.
- Sparse coding allows shapes to be accurately represented with fewer active components.

Example reconstruction



Open questions and future work

- To what degree do these sparse shape components predict shape selectivity of neurons in V4 [Carlson *et al.* 2011] or posterior IT [Connor *et al.* 2007]?
- Does sparse coding of shape facilitate object recognition?

References

Carlson, E. T., R. J. Rasquinha, K. Zhang, and C. E. Connor (2011). "A sparse object coding scheme in area V4". *Current Biology*, 21(4):288–293.

Connor, C. E., S. L. Brincat, and A. Pasupathy (2007). "Transformation of shape information in the ventral pathway". *Current Opinion in Neurobiology*, 17(2):140–147.

Dubinskiy, A. and S. C. Zhu (2003). "A Multi-scale Generative Model for Animate Shapes and Parts". In: *IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*. Vol. 1. 3, pp. 249–256.

Mairal, J., F. Bach, J. Ponce, and G. Sapiro (2010). "Online Learning for Matrix Factorization and Sparse Coding". *Journal of Machine Learning Research*, 11:19–60.

Olshausen, B. A. and D. J. Field (1996). "Emergence of simple-cell receptive field properties by learning a sparse code for natural images", 381(6583):607–609.