Distributed Scheduling of Multiple Bag-Of-Task Applications: Proportionnal Fairness

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Motivation

Large-scale distributed platforms result from the collaboration of many users:

- sharing resources amongst users should somehow be fair.
- Task regularity (SETI@home, BOINC, ...) $\rightsquigarrow$ steady-state scheduling.

Designing a *Fair* and Distributed scheduling algorithm for this framework.

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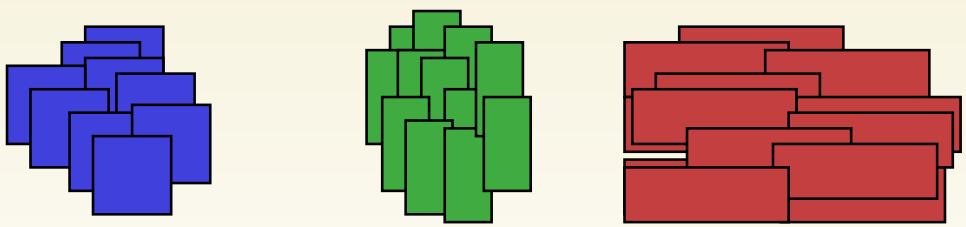
Application model

Multiple applications:

- A set A of K applications $A_1, \ldots, A_K$
- Each consisting in a large number of same-size independent tasks $\rightsquigarrow$ each application is defined by a communication cost $w^{(k)}$ (in MFlops) and a communication cost $b^{(k)}$ (in MB).
- Different communication and computation demands for different applications



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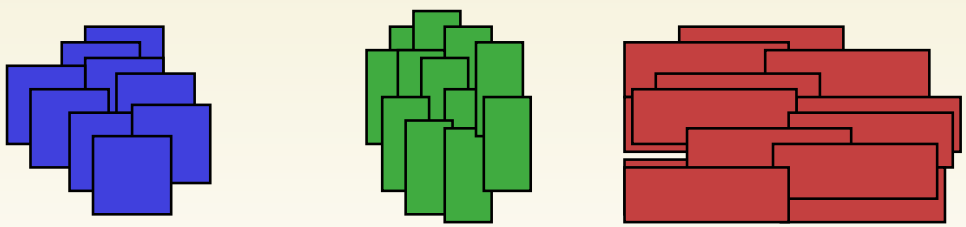

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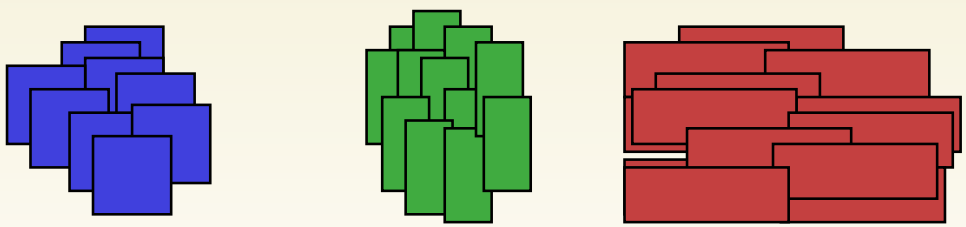

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Steady-state scheduling

- All tasks of a given application are identical and independant
  $\rightsquigarrow$ we do not really need to care about *where* and *when* (as opposed to classical scheduling problems)
- We only need to focus on average values in steady-state.
- Steady-state values:
  - Variables: average number of tasks of type k processed by processor i per time unit: $\alpha_i^{(k)}$
  - Throughput of application k : $D_k = \sum_{i \in P} \alpha_i^{(k)}$

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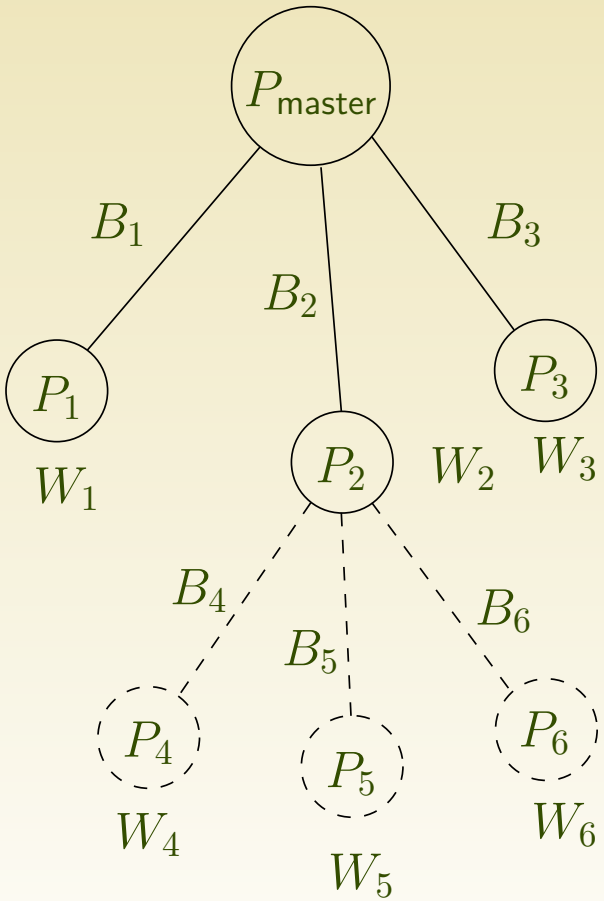

- Target platform: master-worker
  - Master P_{master} holds all tasks initially
  - Workers: $P_1, \ldots, P_p$
  - Father of P_u: $P_{f(u)}$
- Bandwidth of $P_{f(u)} \rightarrow P_u$: B_u (in MB/s)
- Speed of P_u: W_u (in MFlops/s)
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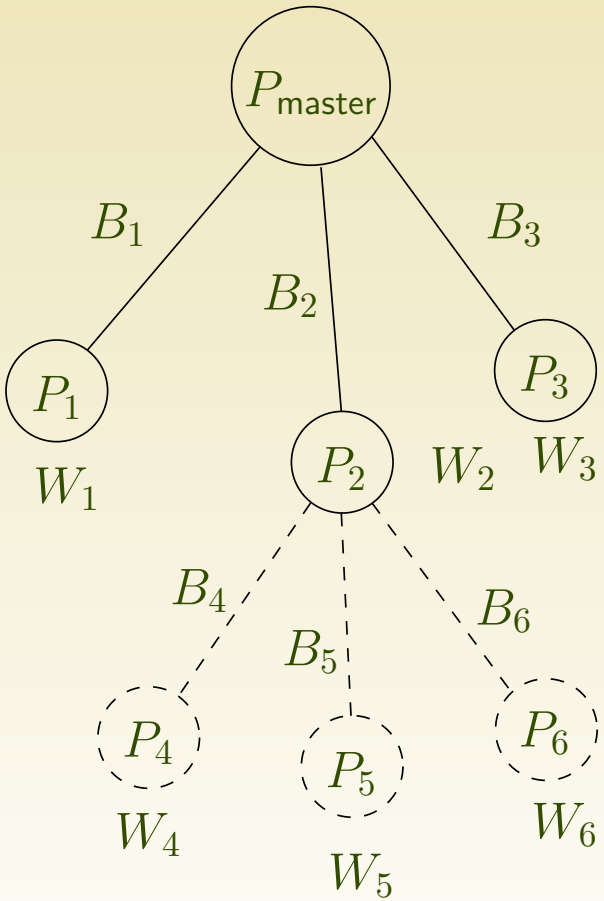

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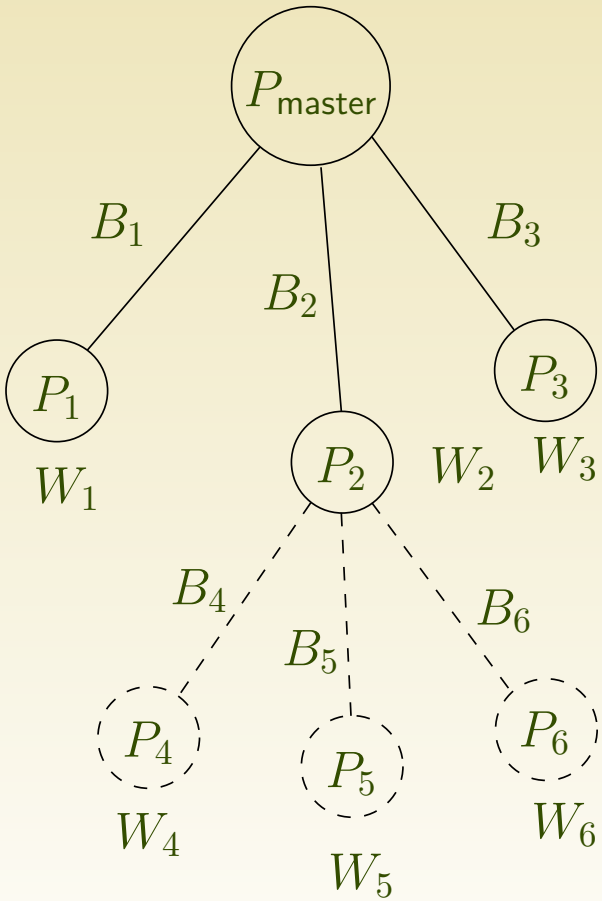

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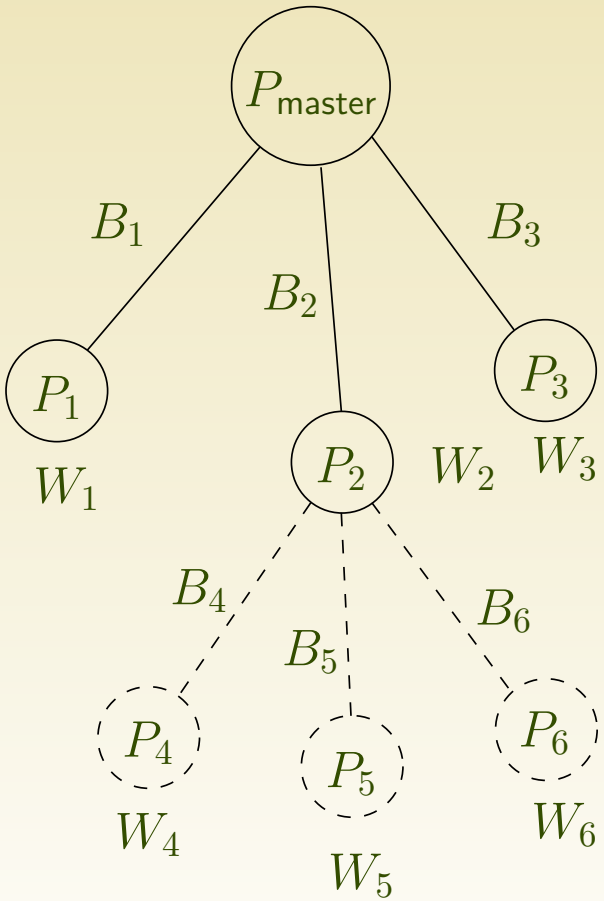

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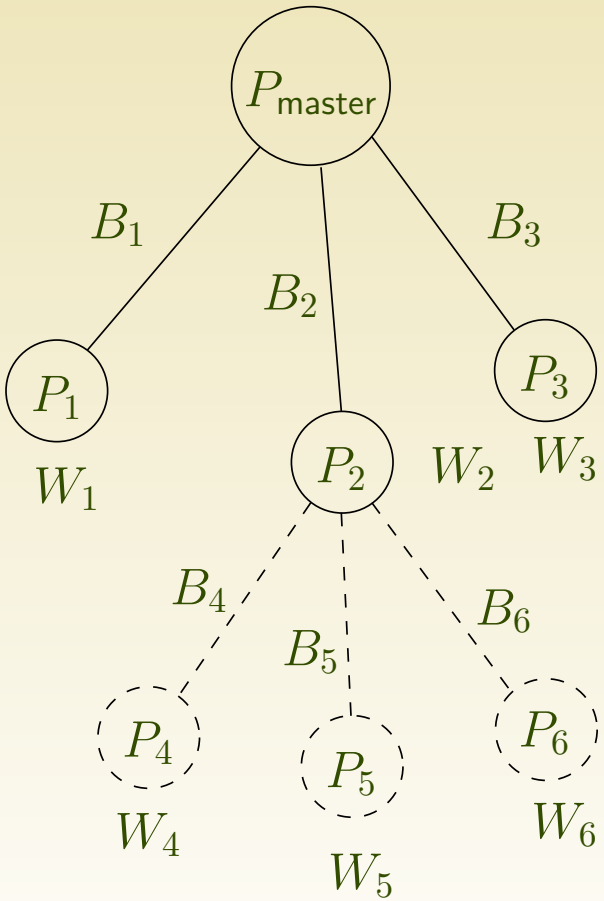

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Steady-state constraints

- Constraint for computations by P_i:

$$\forall i \in P : \sum_{k \in A} \alpha_i^{(k)} \cdot w^{(k)} \leqslant W_i$$

- Number of bytes sent on $P_{f(i)} \rightarrow P_i$:

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$$\forall i \in P, \forall k \in A : \alpha_i^{(k)} \geqslant 0$$

We would like to maximize *all* throughputs D_k.

Defining fairness enables to go from a multi-criteria problem to a more classical mono-criteria problem.

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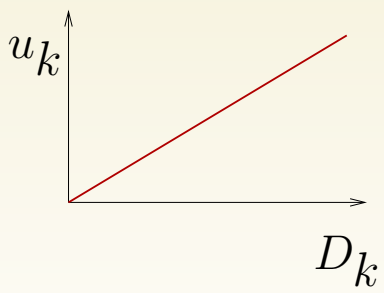
Utility Function

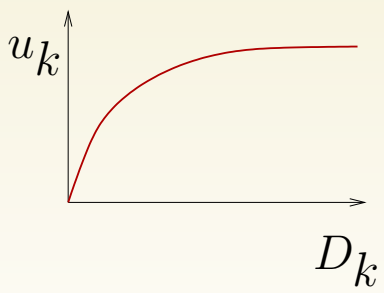
In a general context, each application is characterized by a utility function u_k defined on $(\alpha_n^{(k)})_{1 \leqslant k \leqslant K, 1 \leqslant n \leqslant N}$.
In our context, the utility is simply defined by

$$u_k(\alpha) = \sum_n \alpha_n^{(k)} = D_k$$

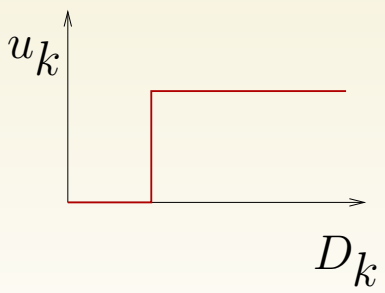
But we could perfectly imagine other utility functions:



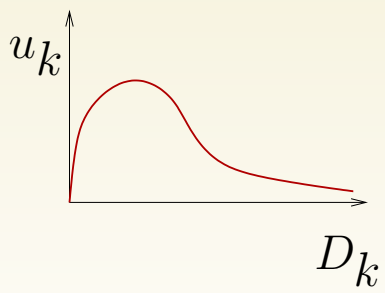
linear



Voice over IP



threshold



price-accounting

Multi-player Optimality

Our goal is to find scheduling strategies such that the utility of each user is maximized.

Definition: Pareto-optimality.

A vector of strategy is said to be Pareto optimal if it is impossible to strictly increase the utility of a player without strictly decreasing the one of another. In other words, $\widetilde{\alpha}$ is Pareto optimal iff:

$$\forall \alpha, \exists i, u_i(\alpha) > u_i(\widetilde{\alpha}) \Rightarrow \exists j, u_j(\alpha) < u_j(\widetilde{\alpha})$$

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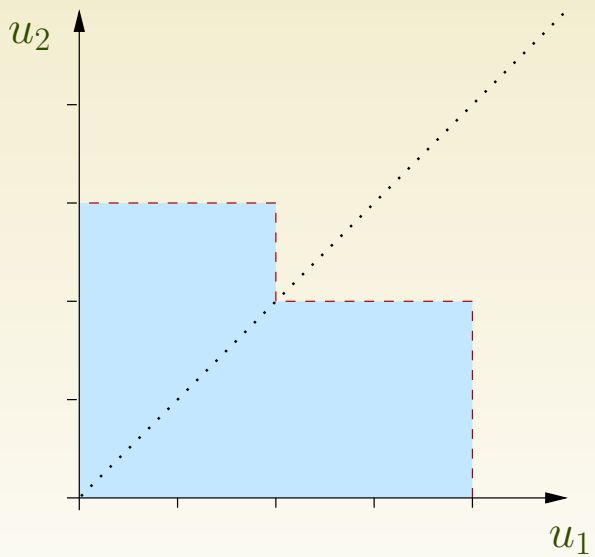

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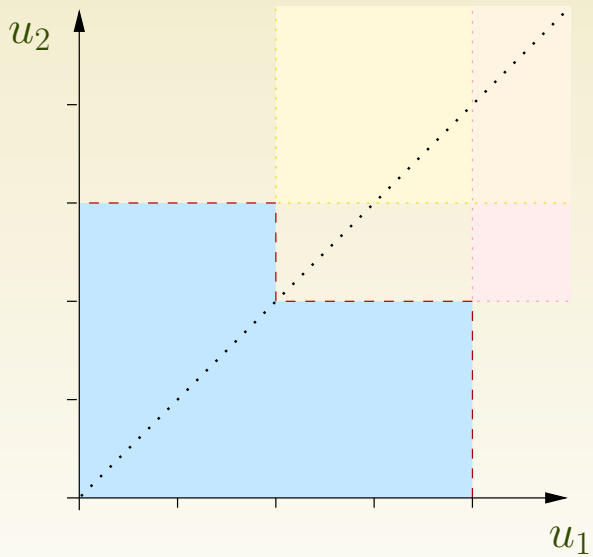

Utility Set and Fairness

How can fair be defined? Does it always mean *"give the same thing to everyone"*? How can efficiency be defined?

$$\begin{aligned}
\alpha^{(1)} \cdot c^{(1)} + \alpha^{(2)} \cdot c^{(2)} &\leqslant W_u \\
\alpha^{(1)} \cdot b^{(1)} + \alpha^{(2)} \cdot b^{(2)} &\leqslant B_u \\
\alpha^{(1)} &\geqslant 0 \\
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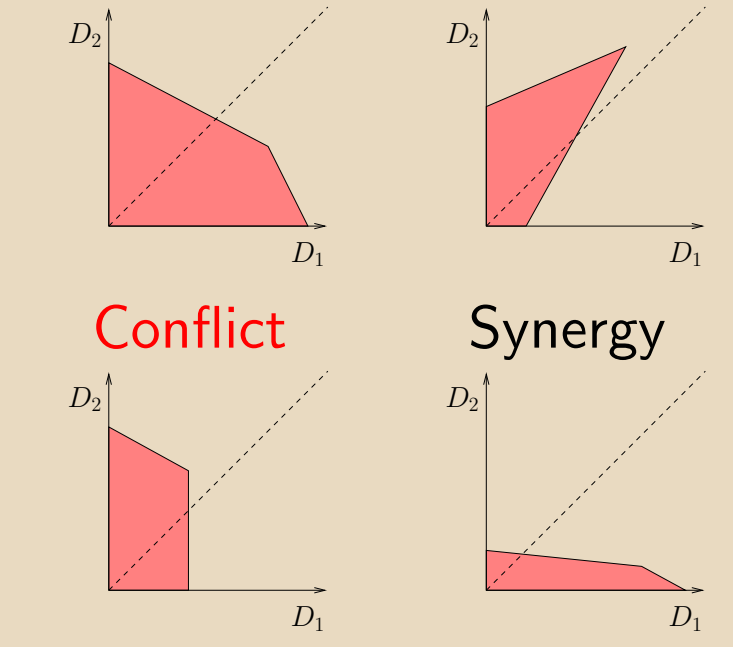
\end{aligned}$$



Fairness can be seen as the trade-off between individual satisfaction and global satisfaction.

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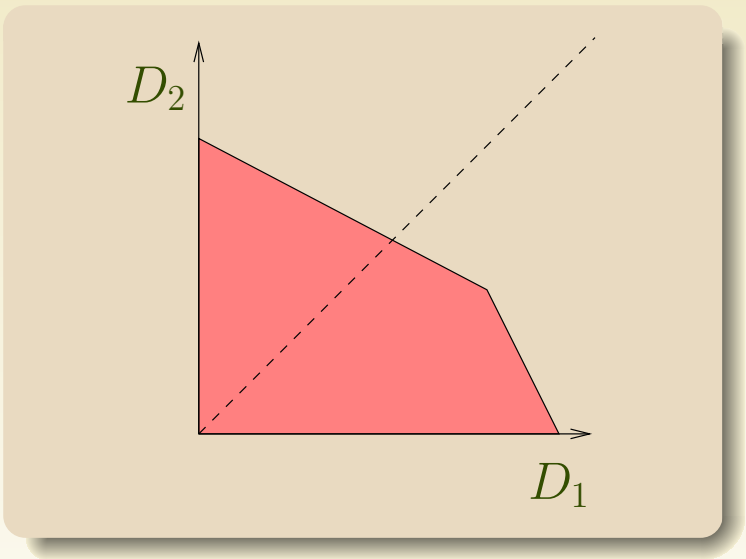

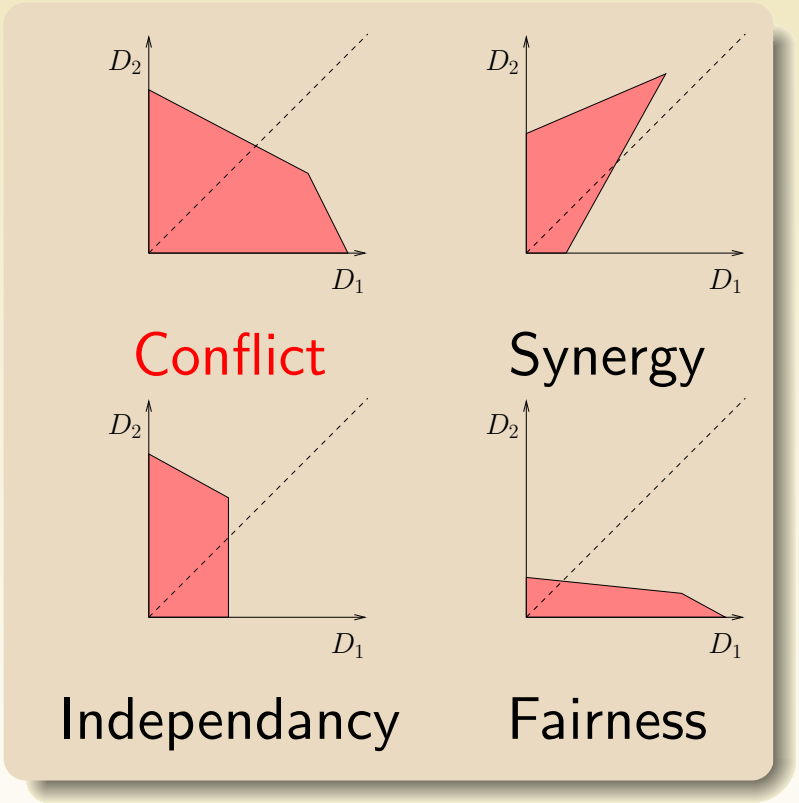

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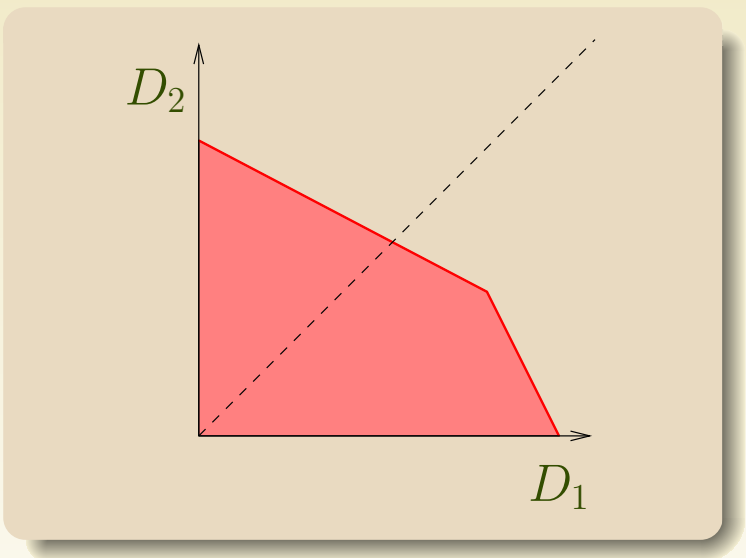

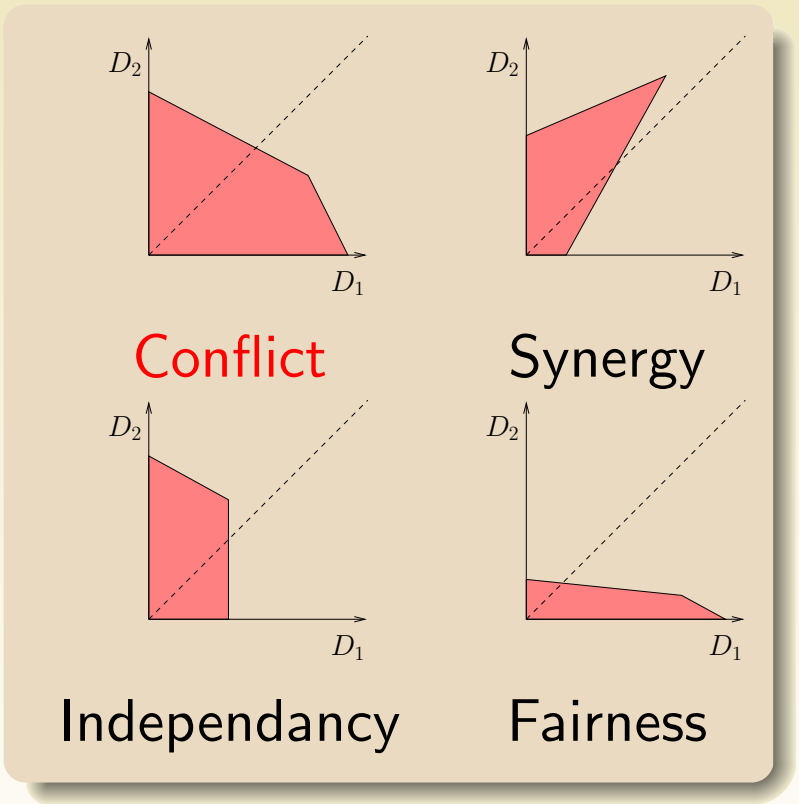

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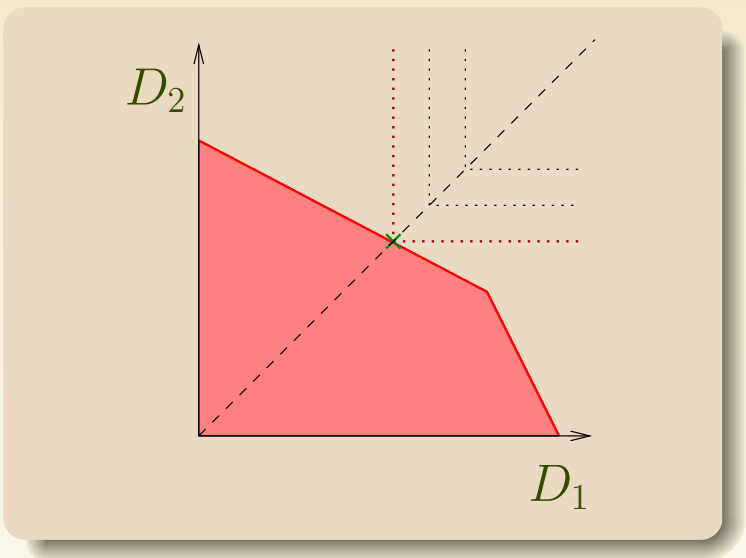

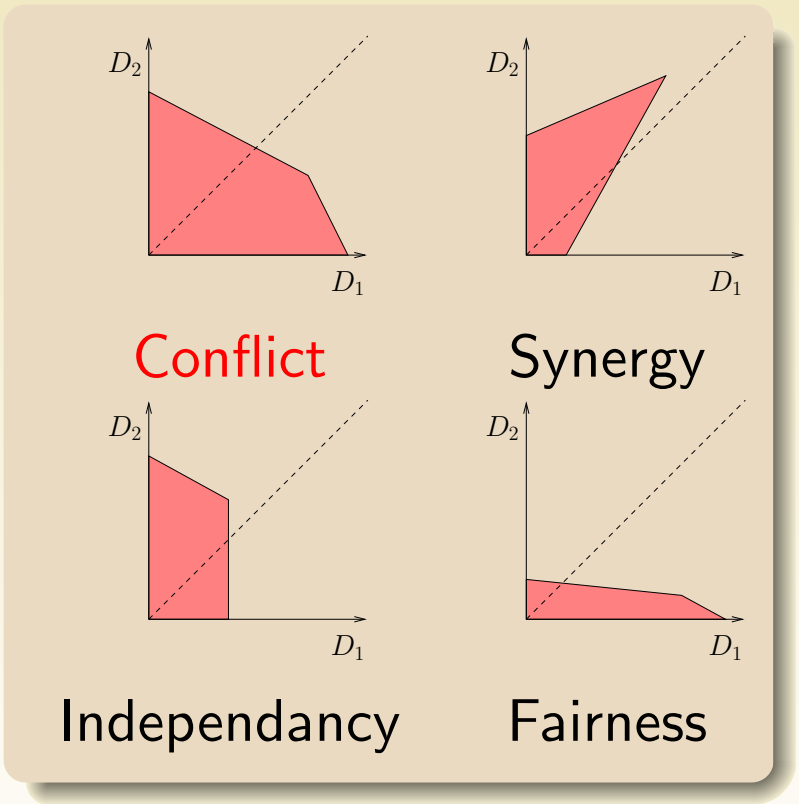

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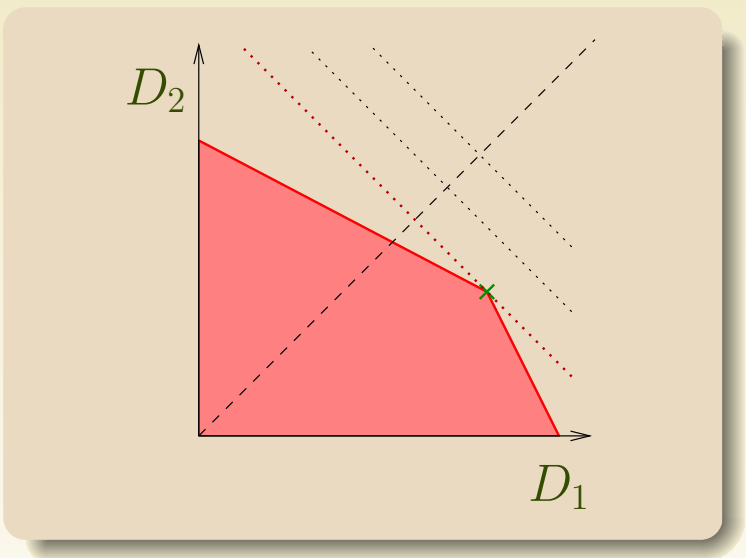

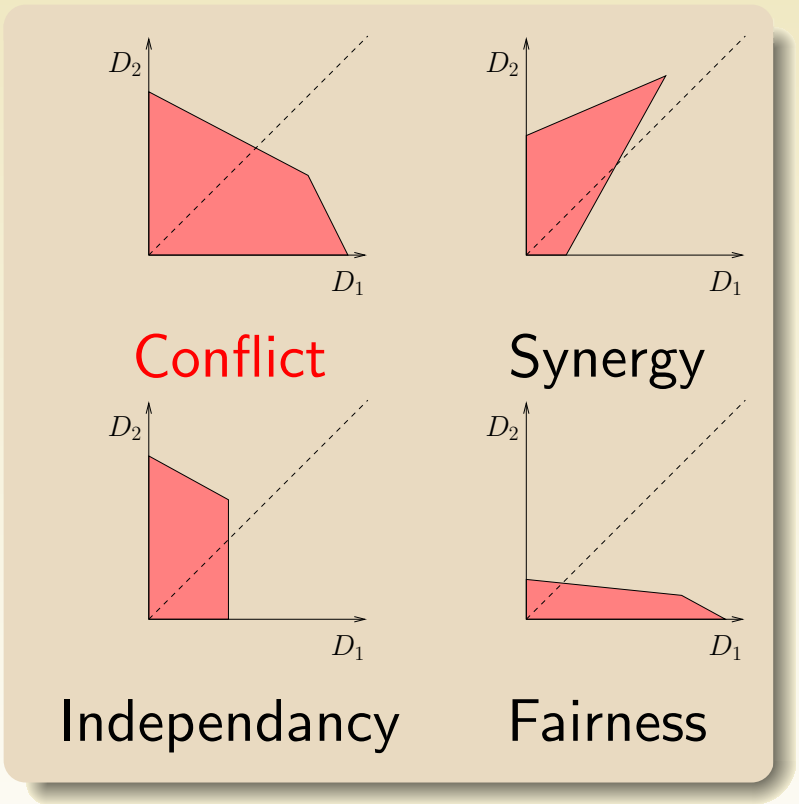

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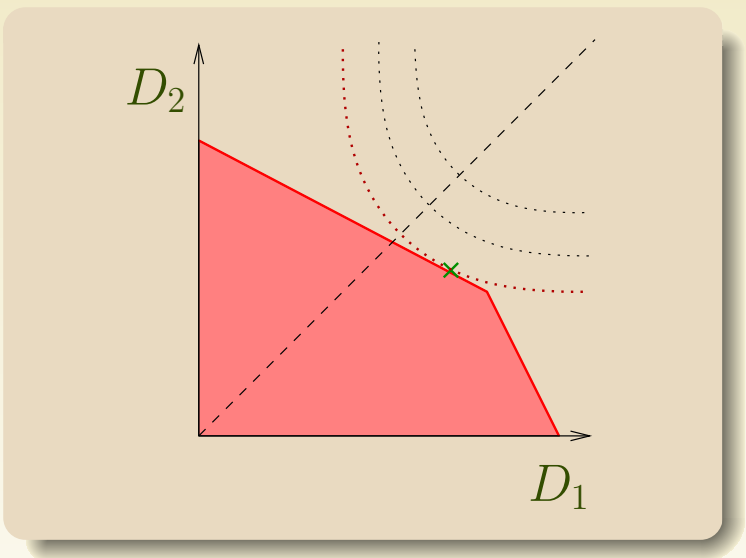

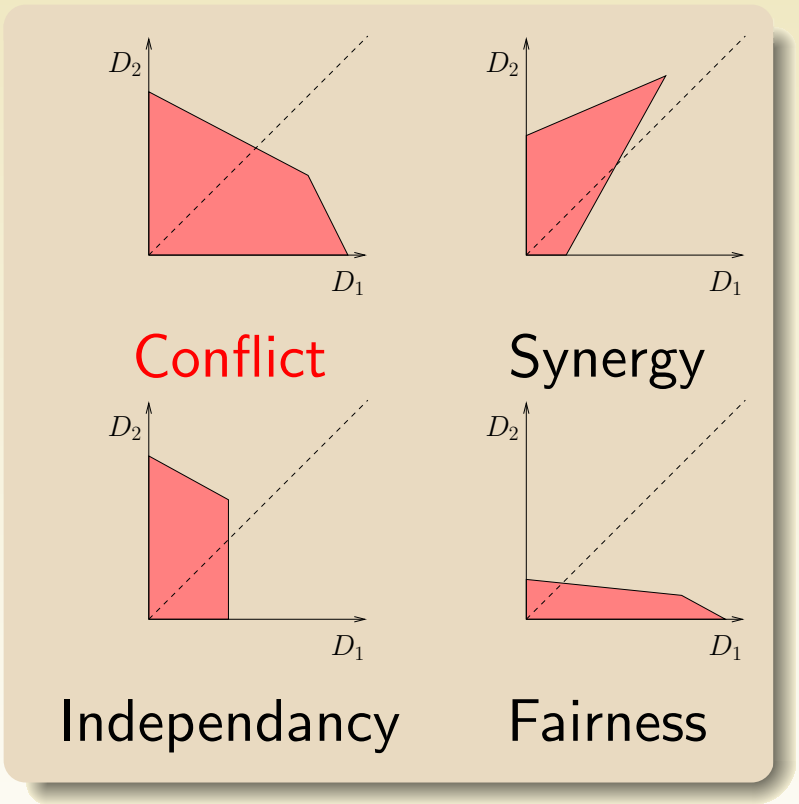

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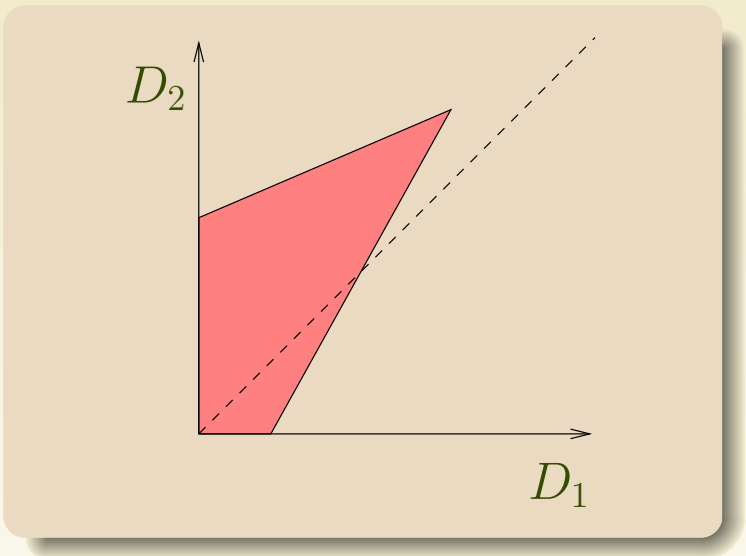

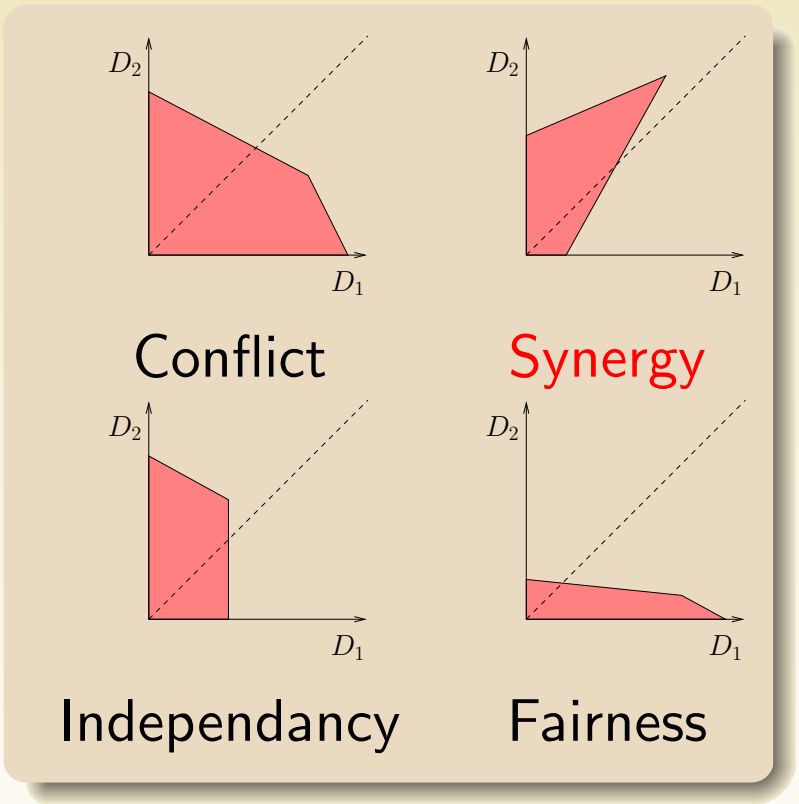

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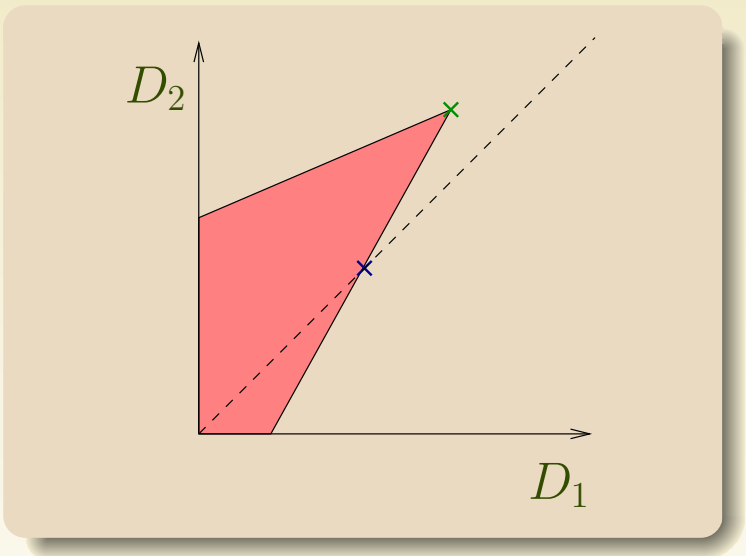

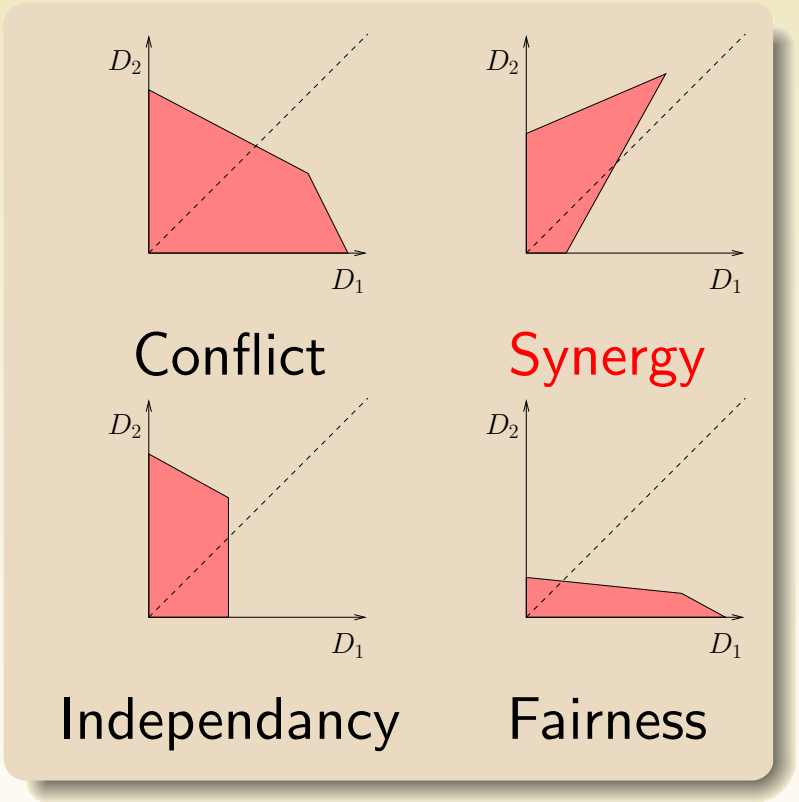

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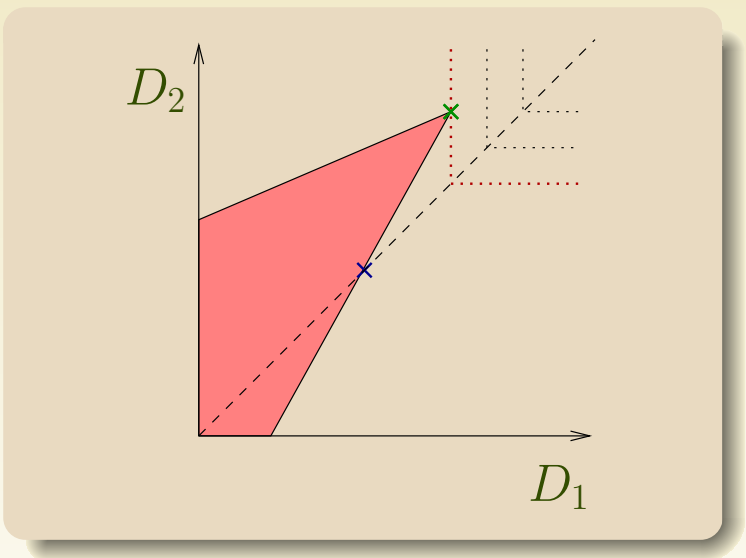

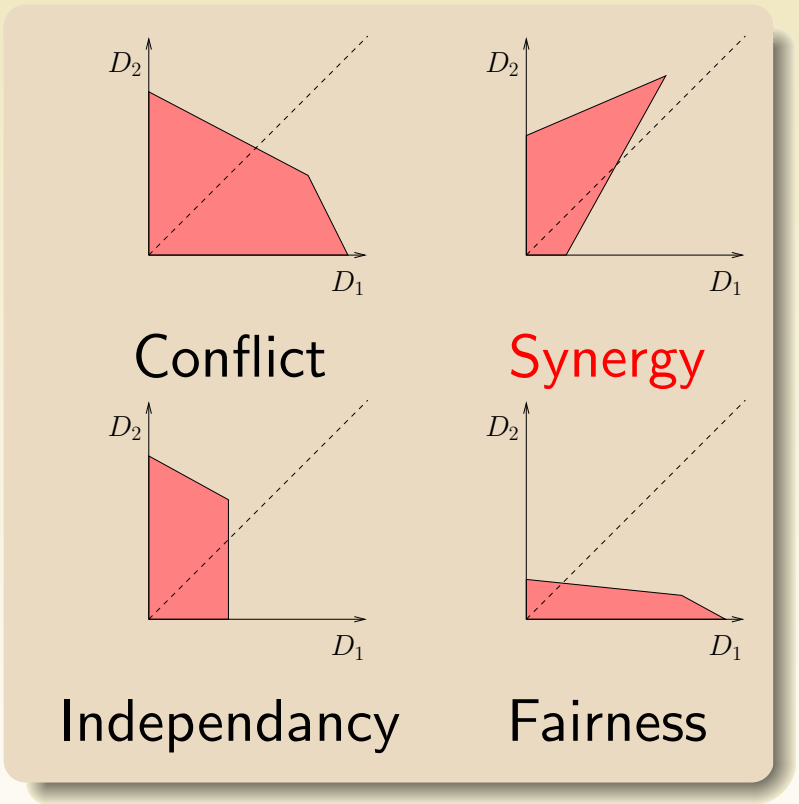

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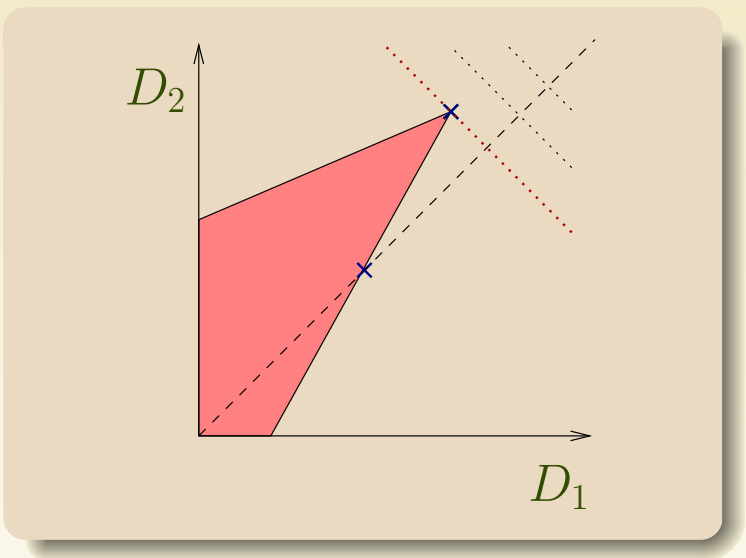

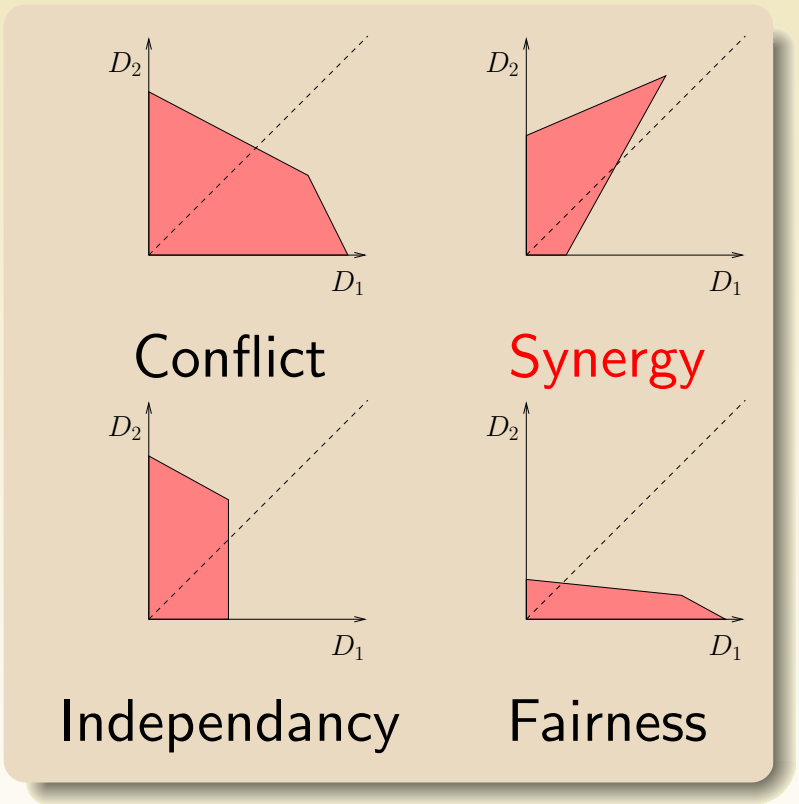

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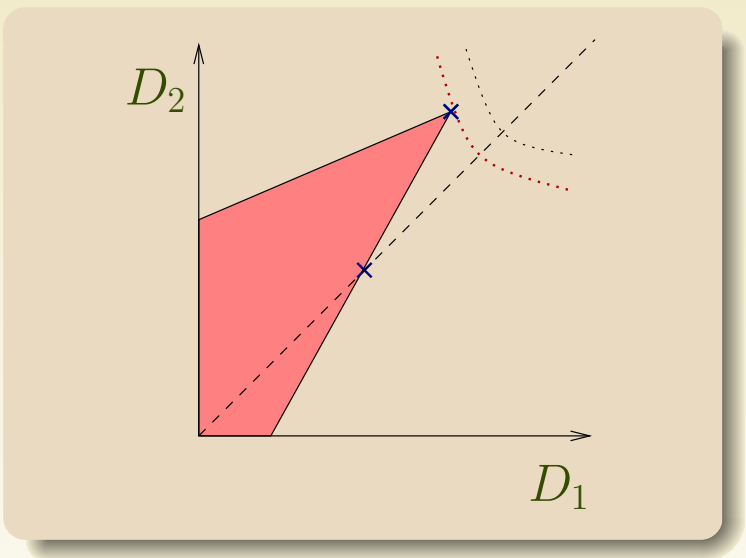

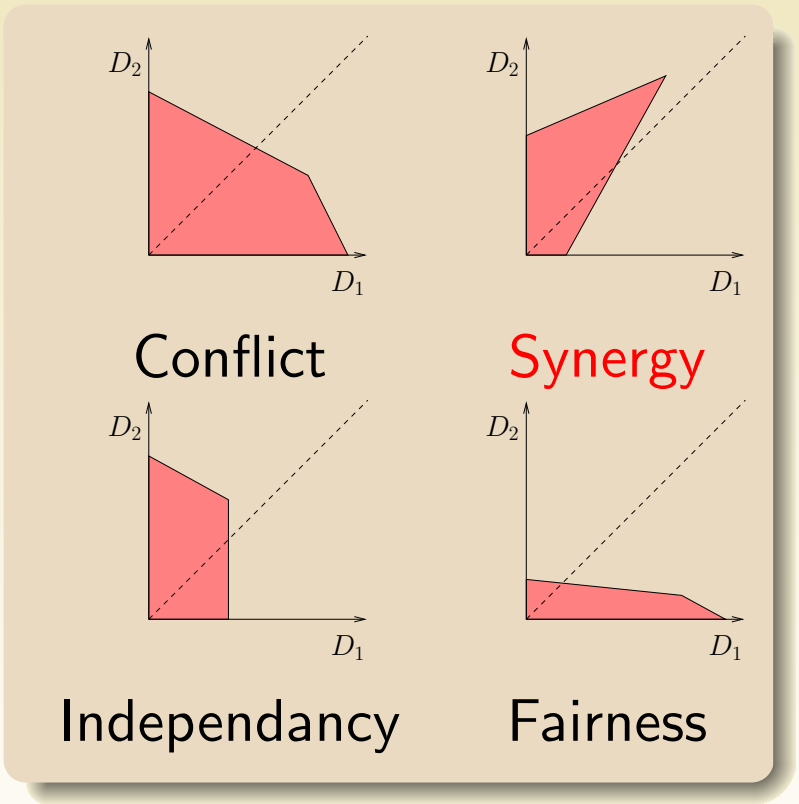

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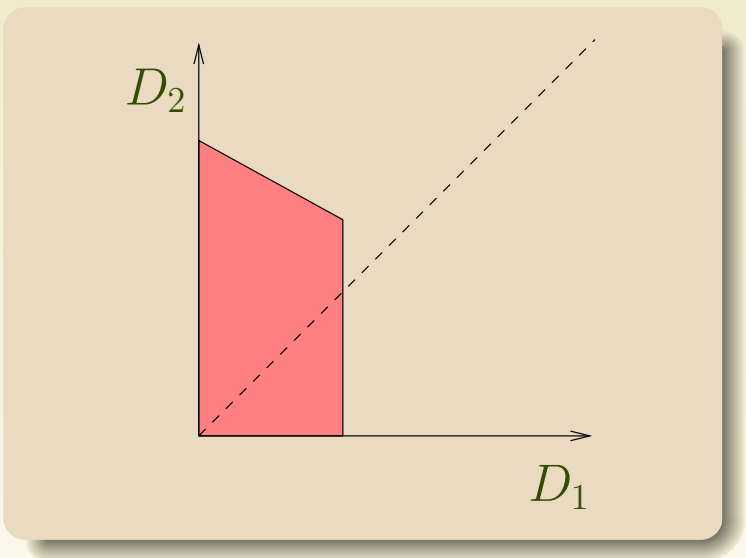

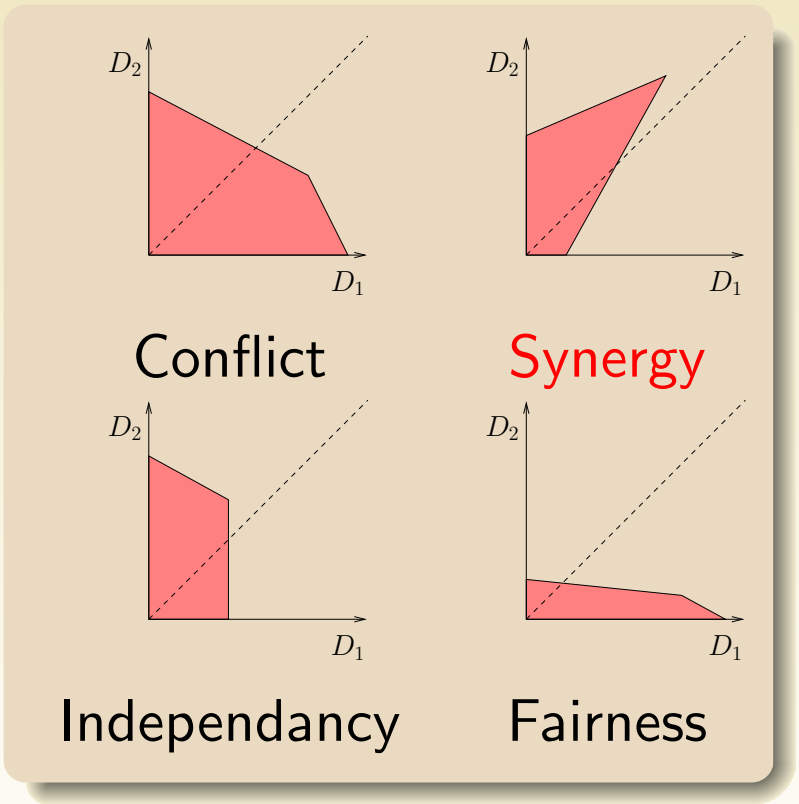

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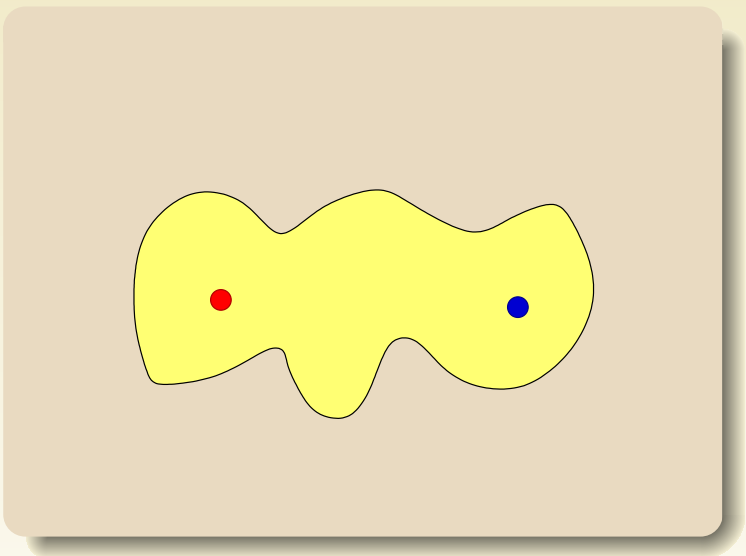

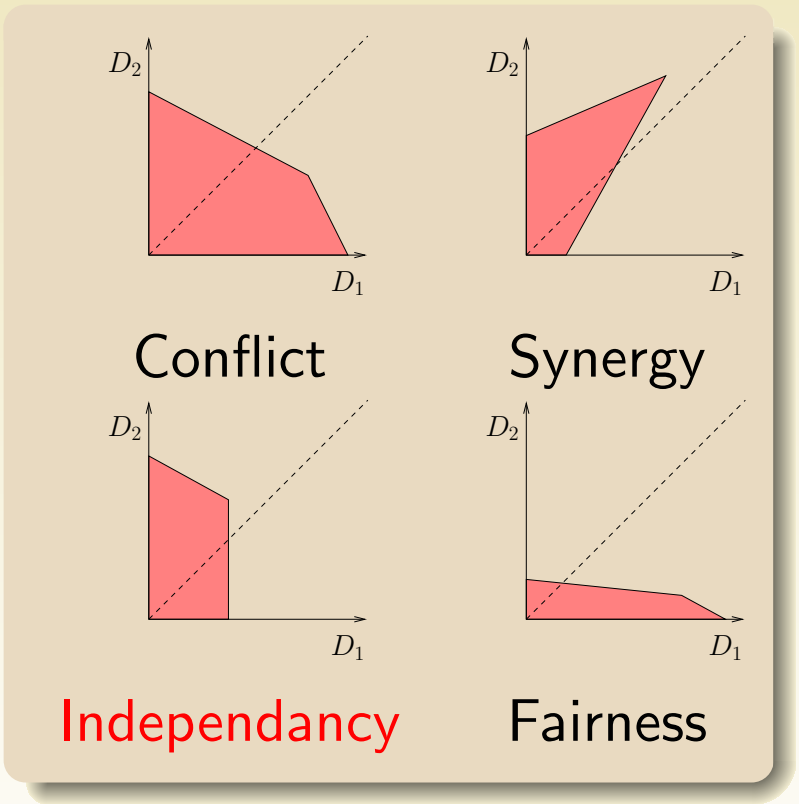

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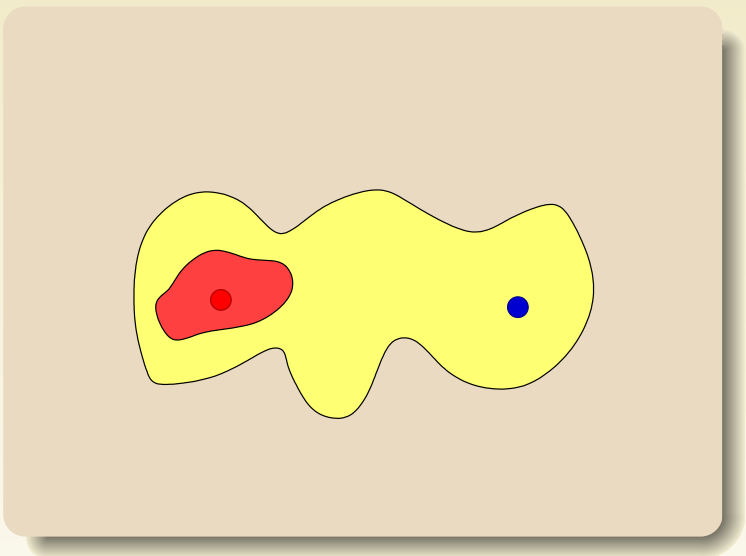

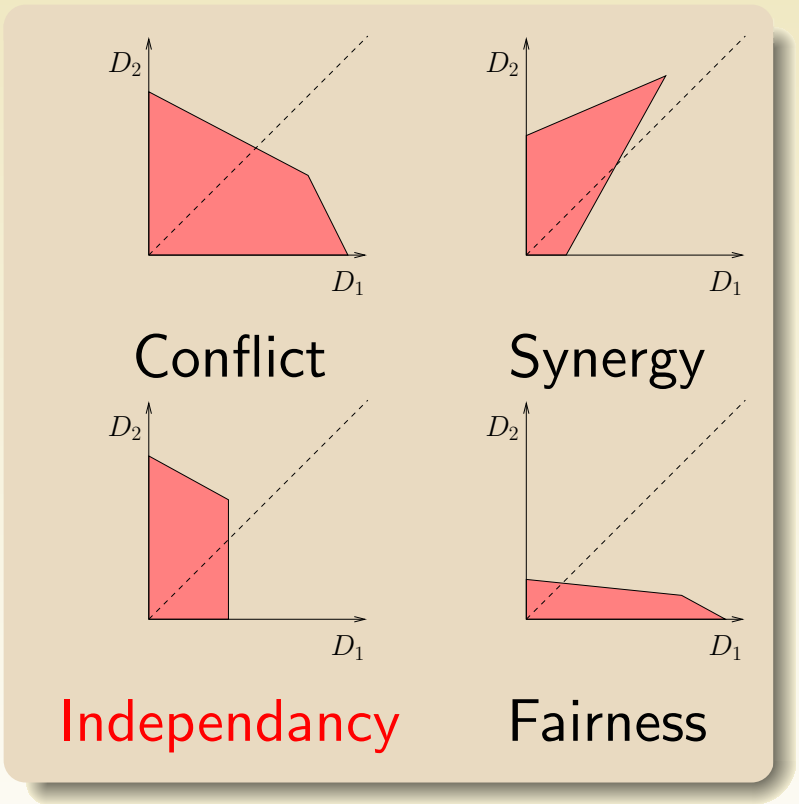

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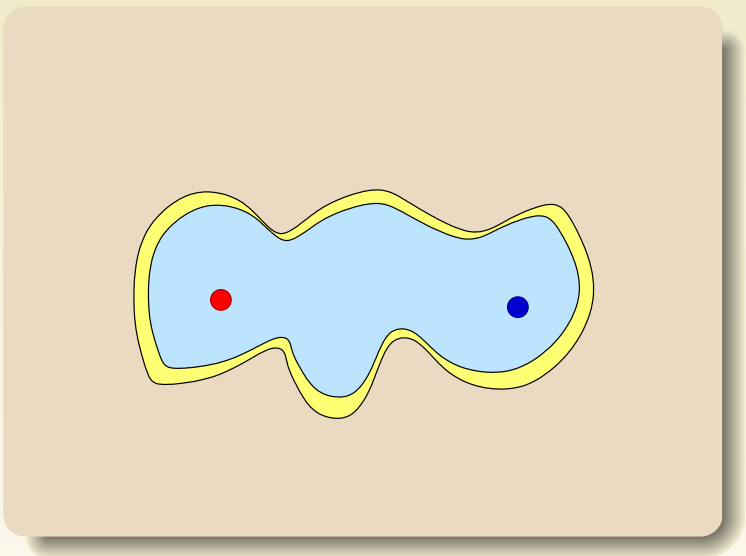

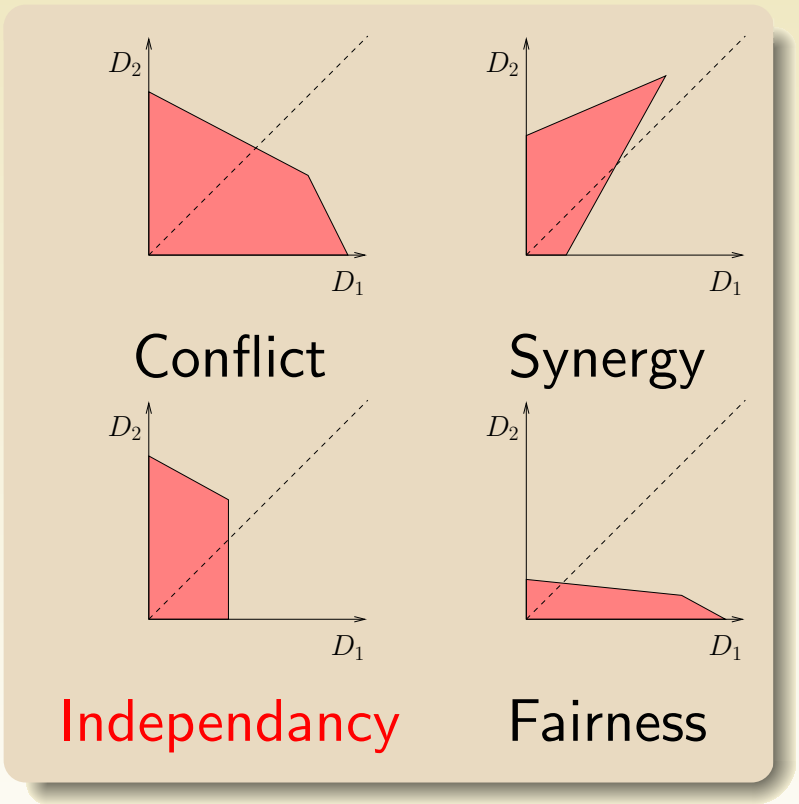

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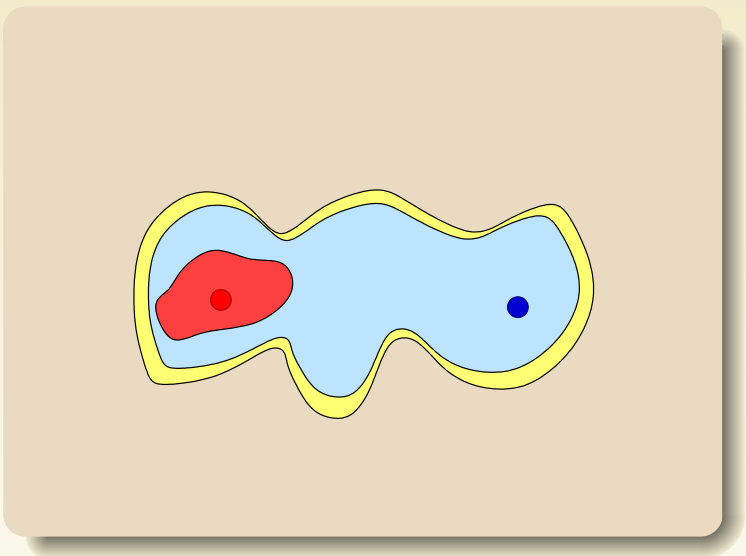

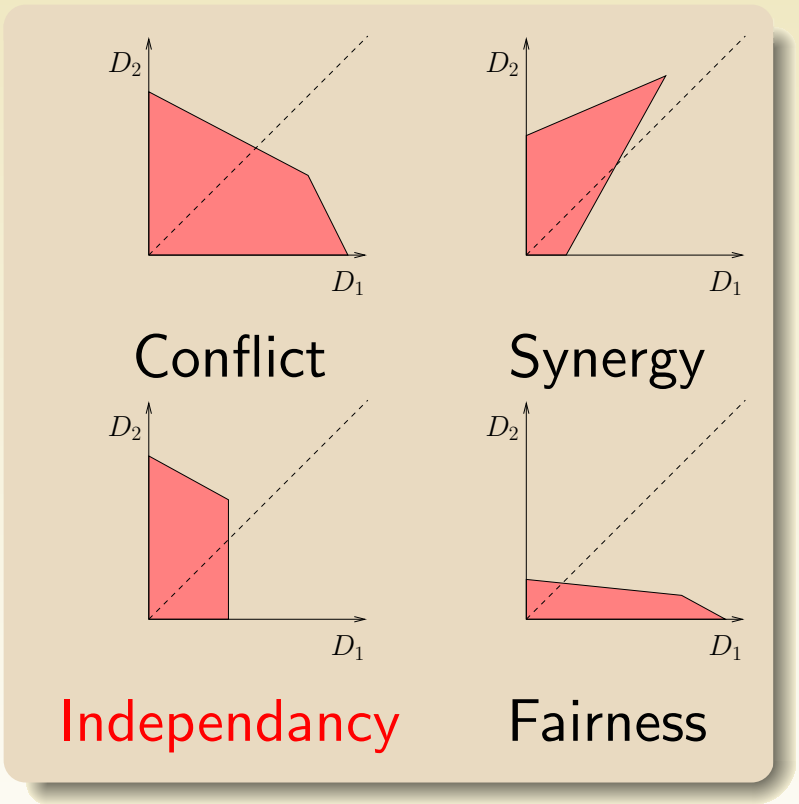

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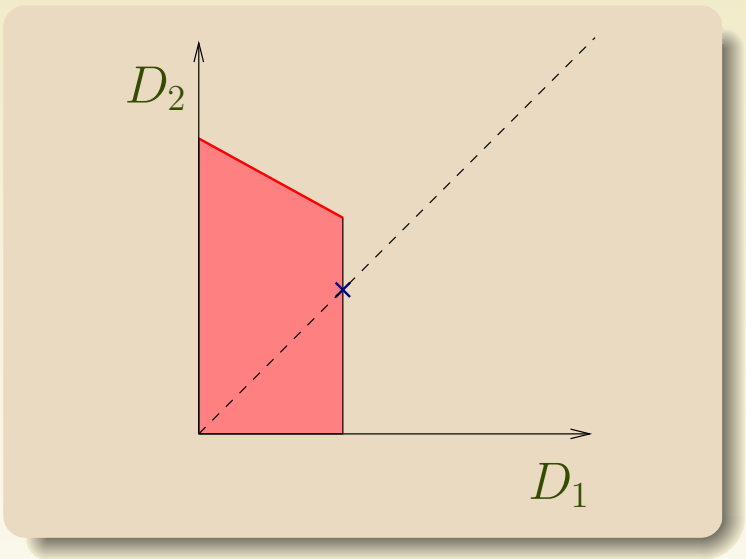

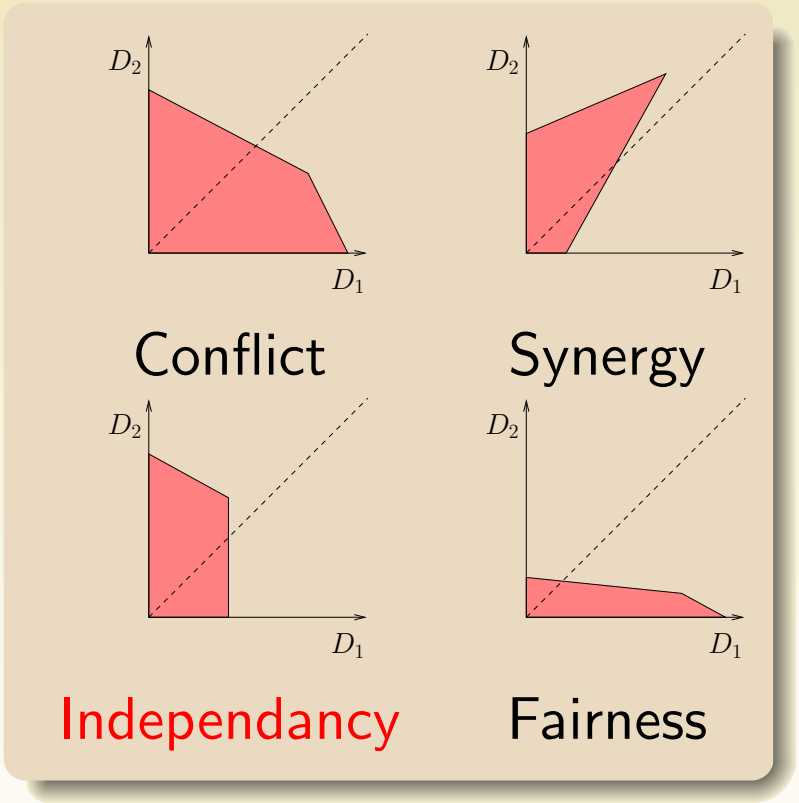

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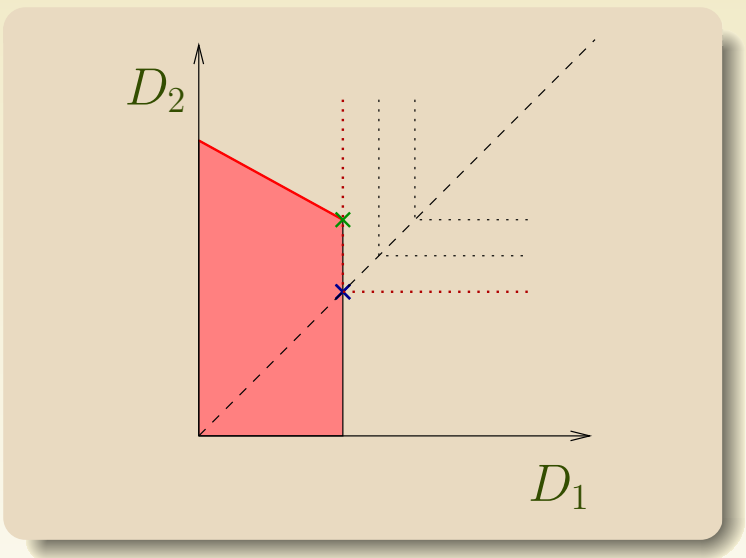

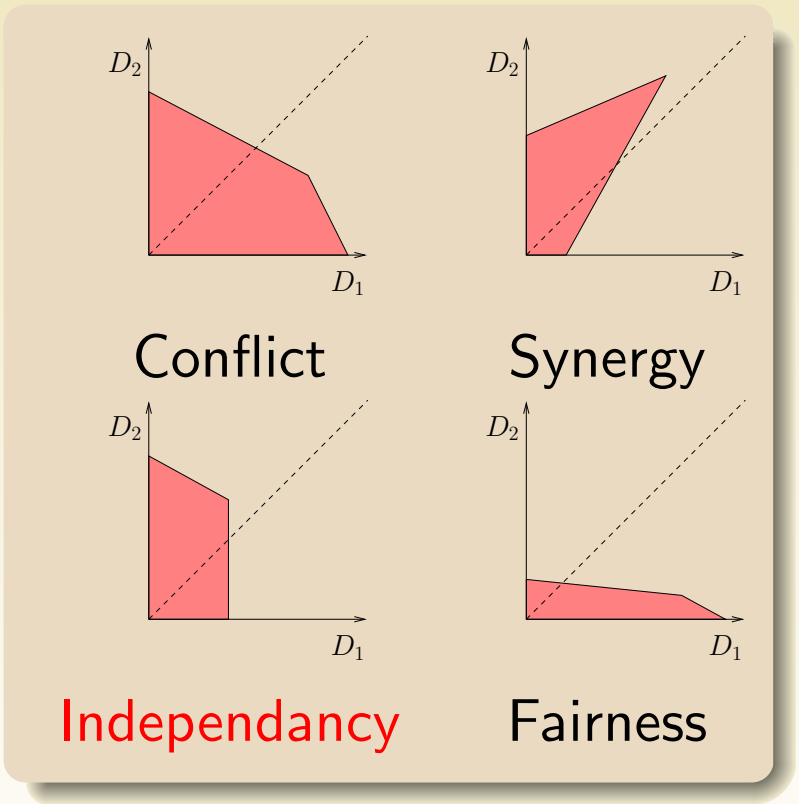

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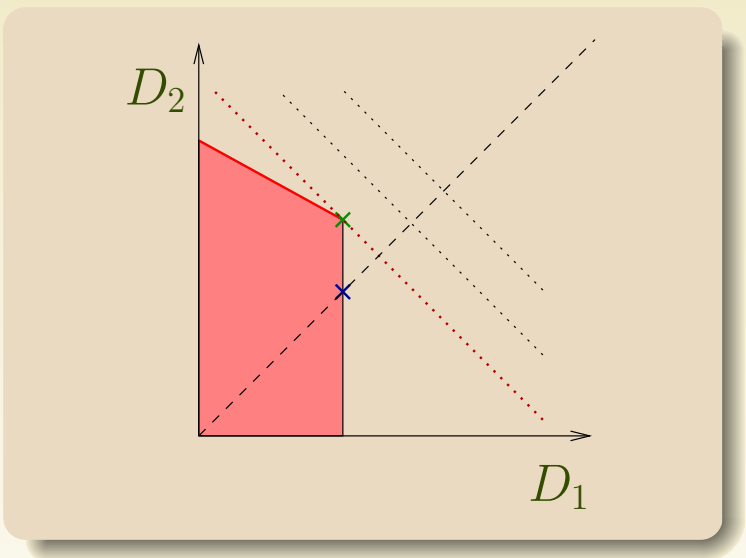

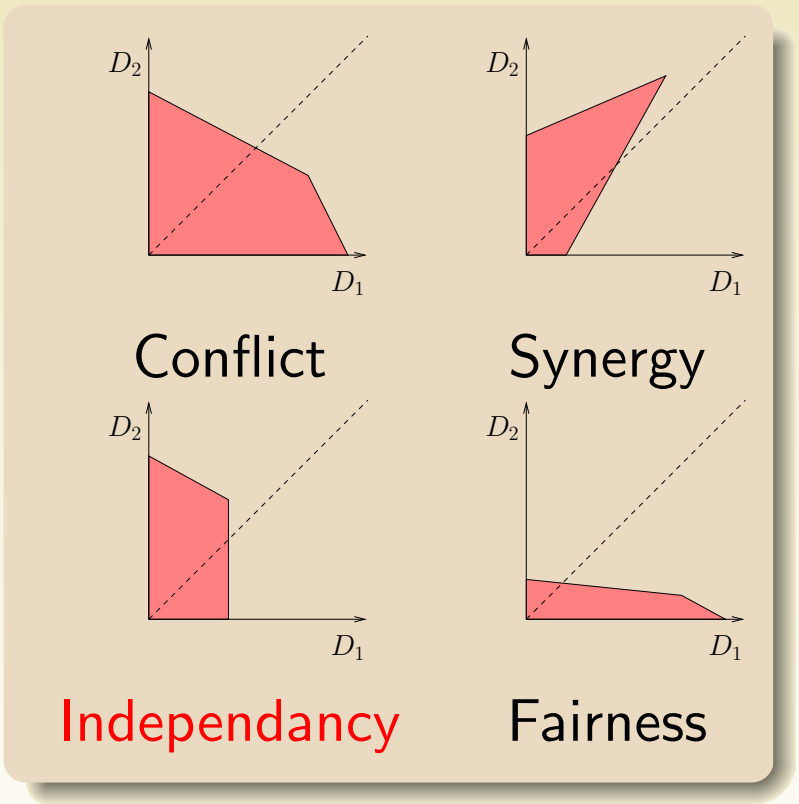

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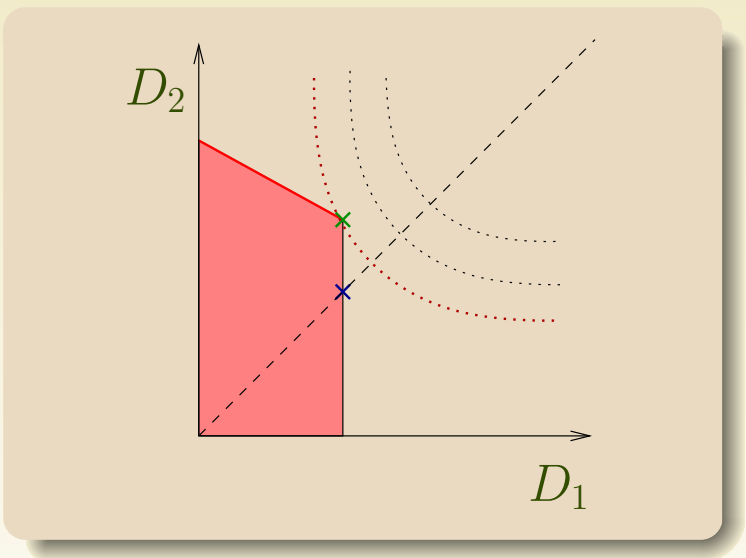

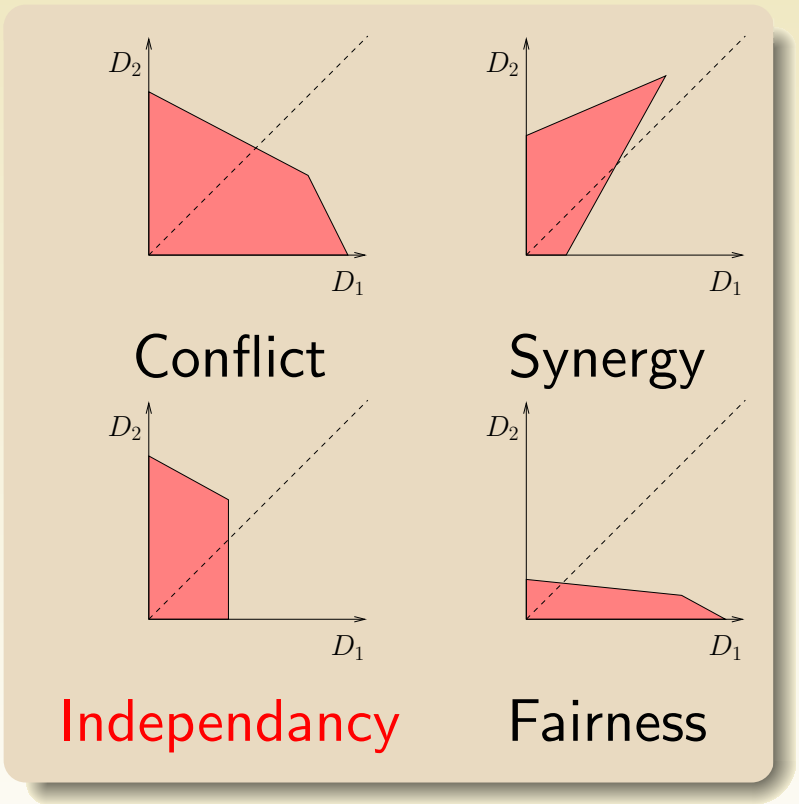

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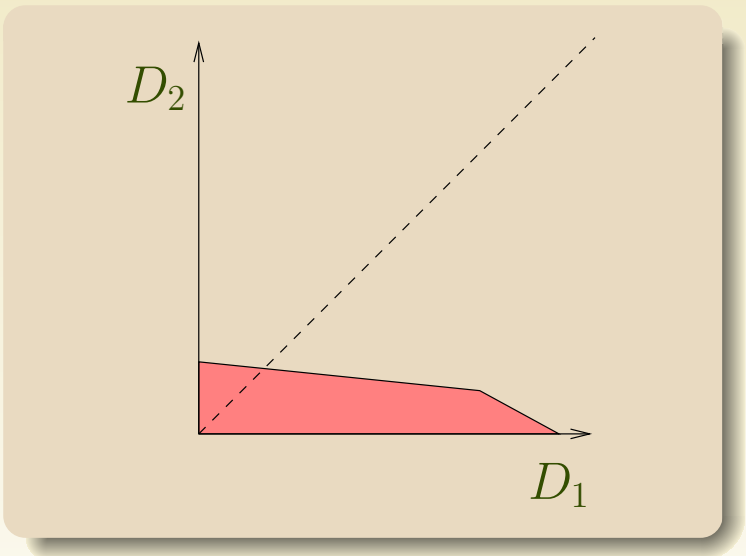

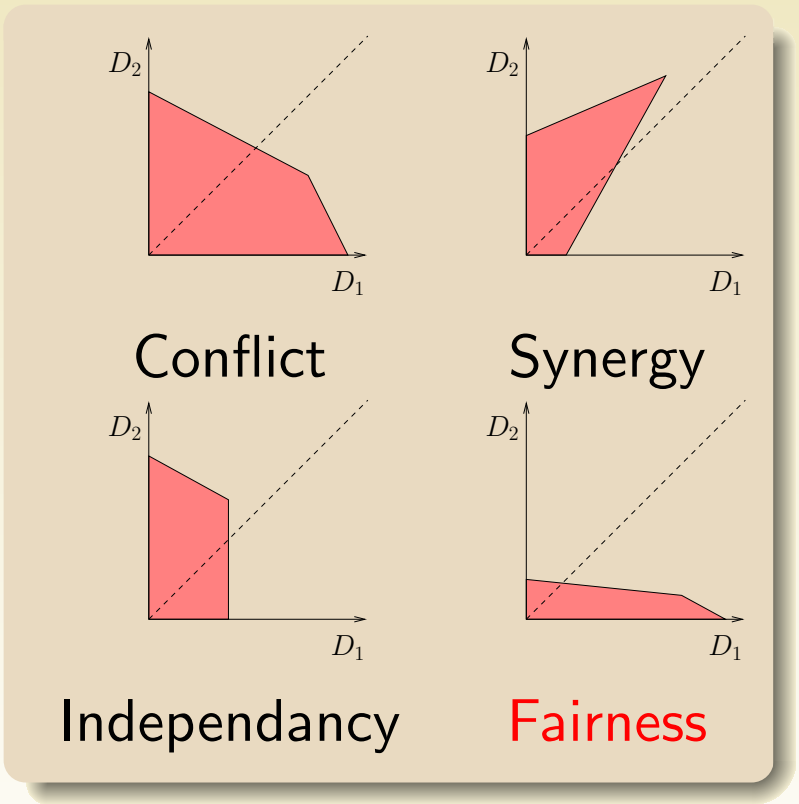

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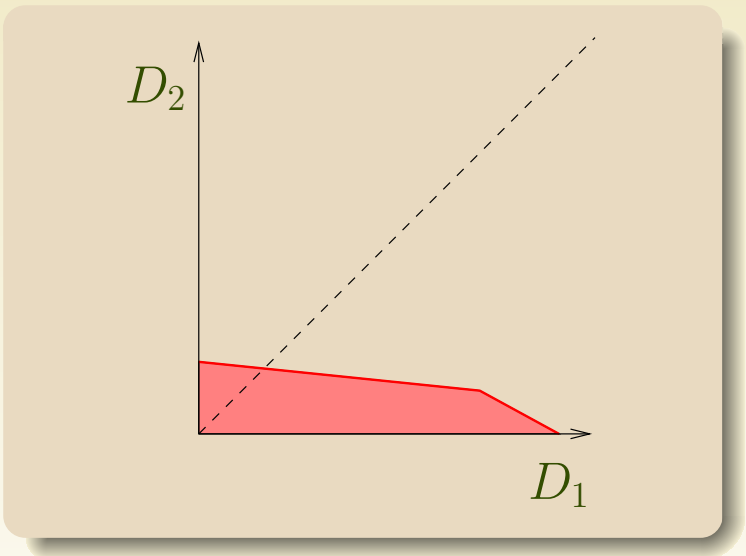

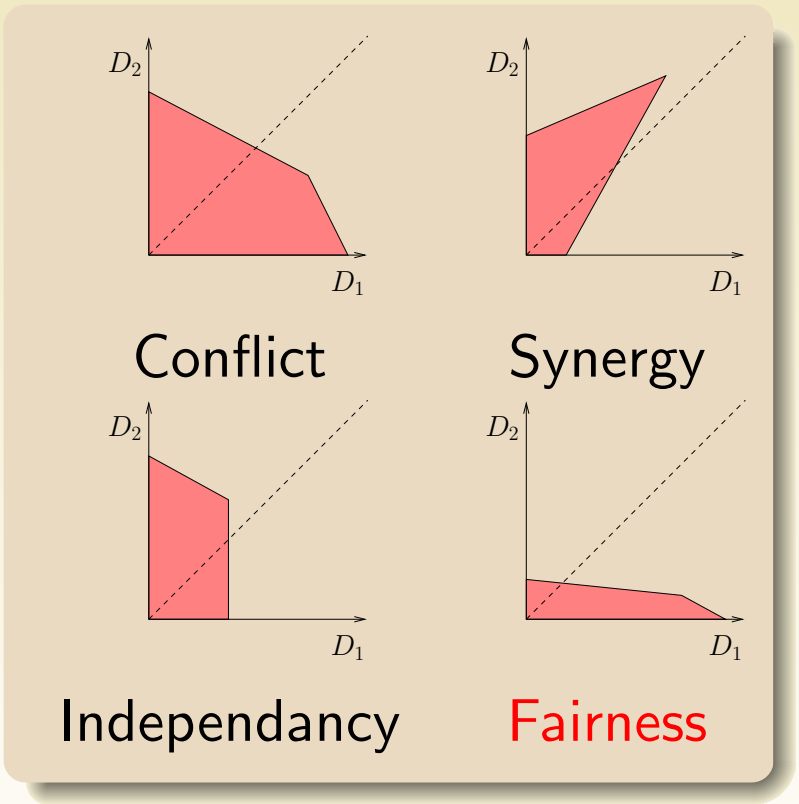

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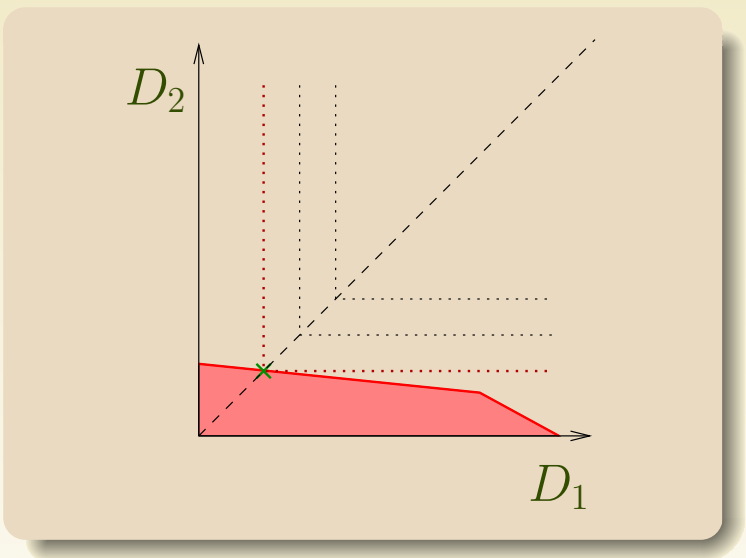

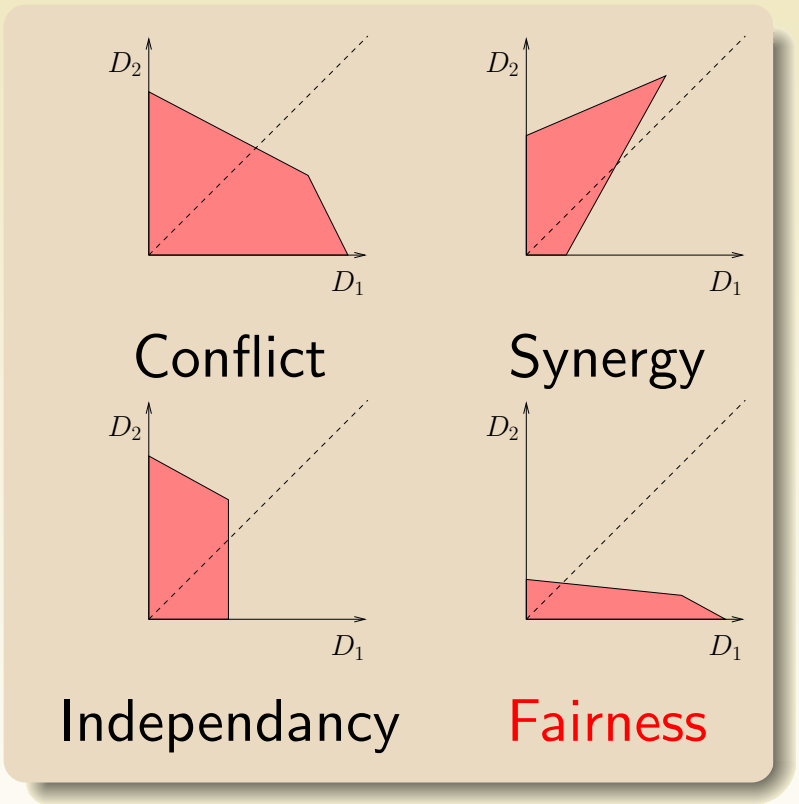

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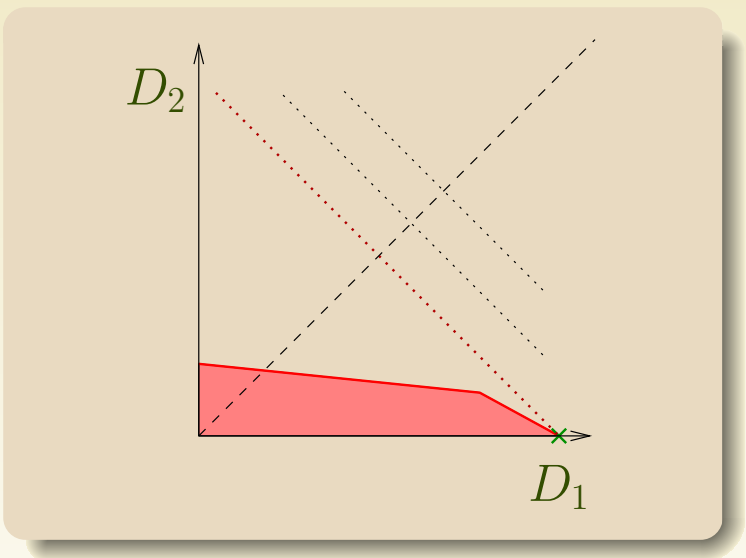

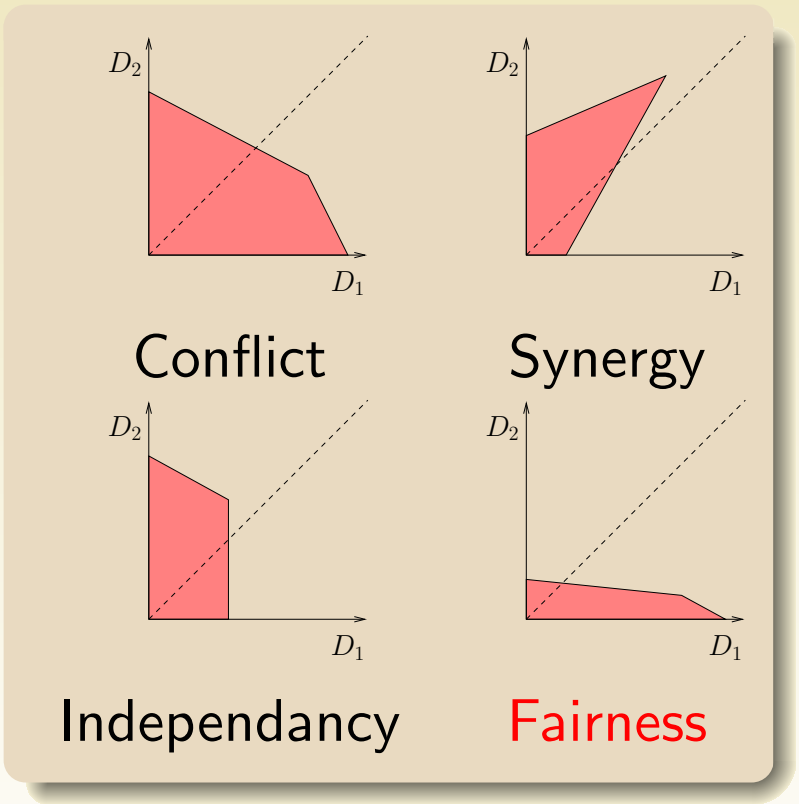

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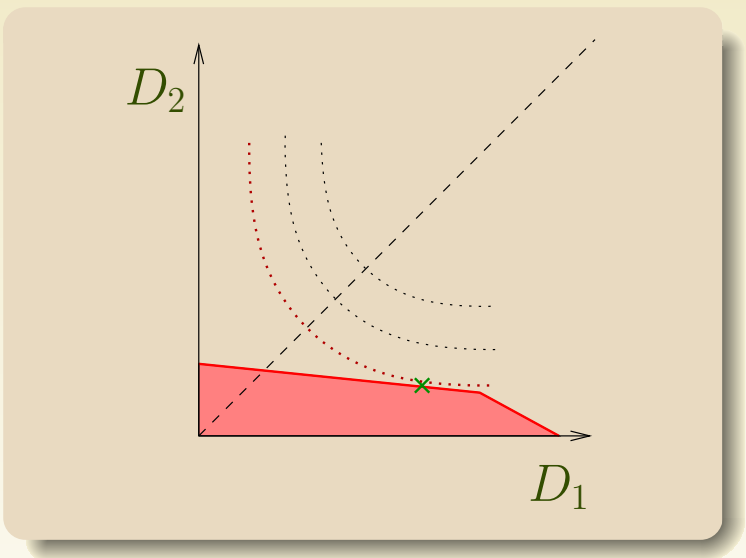

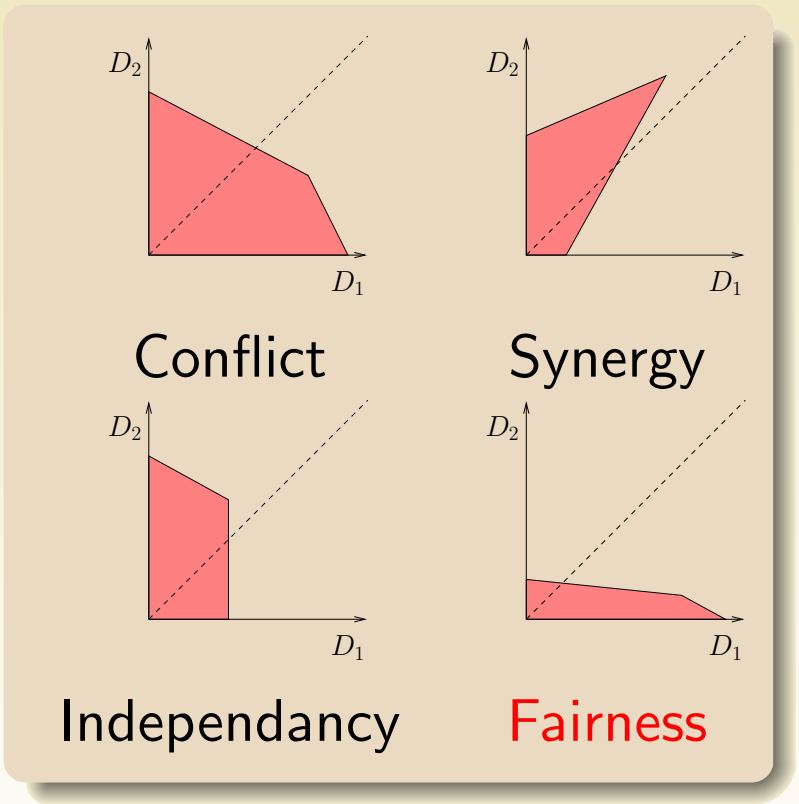

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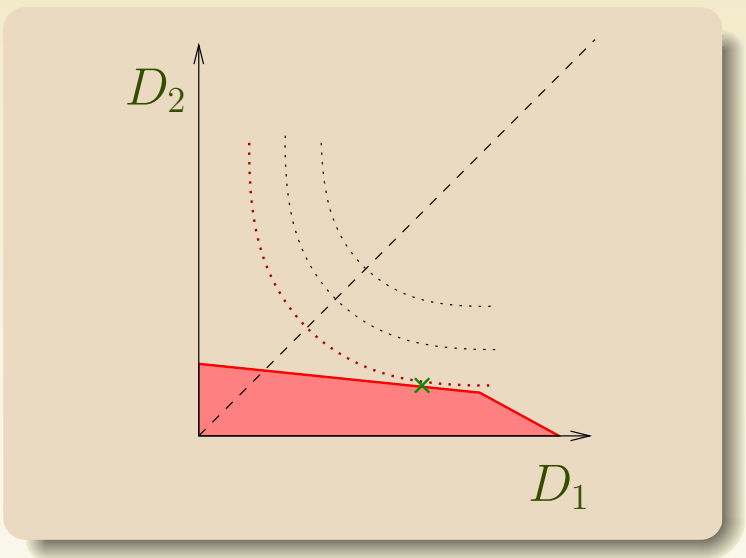

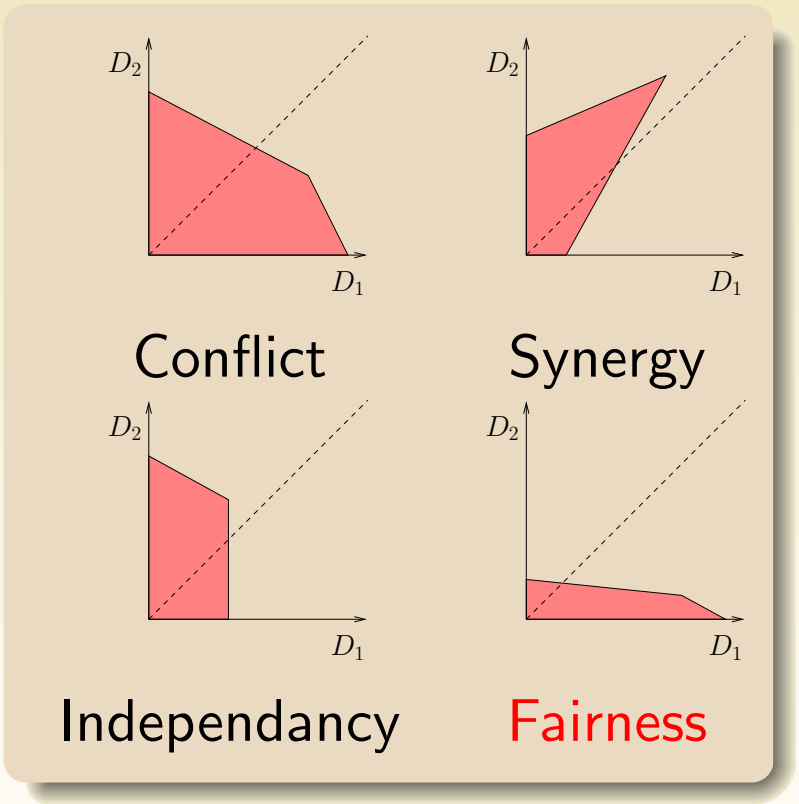

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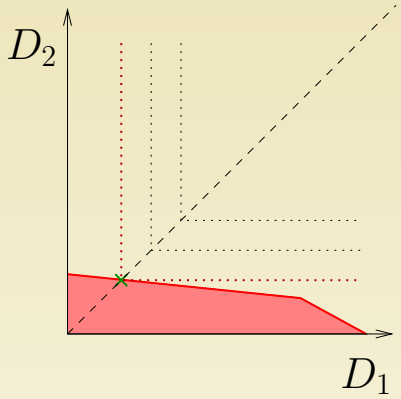

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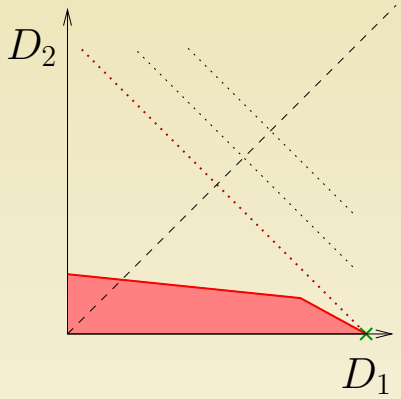
$\min_k D_k$



Social welfare
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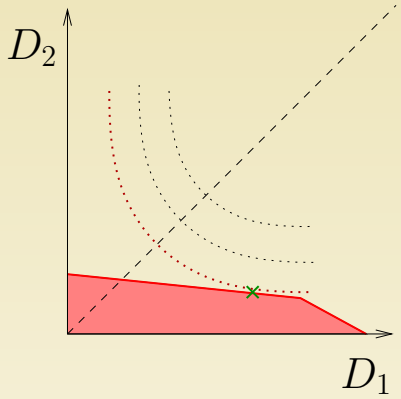

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- A few problems with max-min fairness:
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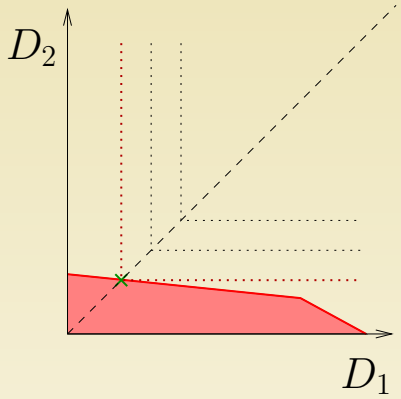

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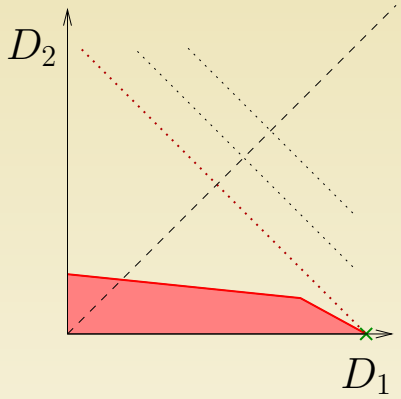

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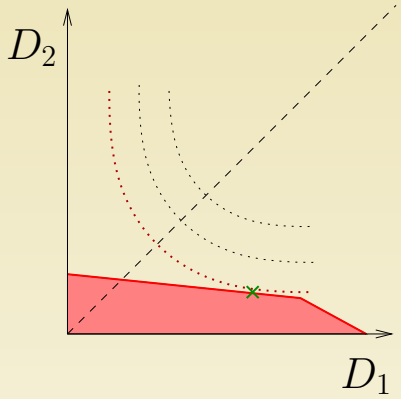

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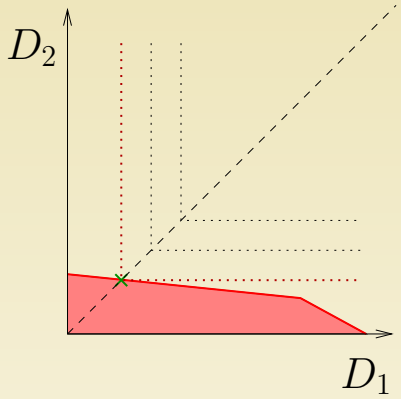

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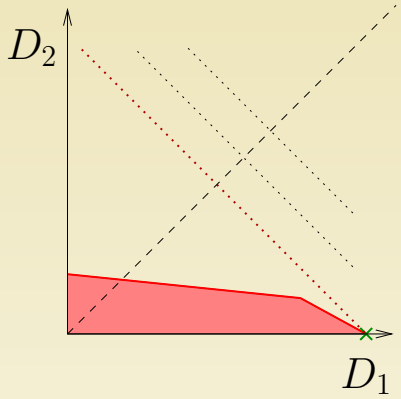

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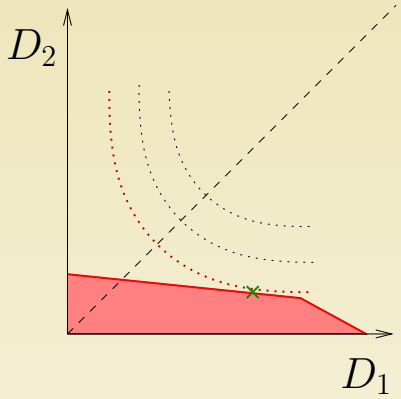

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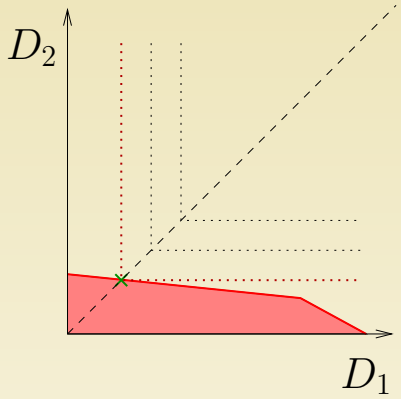

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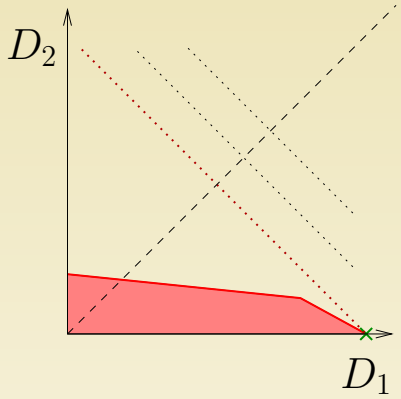

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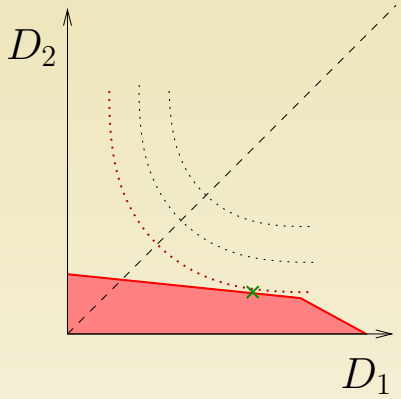

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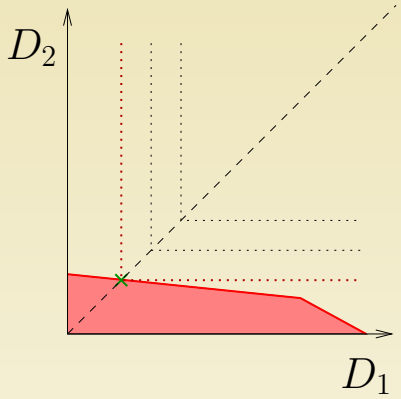

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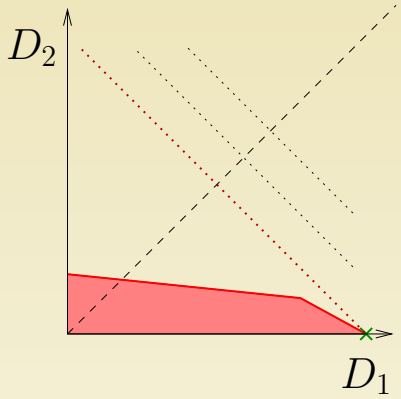

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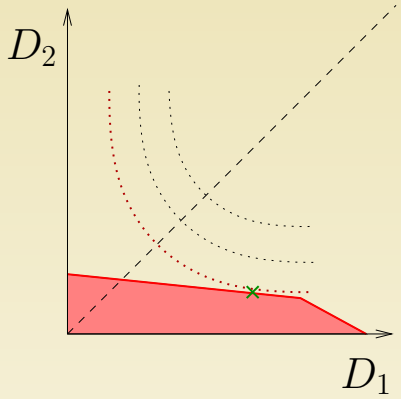

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A new optimization problem

$$\text{MAXIMIZE} \quad f(\alpha) = \sum_{k \in A} \ln \left(\sum_{i \in P} \alpha_i^{(k)} \right)$$

$$\text{UNDER THE CONSTRAINTS} \quad \begin{cases} \forall i \in P : \sum_{k \in A} \alpha_i^{(k)} \cdot w^{(k)} \leqslant W_i & \text{(Computations)} \\ \forall i \in P : \sum_{\substack{j \text{ tel que} \\ i \in M \to j}} \sum_{k \in A} \alpha_j^{(k)} \cdot b^{(k)} \leqslant B_i & \text{(Communications)} \\ \forall i \in P, \forall k \in A : \alpha_i^{(k)} \geqslant 0 & \text{(Feasability)} \end{cases}$$

- Can be solved in polynomial time with semi-definite programming. It is very centralized though.

Can we solve it in a distributed way ?

Plan

Lagrangian Optimisation: basics

- Designed to solve non linear optimization problems:
  - Let $\alpha \to f(\alpha)$ be a function to maximize
  - Let $(C_i(\alpha) \geqslant 0)_{i \in [1..n]}$ be a set of n constraints:
  - We wish to solve:

$$(P) \begin{cases} \text{maximize } f(\alpha) \\ \forall i \in [1..n], C_i(\alpha) \geqslant 0, \text{ and } \alpha \geqslant 0 \end{cases}$$

- The Lagrangian function: $\mathcal{L}(\alpha, \lambda) = f(\alpha) - \sum_{i \in [1..n]} \lambda_i C_i(\alpha)$
- The dual functionnal: $d(\lambda) = \max_{\alpha \geqslant 0} \mathcal{L}(\alpha, \lambda)$
- Under some weak hypothesis, solving (P) is equivalent to solve the dual problem:

$$(D) \begin{cases} \text{minimize } d(\lambda) \\ \lambda \geqslant 0 \end{cases}$$

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Lagrangian Optimisation: basics

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So what?..

- Two coupled problems with simple constraints.
- The structure of constraints is transposed to (D).
- This technique has been used successfully for network resource sharing.

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Trying to use Lagrangian optimization

- What does the Lagrangien function look like ?

$$\mathcal{L}(\alpha, \lambda, \mu) = f(\alpha) - \sum_{i \in P} \lambda_i \cdot \left(\sum_{k \in A} \alpha_i^{(k)} \cdot w^{(k)} - W_i \right) - \sum_{i \in P} \mu_i \cdot \left(\sum_{\substack{j \text{ tel que} \\ i \in M \to j}} \sum_{k \in A} \alpha_j^{(k)} \cdot b^{(k)} - B_i \right)$$

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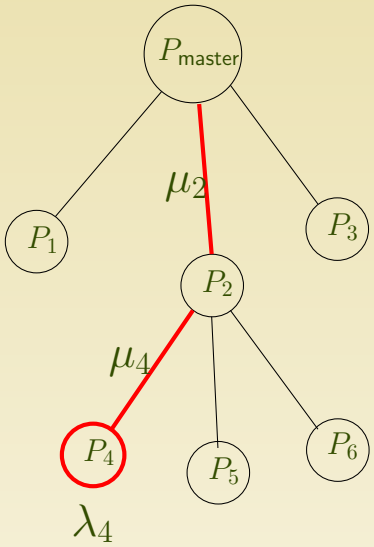

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- d is convex. Distributed gradient methods can be used to minimize d: each resource will update it's price accordingly.
- Minimizing d can be seen as maximizing the global *income* of resources.
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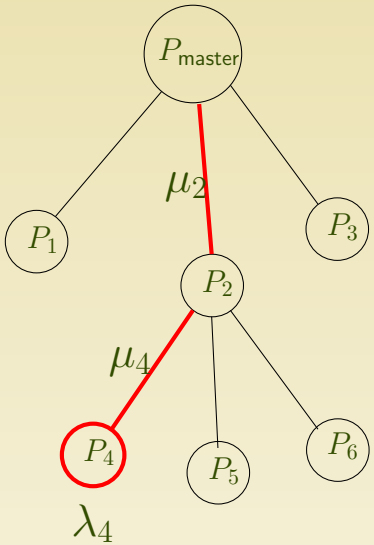

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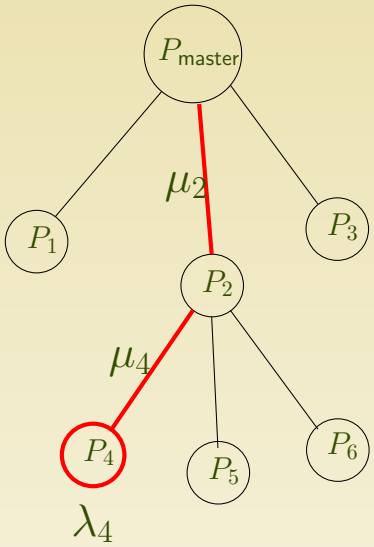

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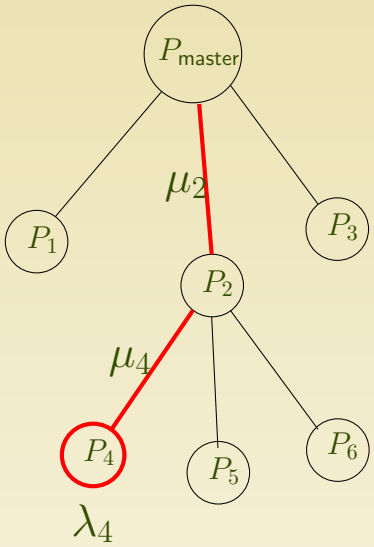

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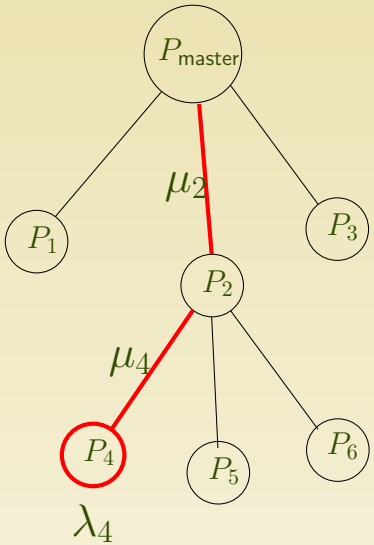

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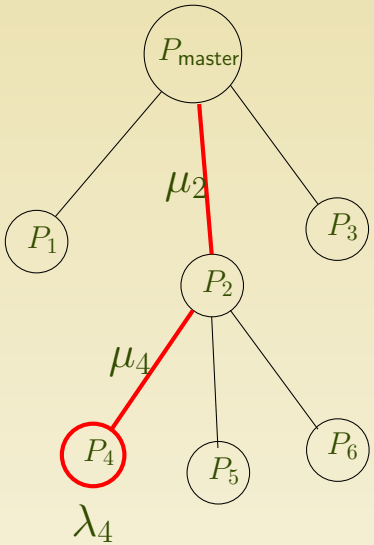

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Computing the prices

- Sketch of the algorithm:
  - Each worker gives *its* best local deal (depending on its own λ and μ and on the best deals of its own workers) to *its* master.
  - Best deals are thus propagated to the *root master* along the tree
  - For each application, each master informs its workers that have the best deal that he wishes to use them.
  - Best workers are thus selected along the tree.
  - Each worker updates its price depending on applications that want to use its resource.
- Main difficulties:
  - The Lagrangian function is not C^2 $\Rightarrow$ there are oscillations around angles.
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Coming back to our original problem

- We found the optimal deal ($\Pi(k)$) and prices (λ's and μ's) but we still don't have the optimal values of the $\alpha_i^{(k)}$'s.
- However, we have the throughput of each application: $D_k = \Pi(k)$
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Plan

SimGrid, MSG, plates-formes tiers

- The simulator is SimGrid.
- The platform generator is Tiers, but very simple trees here.
- Fully synchronous gradient.
- Checking the correctness of the results using semi-definite programming.

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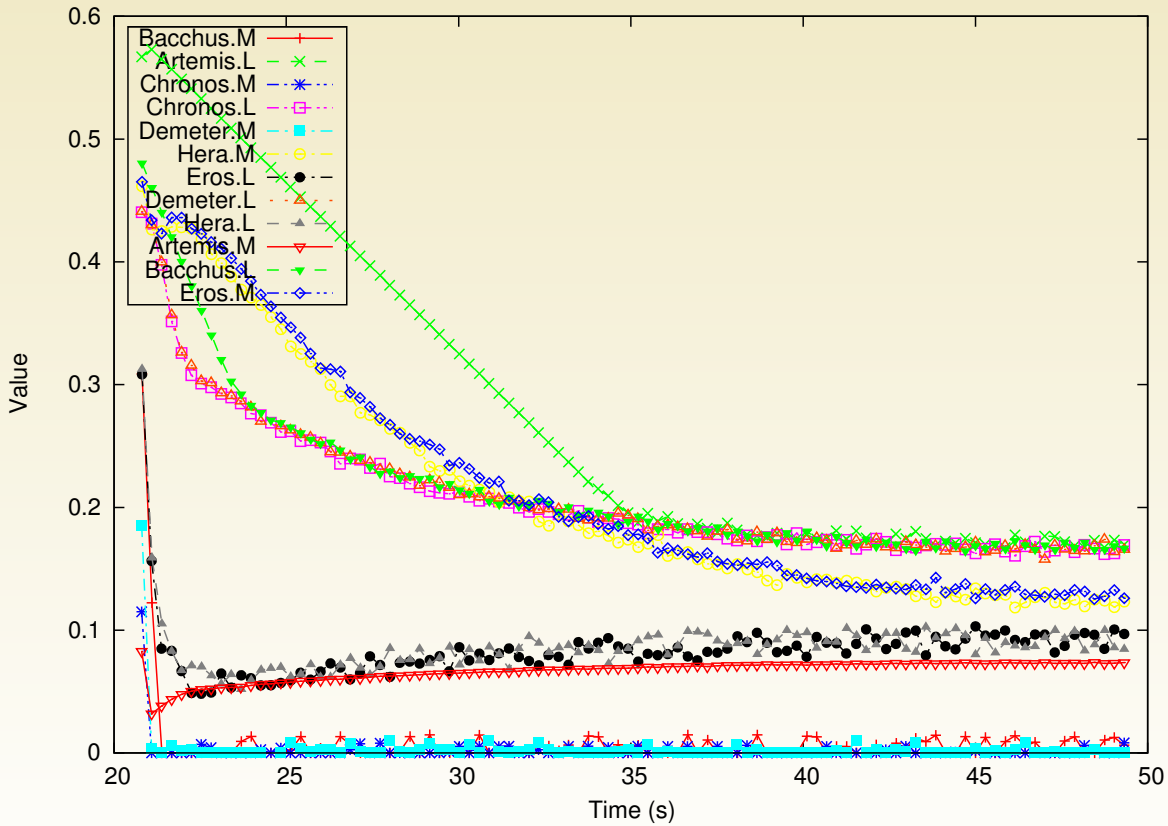
Convergence

- 6 processors, 2 applications
- Trajectories *merge*



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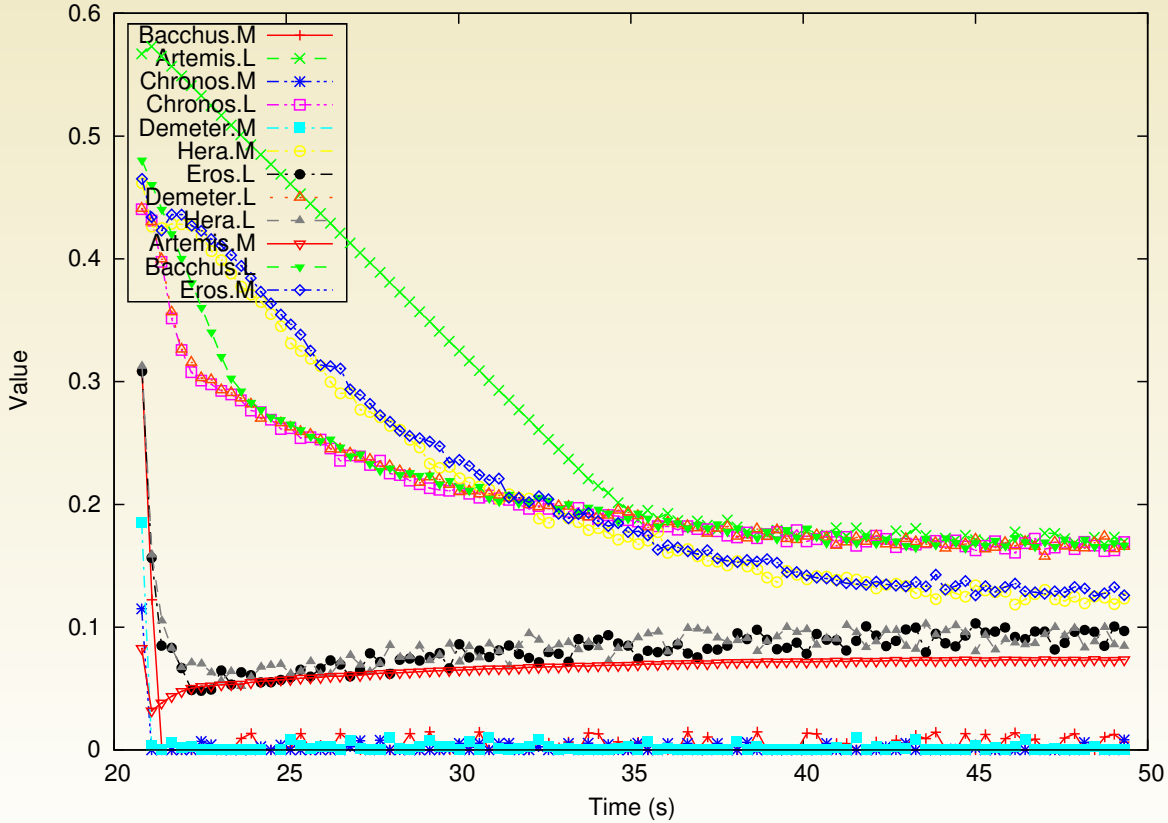
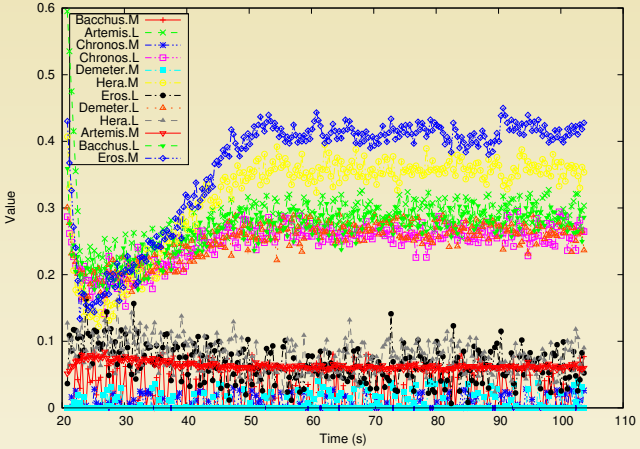
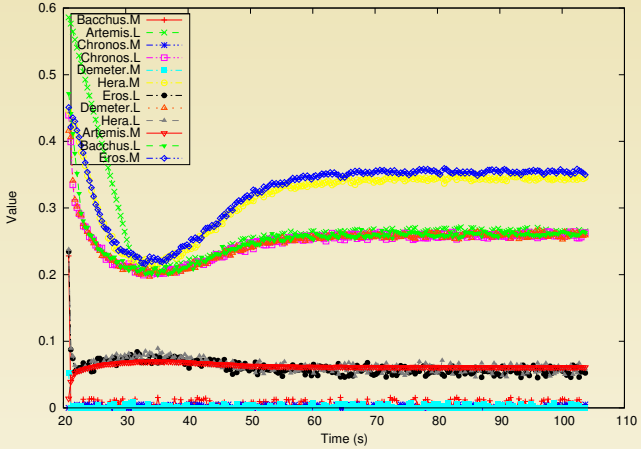


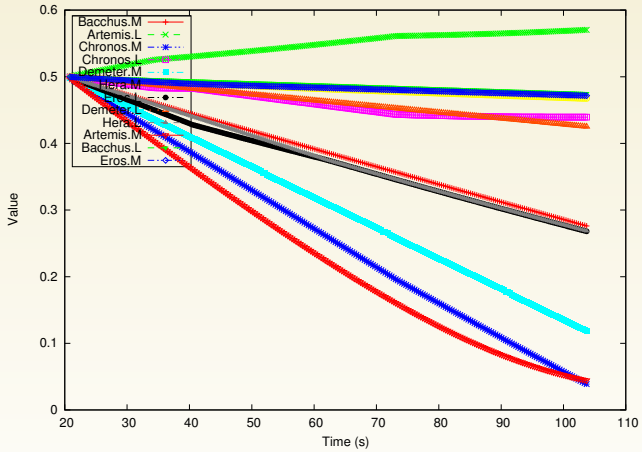
Oscillations: choosing the right step



(a) $\sigma = 1.5.10^{-2}$



(b) $\sigma = 3.10^{-3}$



(c) $\sigma = 1.0.10^{-5}$

Plan

Perpectives

- Speeding up the bargain
  - Trying asynchronous steps.
  - Smoothing the min.
  - Studying the ability to react to platform changes.
- Extending to general platforms with tree deployments is trivial.
- Trying other fairness measures.
- Implementing the Awerbuch Leighton algorithm and coupling with the bargain $\rightsquigarrow$ will it work in practice ?

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Bibliographie

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