

Anonymous Graph Exploration without Collision by Mobile Robots

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Outline

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 - Labeled mobility tree
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Context

(Perpetual) Graph Exploration Problem

- A connected graph
- One or several mobile entities
- Goal: Visit (infinitely often) each vertex by a robot

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What has been studied?

- Different model; synchronous, multiplicity,...
- Different kind of nodes; computational power, communication,...
- Time needed for exploration
- Amount of memory required

Our Problem

Constrained Perpetual Exploration Problem

Each robot has to visit the graph infinitely many times, while avoiding several robots to collide in a same vertex or to concurrently traverse a same edge.

Goal

Find the upper bound k that keep the solvability of the problem.

Model

- Finite connected undirected graph $G = (S, E)$
- Finite set R of robots
- Common clock \Rightarrow synchronous rounds
 - Move to an adjacent vertex
 - Stay on the same vertex
- Omniscient daemon that controls the robots

Given a robot a and a round r , $V(a, r)$ denotes the vertex where the robot a is located at the beginning of round r .

Constrained Perpetual Exploration Problem

- Perpetual Exploration.

$$\forall v \in S : \forall a \in R : \{r \mid V(a, r) = v\} \text{ is not finite.}$$

- Vertex Mutual Exclusion.

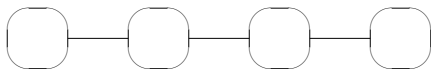
$$\forall r \geq 0 : \forall (a, b) \in R \times R : V(a, r) \neq V(b, r).$$

- Edge Mutual Exclusion.

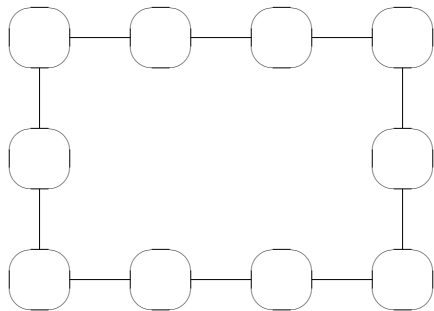
$$\forall r \geq 0 : \forall (a, b) \in R \times R :$$

$$(V(a, r + 1) = V(b, r)) \Rightarrow (V(b, r + 1) \neq V(a, r)).$$

Line

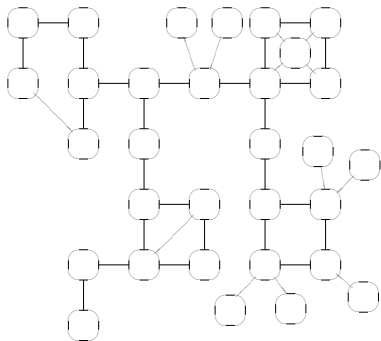


Ring



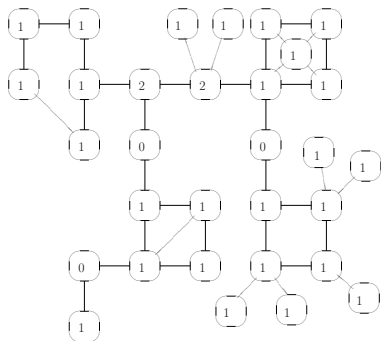
Labeled mobility tree: some definition

- Leaf
- Subgraph
- Bridge, bridgeless

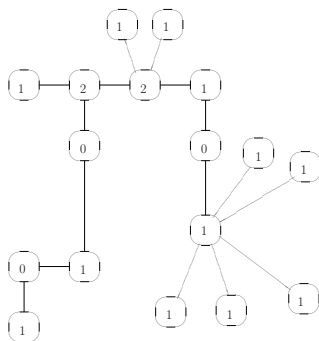
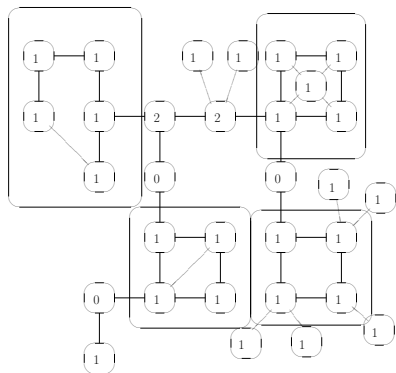


Labeled mobility tree: initial labeling

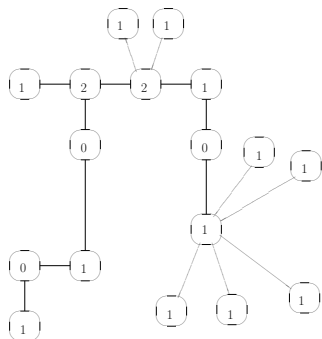
- Label 0: if v does not belong to a bridgeless subgraph of G and its degree is two;
- Label 1: if v is a leaf of G or belongs to a non-singleton bridgeless subgraph of G ;
- Label 2: otherwise.



Labeled mobility tree: compression



Labeled mobility tree: labeled path



Labeled path

- Starts with label 1 or 2
- Ends with label 1 or 2
- Contains only 0 for intermediate labels

Length of a labeled path

Classical length augmented by the number of vertices with label 2

Our result

Upper bound theorem

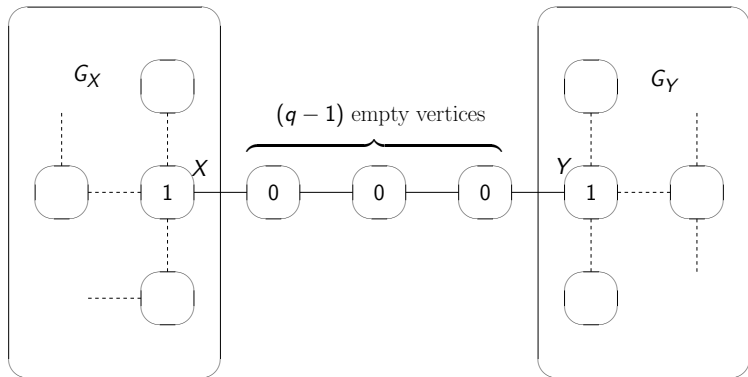
If G is a graph with p vertices and q is the maximal length of the labeled paths of the mobility tree associated with G , then there exists no algorithm that solves the Constrained Perpetual Graph Exploration problem for G from an arbitrary configuration including more than $k = p - q$ robots.

Proof

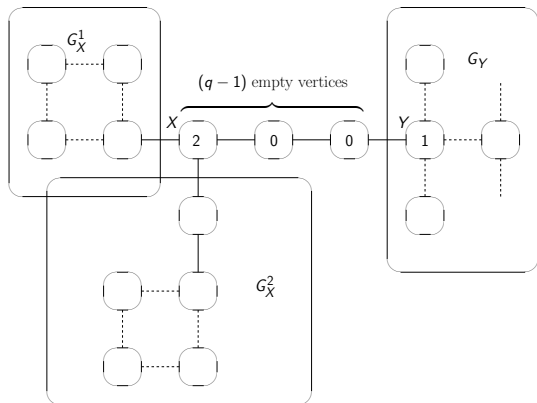
Based on the type of maximal labeled path:

- Case 1: From a label 1 to a label 1
- Case 2: From a label 1 to a label 2
- Case 3: From a label 2 to a label 2

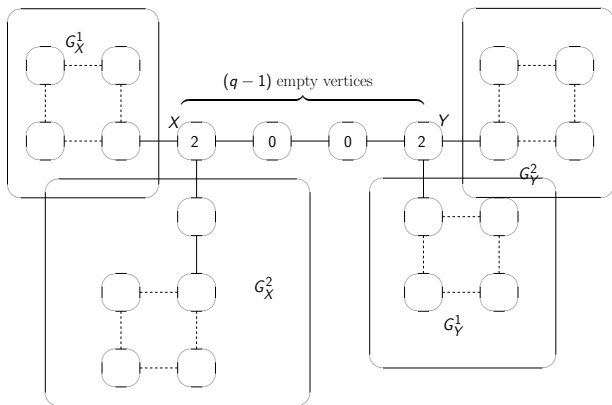
Case 1



Case 2



Case 3



Current/future work

Reach the bound ?

- in any case?
- for some restrictions of graphs?
- if robots have global/local vision of the graph?
- if they are oblivious?
- ...

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Preliminary results

- Build the map of the graph is THE key...

Questions ?