Grid Brokering of Batch Allocation using Indexes

Vandy BERTEN[1] Bruno GAUJAL[2]

[1]Université Libre de Bruxelles and FNRS
vandy.berten@ulb.ac.be

[2]INRIA and LIG (CNRS, INPG, UJF)
bruno.gaujal@imag.fr

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Outline

Motivations

On a methodological point of view :

- Use statistical data and predictions based on a stochastic model to propose an alternative to usual Grid brokering strategies.

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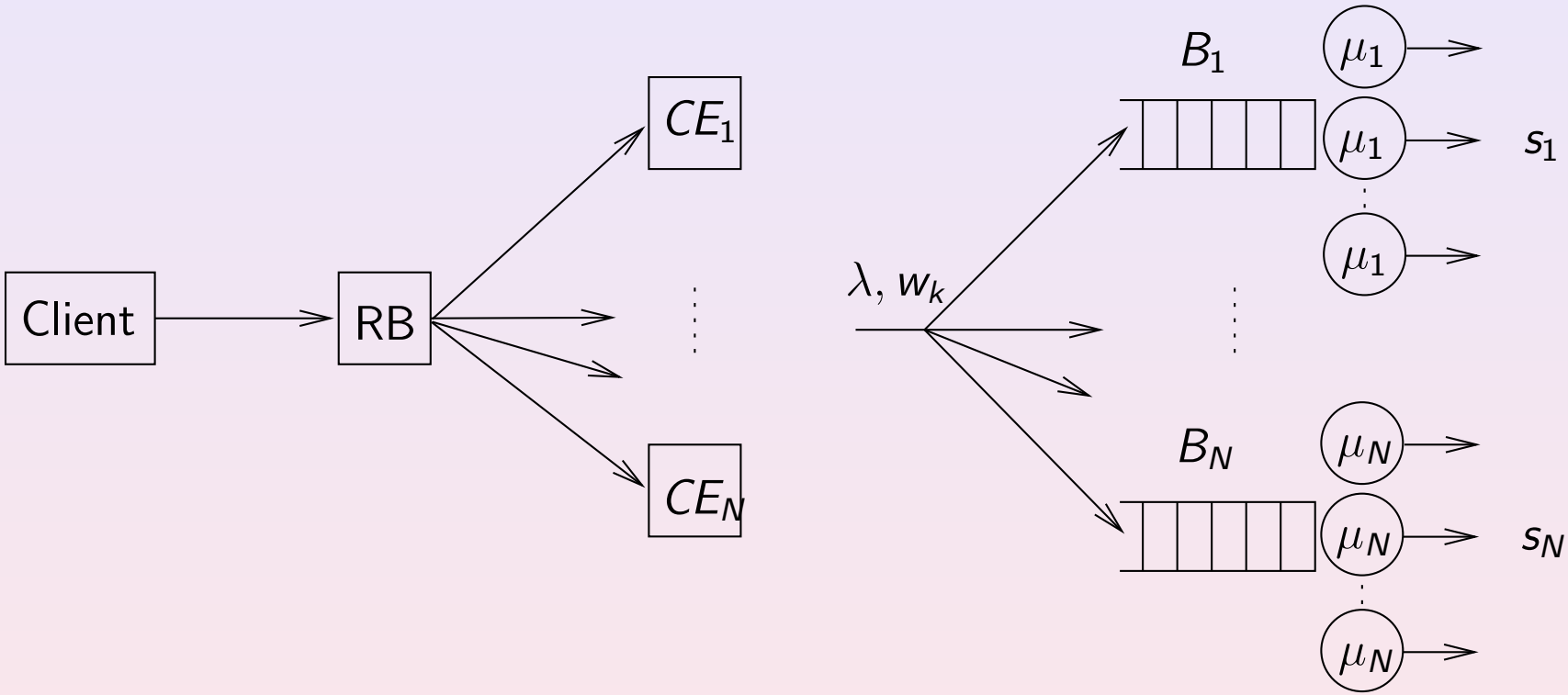
On a technical point of view :

- Propose an alternative algorithm allowing to compute indexes.
- Extend the classical models to batch jobs, with or without information on the size of the batches.

Model



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- If the size of a batch is known at the brokering time, this information can be taken into account.
- Using Index heuristics (Gittins, Whittle, Mitrani, Nino-Mora, ...), we send a job to the "cheapest" queue.

Markovian Model

The jobs are made of several parallel processes, to be executed on the same cluster. We do not model precisely the synchronizations and communications between the processes belonging to the same job.

- The state $(x_1, \ldots, x_N)$ in S is the number of processes in clusters $1, \ldots, N$ ($x_i \in \{0, \ldots, B_i\}$).
- An action (u in U) is either i (cluster i is chosen), or 0 (rejection of a job). Rejection is only allowed when all queues are "full".
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We look for a routing policy minimizing the expected discounted workload in infinite horizon.

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The cost under policy $\pi = (u_0, u_1, \ldots)$ with initial state $x^{(0)} = (x_1^{(0)}, \ldots, x_N^{(0)})$ is

$$J_\pi(x^{(0)}) = \limsup_{n \to \infty} \mathbb{E} \sum_{m=0}^{n-1} \alpha^m h(x^{(m)}, u_m(x^{(m)})),$$

with discounting factor α and immediate cost

$$h \stackrel{def}{=} \sum_{i=1}^{N} c_i((x_i^{(m)} + \delta_{\{\text{arrival at step } m\}} \delta_{\{u_m(x^{(m)})=i\}})),$$

where c_i is the cost in the i-th cluster per customer per time unit.

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As the cost is uniformly bounded over the state space, we consider time and initial state independent routing policies : u says which action to take in each state ($u : S \to U$).

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This problem is P-space hard in general.

Routing based on a Local Criterion

Routing policies based on a Local Criterion (LCR) are a subset of routing policies. They are defined as follows :

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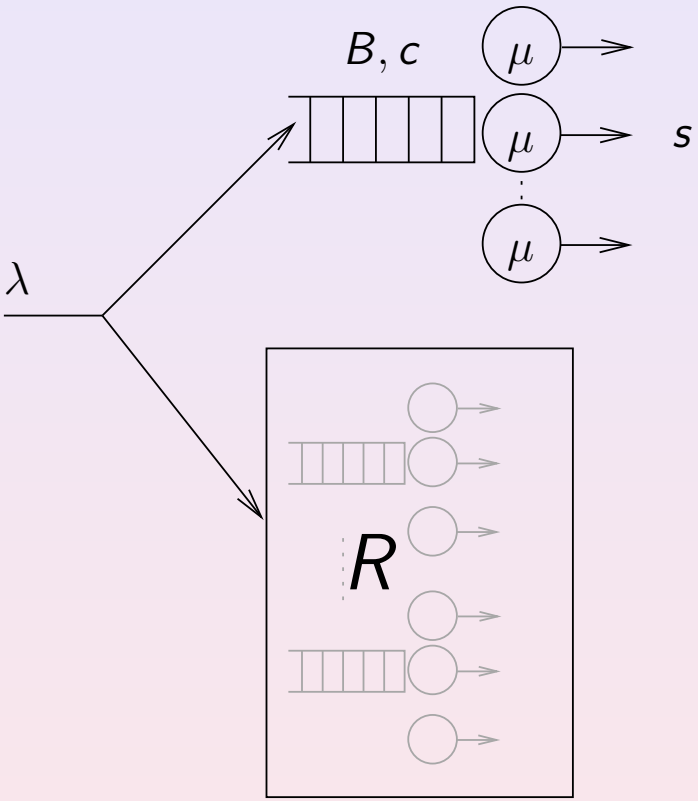
- The routing policy is the following :

$$u(x_1, \ldots, x_N) = \underset{x_i < B_i}{argmin} \{L_1(x_1), \ldots, L_N(x_N)\}$$

An arriving job is sent towards the queue with the smallest current local index. If the results of the argmin is empty, the incoming job is rejected.

Index heuristics



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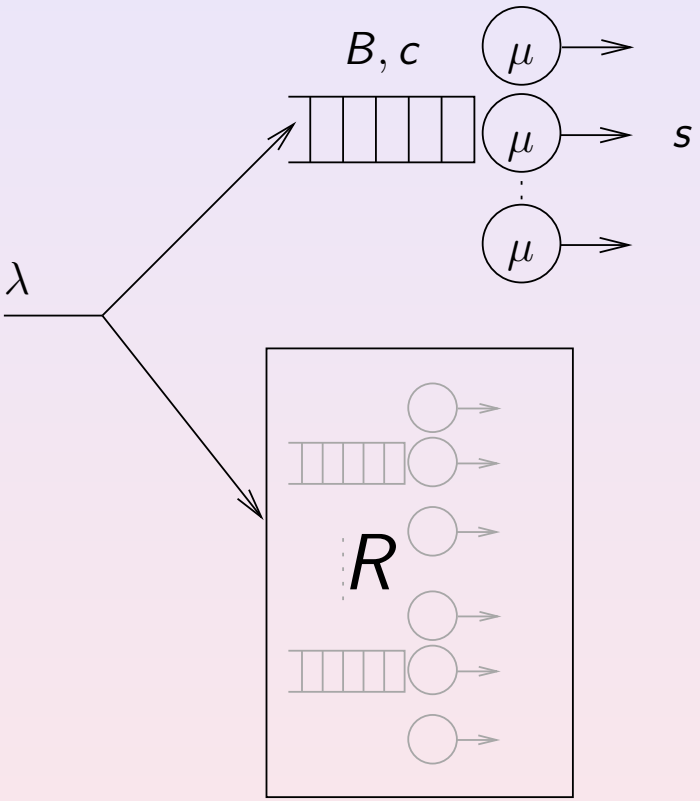

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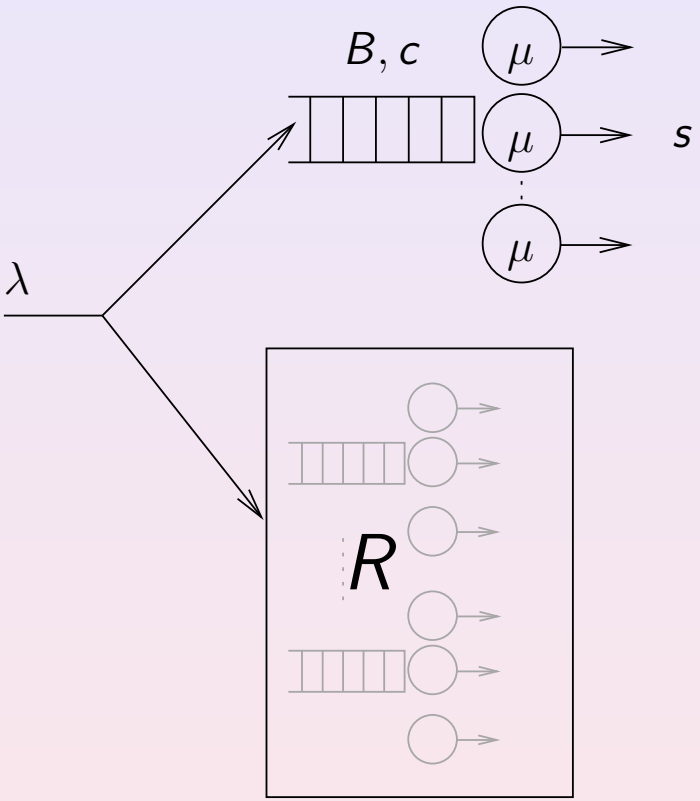

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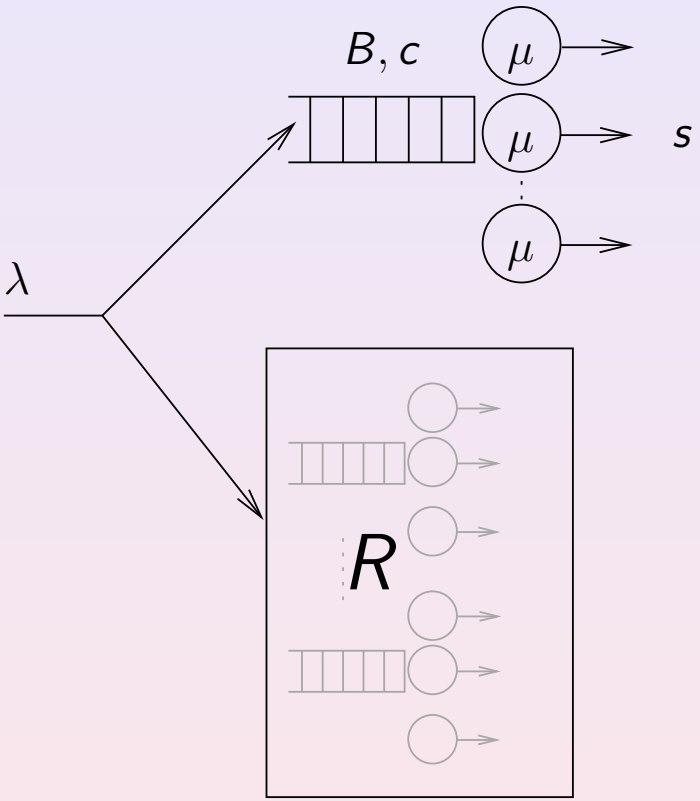

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- The effect of routing a job to queue i is rather intricate to measure in the long term. The idea is to come up with a real valued cost of this decision.
- To do so, we concentrate on one queue, other ones being considered as a black box costing R (degree of freedom).

Computing the Local Index

The cost in one queue under policy u is :

$$J_u(x^0) = \limsup_{n\to\infty} \mathbb{E} \sum_{m=0}^{n-1} \alpha^m (cx^{(m)} + kR\delta_{\{u_m=0\}}\delta_{\{\text{arrival at step } m \text{ of a job of size } k\}}).$$

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The optimal cost J^* is the unique solution of the Bellman Equation,

$$\begin{aligned} & J^*(x) \\ = \; & \alpha\Big(\lambda' \textstyle\sum_k P_k \min\{kR + J^*(x), J^*(x+k)\} \\ & \quad + \mu' \min\{s, x\} J^*(x-1) \\ & \quad + (1 - \lambda' - \mu' \min\{s, x\}) J^*(x) \Big) \\ & + cx. \end{aligned} \tag{1}$$

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There is an optimal policy minimizing the α-discounted cost in infinite horizon for one queue of threshold type : $u^*(x) = 1$ if $x < \Theta(R)$ and $u^*(x) = 0$ otherwise.

Algorithms

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Computing the cost can be done by using the cost equation and its derivatives [Palmer, Mitrani 2005] or using polytope problems [Nio-Mora 2002]
We use an alternative approach, based on policy iteration.
Let $J_{\theta,\theta'} \stackrel{def}{=} (\alpha F_{\theta'} J_\theta + S_{\theta'})$ be the cost obtained by using values found for J_θ under policy $u_{\theta'}$.

1. choose θ
2. solve $J_\theta = (\alpha F_\theta J_\theta + S_\theta)$.
3. find θ^* such as $\theta^* = argmax_{\theta'} \sum_{x=0}^{B} J_\theta(x) - J_{\theta,\theta'}(x)$.
4. restart in 2 with θ^* as long as $\theta \neq \theta^*$.

Index heuristics

The local index L is defined as follows (inverse of Θ) :
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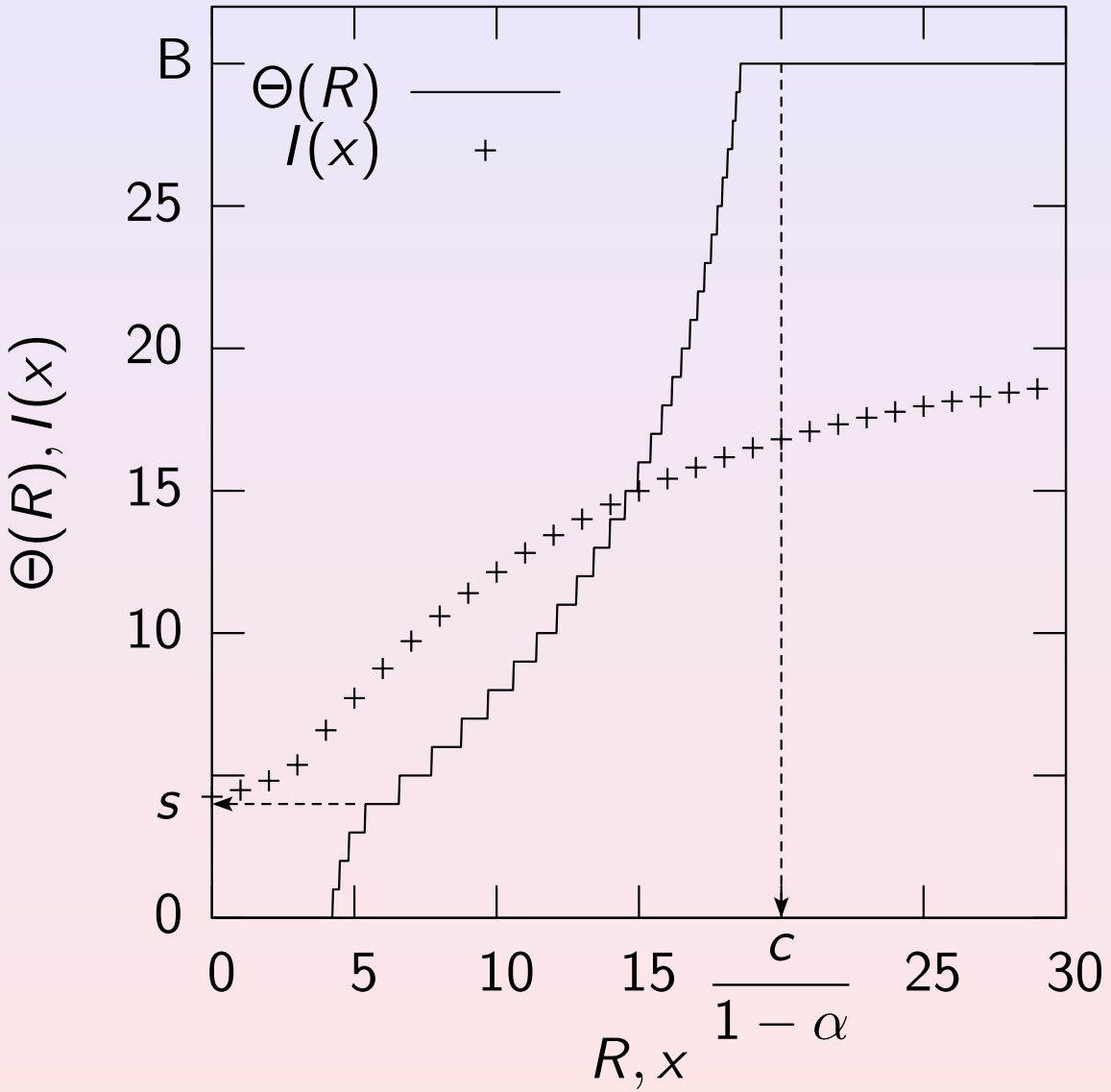
- Computing L can be done in a few seconds in the mono-dimensional case : $O(eB(B+K)K + (B+K)K \log B)$, where $e = -log(\epsilon)$, the precision chosen in the previous algorithm.

Threshold and Local Index



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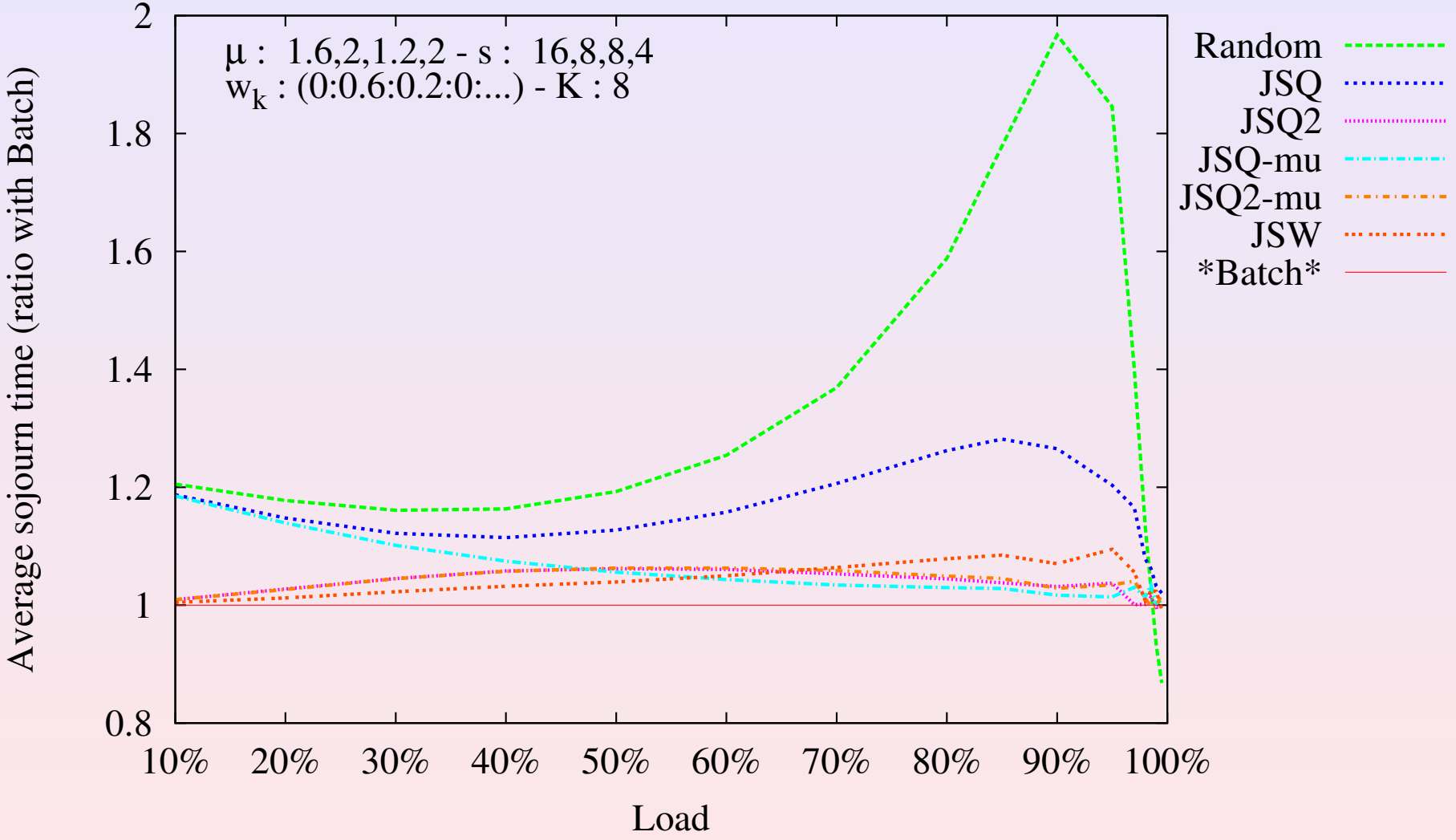
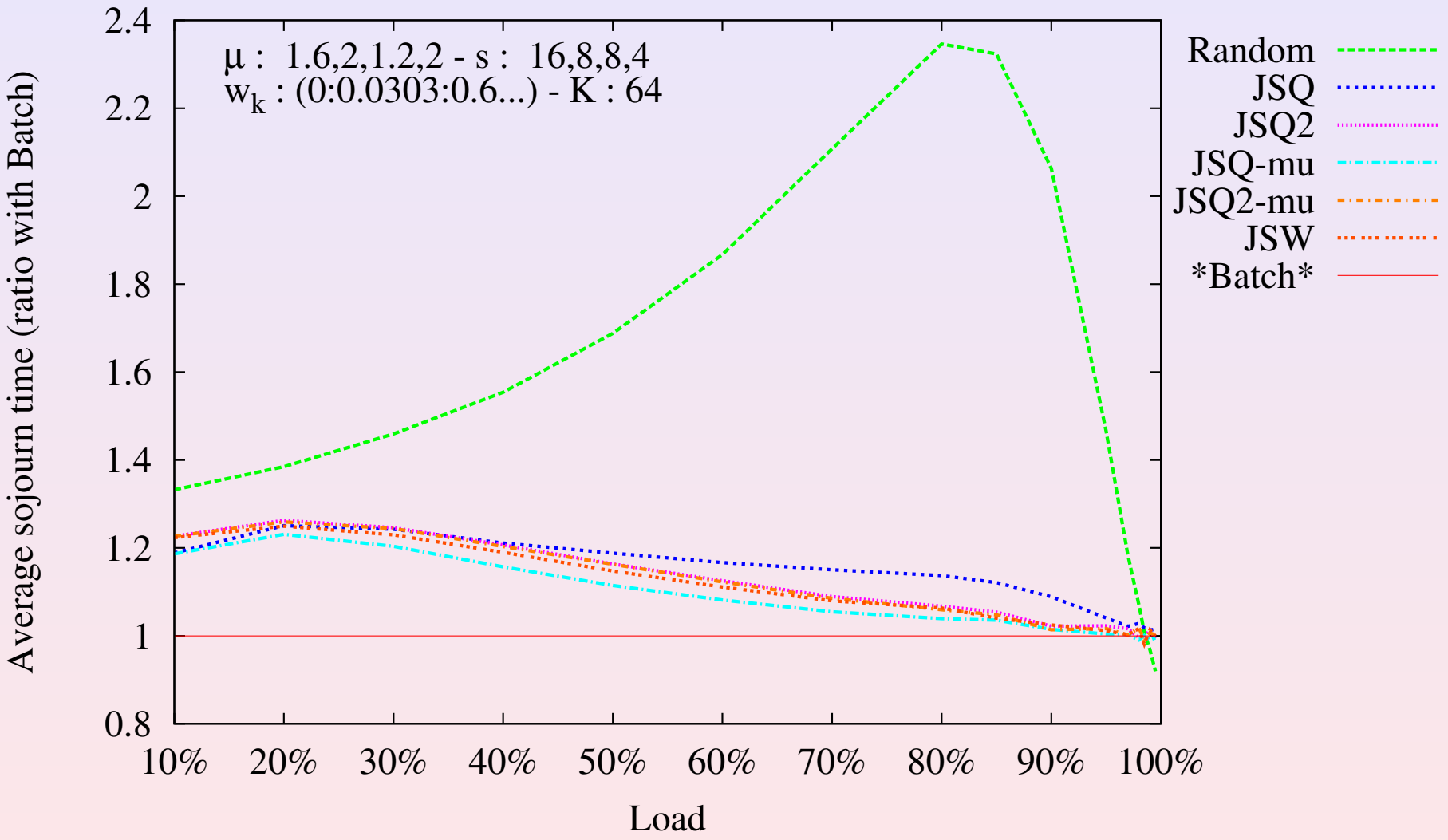
- `Batch` : index without width knowledge.

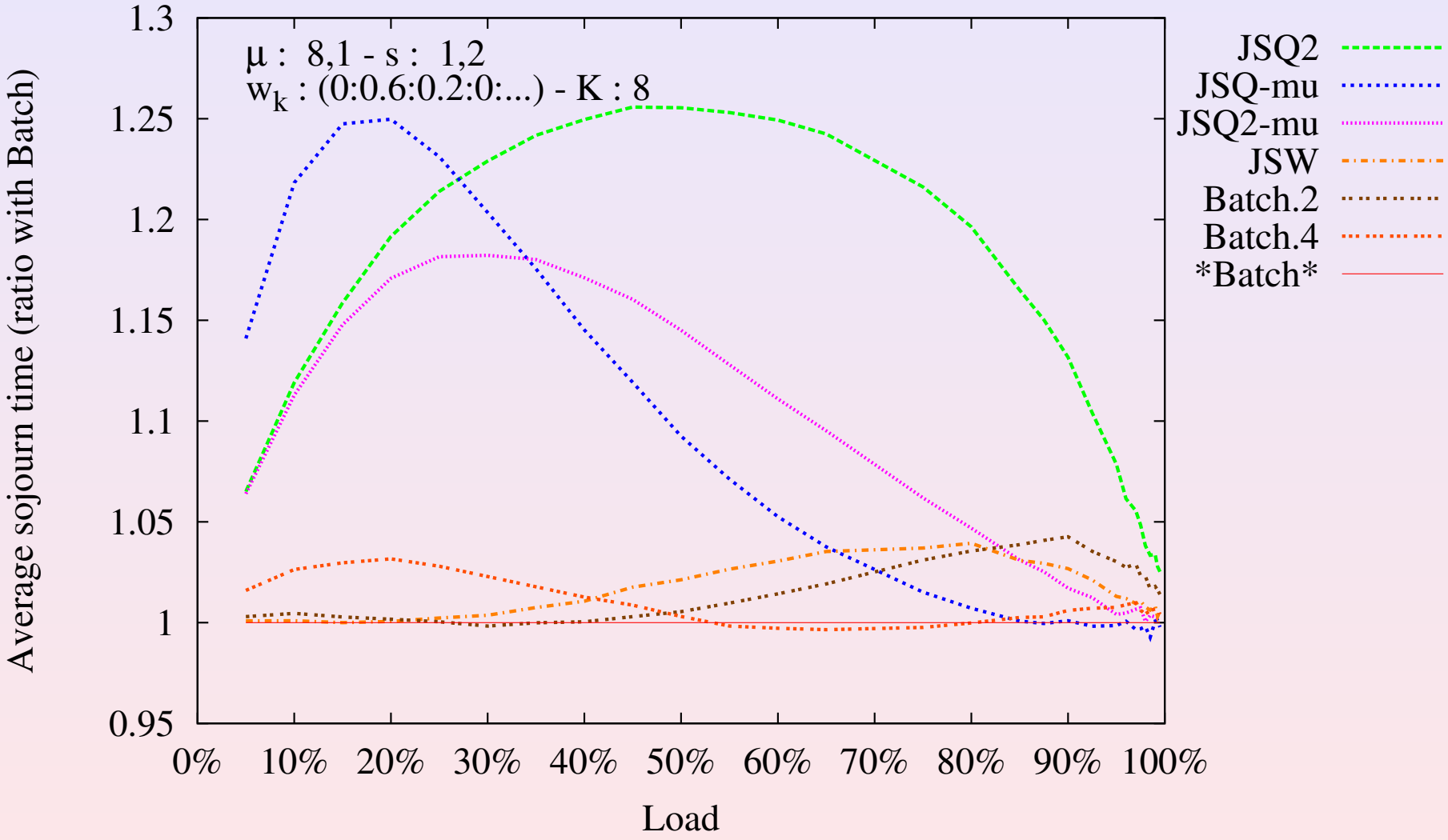
Impact of job width distribution (K=8)



Impact of job width distribution (K=64)



Robustness on load variations



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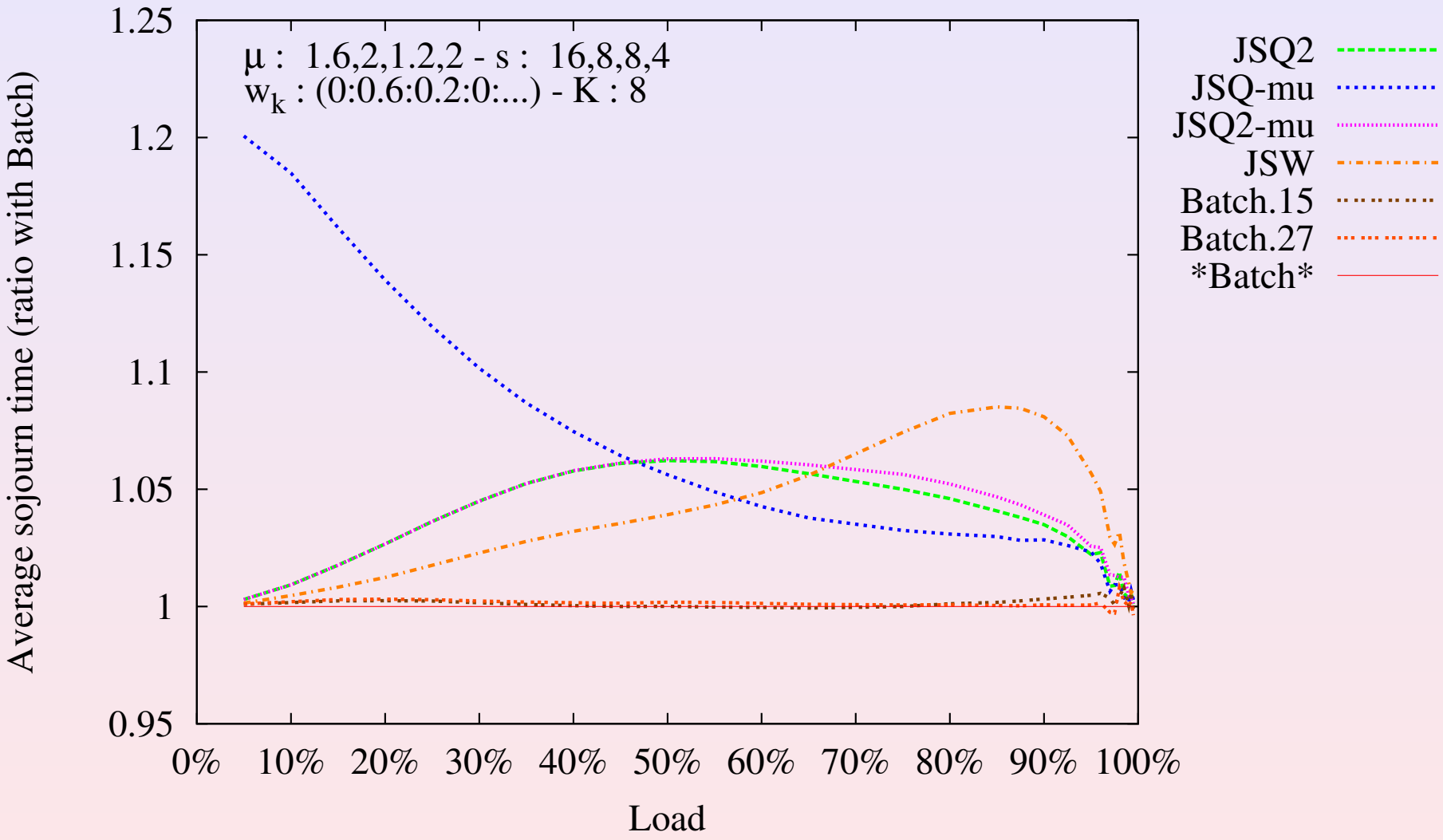

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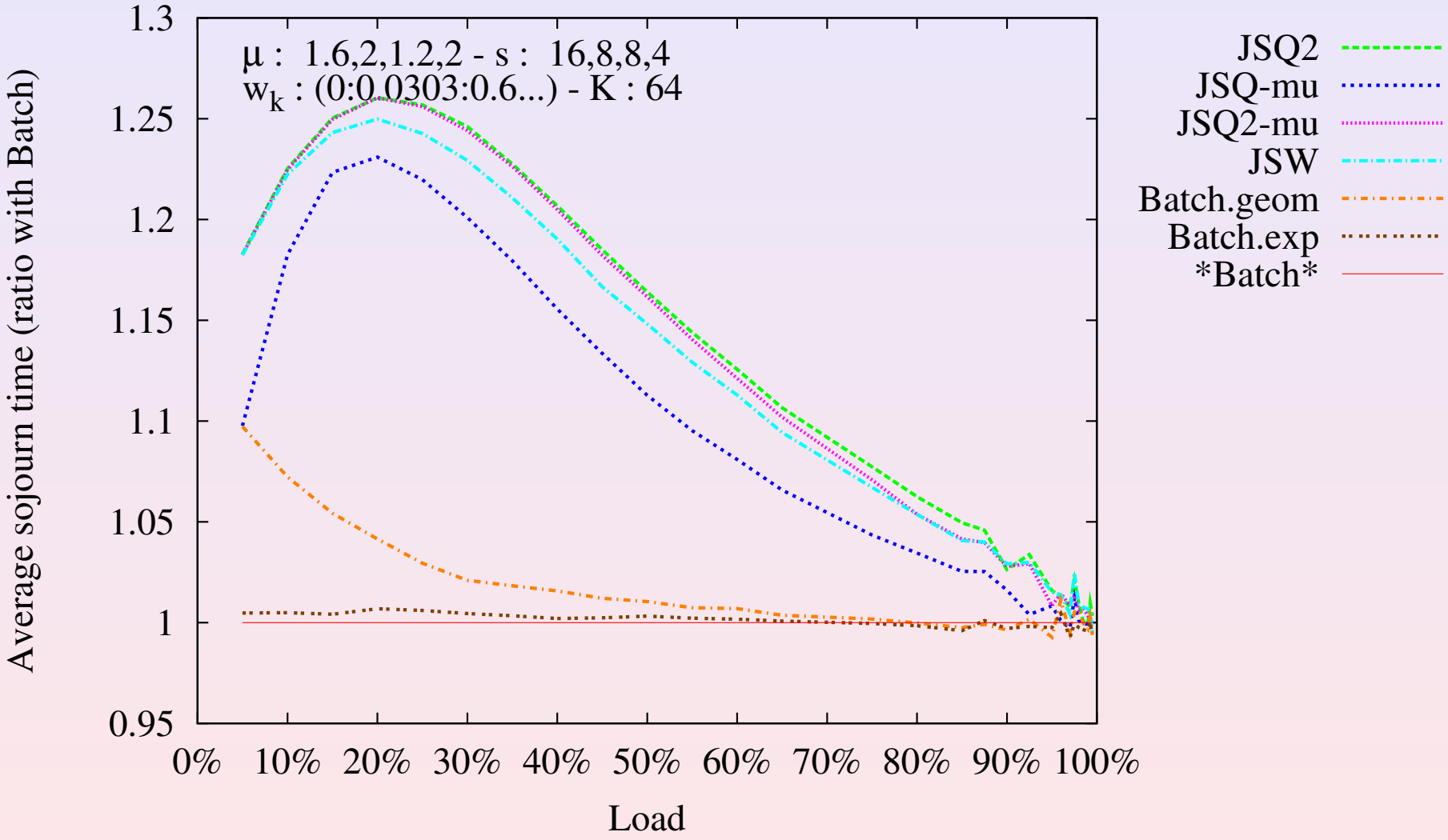

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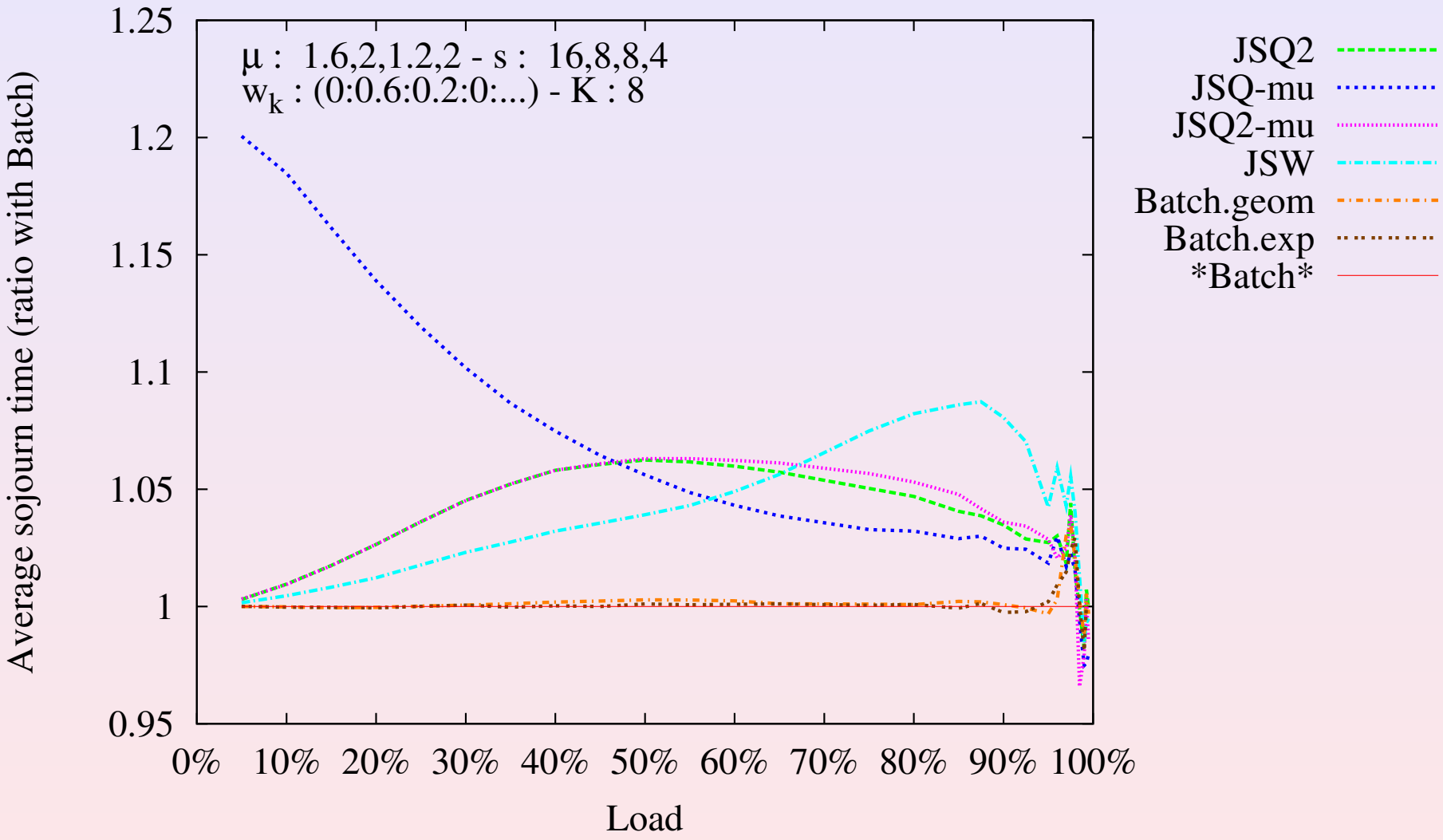

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- Large robustness to parameters variation

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- Job width must be taken into account by the resource broker, as any additional information.
- Index becomes *bi-dimensional* : they depend on the state and the job size.
- As seen later, computations will become much more complex.

Index brokering of known batches

Once again, we focus on local index based policies.

The optimal policy is of threshold type, depending on k the size of the incoming job : If $x < \theta(k)$ the job is rejected and accepted otherwise.

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With unknown batches, the optimal cost is

$$J(x) = \min_{\theta \in \{0,\dots B\}} \alpha\Big(\lambda' \sum_k P_k J(x + k\delta_{x<\theta}) + \mu' \min\{x, s\} J(x-1)$$
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Here the optimal cost is

$$J(x,k) = \min_{\theta \in \{B\}^{\{K\}}} \alpha\Big(\lambda' \sum_{k'=1}^{K} P_{k'} J(x + k\delta_{x<\theta(k)}, k') + \mu' \min\{x, s\} J(x-1,k) + (1 - \lambda' - \mu' \min\{x, s\}) J(x,k)\Big) + cx + \lambda' Rk \delta_{x \geq \theta(k)}$$

This makes the computation of the optimal index possible (but much harder than in the unknown case).

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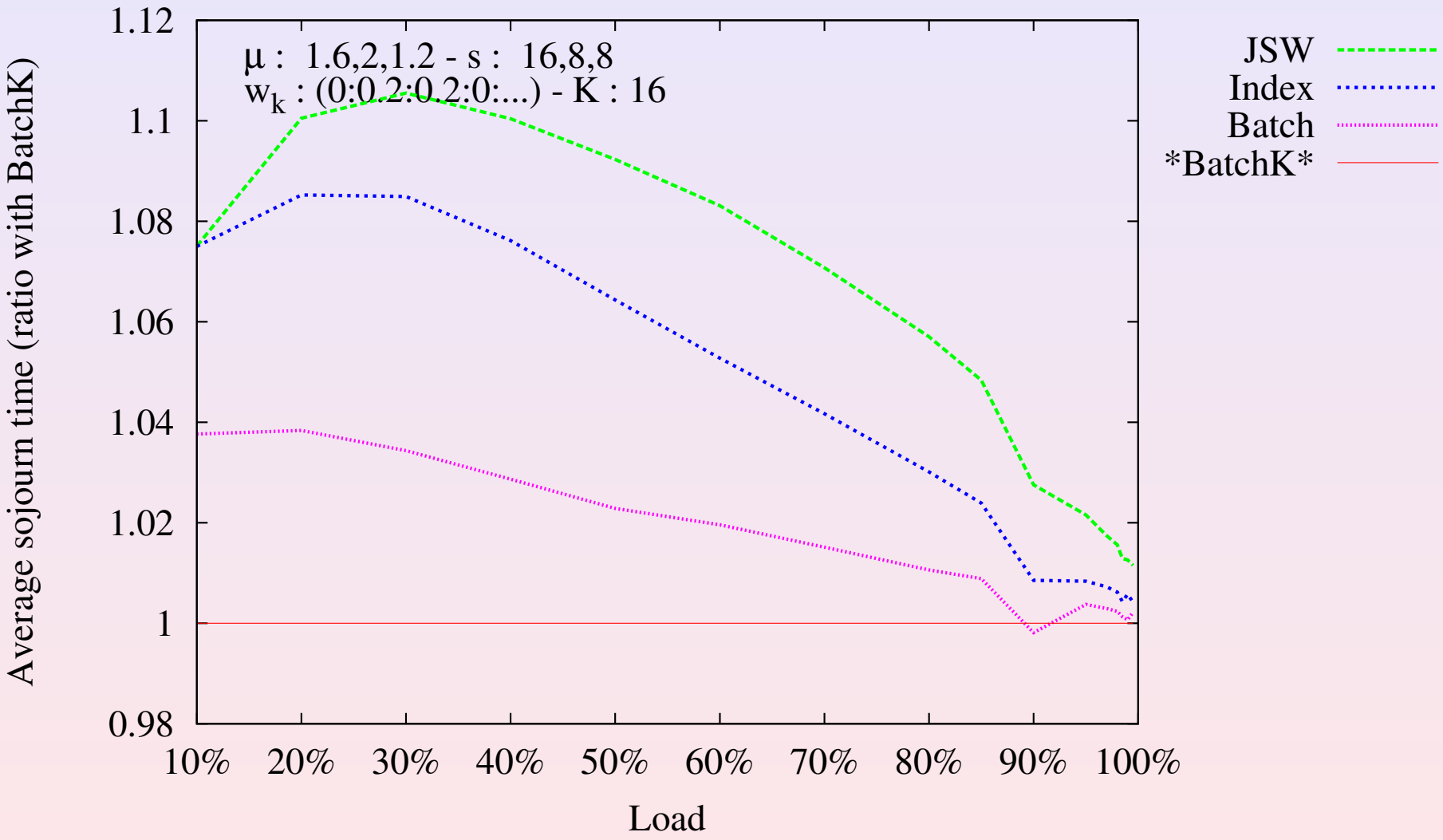
The local index is : $\forall k, \quad L(x, k) = \sup\{R | \Theta(R, k) \leq x\}$.
$L(x, k)$ is the cost of accepting a job of size k in the queue if the state is x.
Now, the LCR policy is : an upcoming job of size k is sent to the queue with the smallest index, $L_i(x_i, k)$.
This new policy should improve on all policies discussed before. The question is by how much. This is usually called the value of information.

Bi-dimensional indexes



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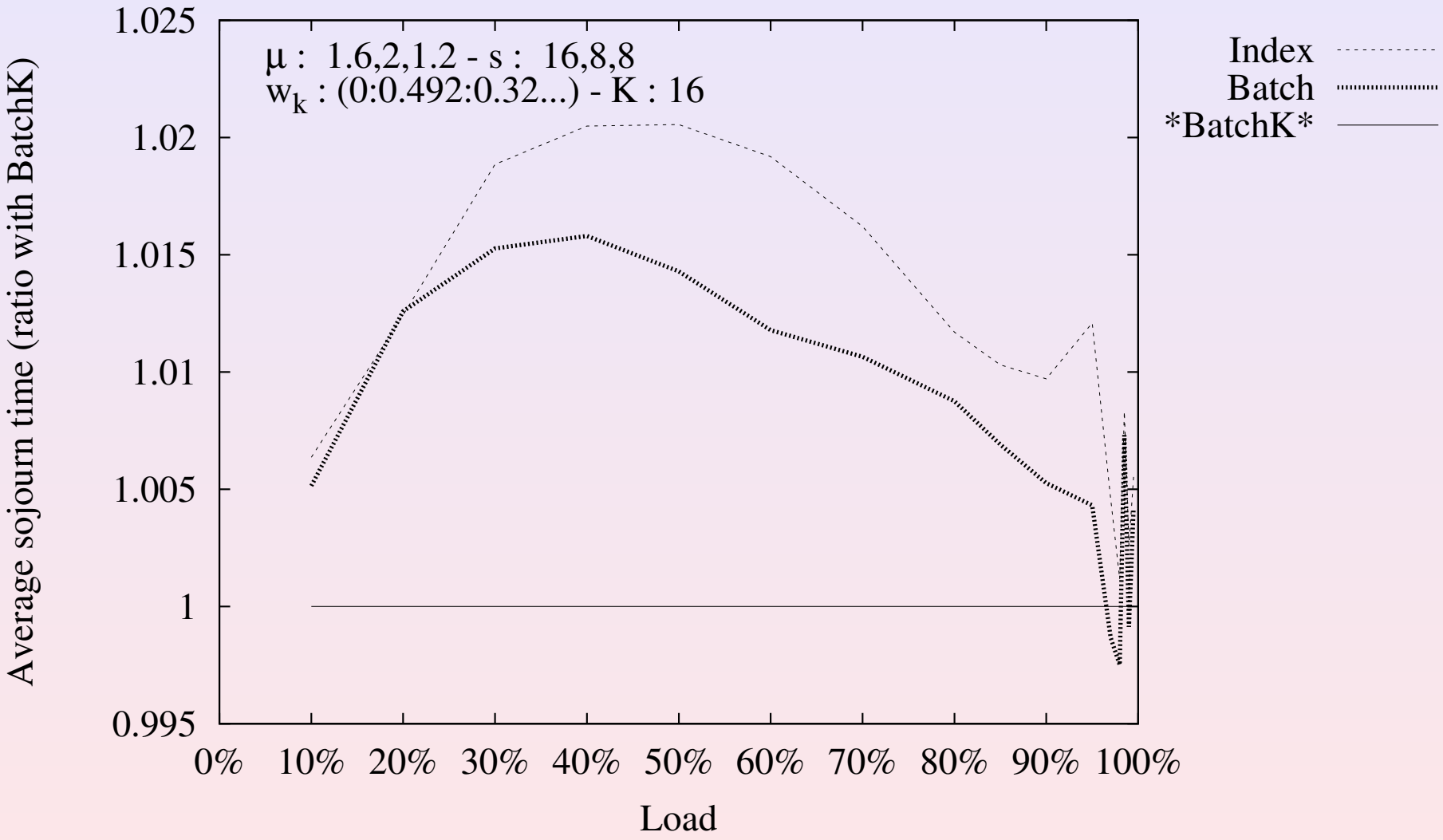

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The End