Labeling schemes and Border automata

- A labeling scheme on a set $A = \{a_1, \dots, a_d\}$ of formal generators is given by :
  - a description of the ordering of the generators around any vertex
  - a description of the generators's inverses
  - a boolean indicating whether the generator inverses the order of rotation or not

For example :

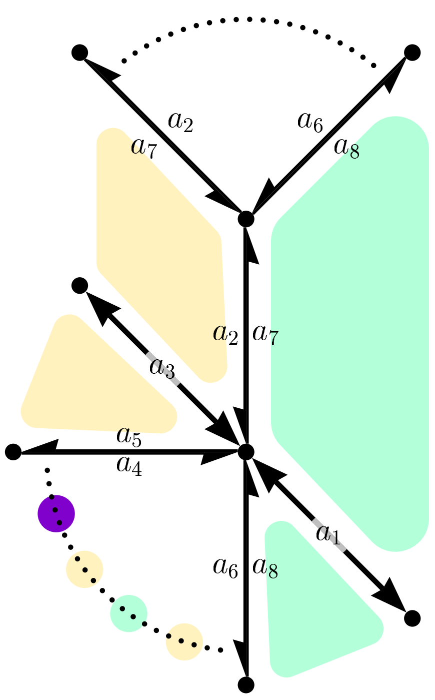
$p = [1, 2, 3, 4, 5, 6, 7, 8]$

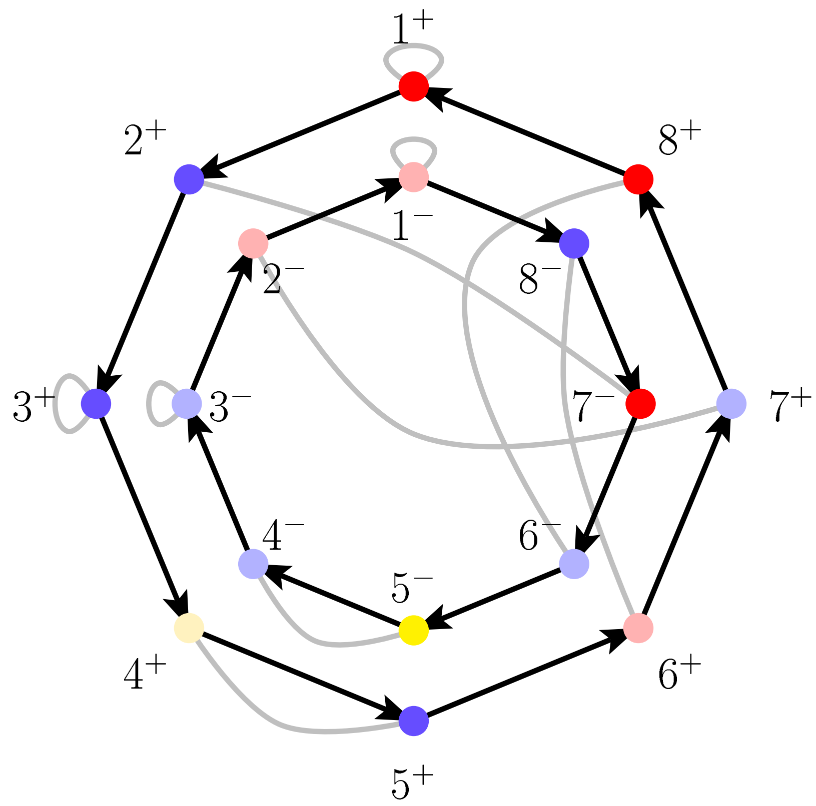
$\sigma = (1)(27)(3)(45)(68)$

$\tau = [0, 1, 0, 0, 0, 1, 1, 1]$

- With these descriptions, it is possible to compute finite balls of G. A border automaton is an automaton whose states are labeled by elements of A and whose orbits, depending on the starting state, describe the borders of the faces of G. ⋄

Local Computation





Border Automaton

- For this specific automaton, the orbits are : $\begin{bmatrix} (\,1^+\ 7^-\ 8^+\,) & (\,2^+\ 3^+\ 5^+\ 8^-\,) & (\,4^+\,) \\ (\,6^+\ 2^-\ 1^-\,) & (\,7^+\ 6^-\ 4^-\ 3^-\,) & (\,5^-\,) \end{bmatrix}$ (same color $\Rightarrow$ dual orbits) ⋄

- The primitive type vector describes the set of valid ($i.e$ possible) type vectors for a given labeling scheme. In our case, this vector is $[3p, 4q, 4q, r, 4q, 3p, 4q, 3p]$, where $(p, q, r) \in \mathbb{N}$. ⋄

Theorem (Existence) To any locally finite Cayley graph corresponds a unique labeling scheme and type vector.

Theorem (Unicity) Given a labeling scheme and a valid type vector, there exists a unique locally finite Cayley graph possessing the scheme and type vector (up to isomorphism of graph)

Labeling scheme + Type vector = Cayley graph